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Partial wave analysis of $\pi\pi$ scattering below 2 GeV

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Abstract. All resonances included in multi-channel D ($\pi\pi$, 4π, $K\bar{K}$ and $\eta\eta$) and F ($\pi\pi$, 4π, $\omega\pi$ and $K\bar{K}$) waves are analyzed. Results of the analysis are based on a set of well determined $\pi\pi$ scattering amplitudes for S - F waves in a unitary multi-channel approach. The amplitudes are refined and re-fitted to the dispersion relations up to 1.1 GeV, and to the experimental data in the effective two pion mass from the threshold to 2.7 GeV and 1.9 GeV for D and F waves, respectively.

Many existing parameterizations of the $\pi\pi$ scattering amplitudes still significantly do not satisfy the crossing symmetry condition or do not describe the $\pi\pi$ threshold region. Since they are used in some analyses, having a correct determination of $\pi\pi$ scattering amplitudes is essential to achieve a proper result.

In our previous work [1] we presented a general method of refining the $\pi\pi$ amplitudes by fitting them to the GKPY equations [2, 3] which are once-subtracted dispersion relations with imposed crossing symmetry condition. The method was demonstrated on refining of the unitary multi-channel S ($\pi\pi$, $K\bar{K}$ and $\eta\eta$) and P ($\pi\pi$, $\rho 2\pi$, and $\rho\sigma$) wave amplitudes from [4] mostly below 1.1 GeV with no change in their original mathematical structure. In Ref. [5] we included the multi-channel D ($\pi\pi$, effective $(2\pi)(2\pi)$, $K\bar{K}$ and $\eta\eta$) and F ($\pi\pi$, effective $(2\pi)(2\pi)$, $\omega\pi$ and $K\bar{K}$) wave amplitudes from [6]. The S-F amplitudes were refined and then re-fitted to the available experimental data and to the GKPY equations [2, 3]. In Ref. [5] we presented some part of the results of analysis for the D and F wave amplitudes especially, details on formalism of the analysis and precise determination of $\pi\pi$ scattering amplitudes for D and F waves. In this paper we discuss on the resonances included in the analysis and show their importance separately in the corresponding partial waves.

The method of analysis is generally the same as in Ref. [1]. To construct the $D0$ and $F1$ waves, we have used the same formalism as in Refs. [4, 6] and updated the list of contributing resonant states for the $D0$ wave according to the latest issue of PDG [7]. Then, a correct behavior of the phase shift near the threshold was controlled by the pseudo data (see Secs.II in [5]). The initial amplitudes for the $S0$ and $P1$ waves were from [1], both with added polynomial near the threshold (the "extended" amplitudes in [1]). Since there is no any resonance for $S2$ and $D2$ waves, the amplitudes for these isotensor waves were taken from [2] and kept unchanged during the analysis.

In the modification of the original amplitudes only those parameters that were expected to contribute significantly to the dispersive integrals, i.e. the low-energy resonances and the $\pi\pi$ background were allowed to change. The results provided precise determination of $D0$ and $F1$ partial wave amplitudes which satisfy the crossing symmetry condition and describe the experimental data very well.

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In the construction of the partial waves, apart from background part which is relatively small, the resonance contribution described by the multi-channel Breit-Wigner form determines the main structure of the amplitudes. Concerning the dominant, important and insignificant resonances of the $D0$ wave in the multi-channel analysis approach, various fits have been performed. Results show that not all first four lightest states, i.e. $f_2(1270)$, $f_2(1430)$, $f_2(1525)$, and $f_2(1640)$, are the most important states in the analysis, as it was expected at the beginning. For this, Table 1 demonstrates the significance of each resonance in the analysis separately. It shows the χ^2 for the fits when one particular resonance was omitted. The χ^2 was only composed of the χ^2_{Data} for $D0$ wave in the Eq. (16) in [8] and χ^2_{DR} for all partial waves in the Eq. (17) in [8]. Nevertheless, conditions are the same for all fits and they just differ by omitted resonances. Values of the χ^2 achieved from fits 9, 11 and 12 are almost equal to the value in the fit 1 where all resonance states are included. Accordingly, from the point of view of the χ^2 one can omit the $f_2(2010)$, $f_2(2300)$ and $f_2(2340)$ resonances and conclude that these resonances are insignificant in the description. Value of the χ^2 in the fits 3, 5, 7 and 8 show that the $f_2(1430)$, $f_2(1640)$, $f_2(1910)$ and $f_2(1950)$ don't play a very important role on the results but one should keep them. Additionally, the $f_2(1525)$, $f_2(1810)$, and $f_2(2150)$ resonances which strongly influence behavior of the amplitude and therefore value of the χ^2 are the most important resonances in the $D0$ wave and obviously the dominant resonance which dominate the behaviour of the amplitude is $f_2(1270)$ resonance.

From two resonance states $\rho_3(1690)$ and $\rho_3(1990)$ listed in PDG [7] for the isovector $F1$ wave only the $\rho_3(1690)$ resonance state has been considered in the analysis. For the $\rho_3(1990)$ resonance state, the partial widths are not known. In Refs. [5, 8] we have found that enough is only one resonance state $\rho_3(1690)$ which can satisfactorily describe the available experimental data in [9, 10] and give a reasonable description in the threshold region.

Table 1. Values of χ^2 after fitting when some specific resonance states are omitted in the $D0$ wave.

Fit No.		$\chi^2/n.d.f.$
1	All stated are included	326.21/330=0.9885
	Omitted Resonances	
2	$f_2(1270)$	5838.8/330=17.69
3	$f_2(1430)$	327.39/330=0.9921
4	$f_2(1525)$	336.97/330=1.0211
5	$f_2(1640)$	327.76/330=0.9932
6	$f_2(1810)$	348.07/330=1.0548
7	$f_2(1910)$	327.36/330=0.9920
8	$f_2(1950)$	327.26/330=0.9917
9	$f_2(2010)$	326.68/330=0.9899
10	$f_2(2150)$	368.81/330=1.1176
11	$f_2(2300)$	326.54/330=0.9895
12	$f_2(2340)$	326.64/330=0.9898

A figurative comparison of the phase shift of the resonances in $\pi\pi \to \pi\pi$ channel is presented in the Figs. 1-3 for $D0$ and $F1$ waves individually. The $D0$ wave consists of 11 resonances ($f_2(1270)$, $f_2(1430)$, $f_2(1525)$, $f_2(1640)$, $f_2(1810)$, $f_2(1910)$, $f_2(1950)$, $f_2(2010)$, $f_2(2150)$, $f_2(2300)$, $f_2(2340)$) from which the $f_2(1270)$ is the most dominant one. This can be seen in the Fig. 1 where the $f_2(1270)$ determines structure of the full phase shift. Effect of other phase shifts are comparably small and influence of the background part on the full phase shift notably appears above ~1600 MeV which describe the data very well. Removing the full phase shift of the $D0$ wave (Full phase), $f_2(1270)$, background (Bkg.) and experimental data in the Fig. 1 one can see the differences between other phase shifts in the Fig. 2. Influence of these phases on the full phase shift is less than 1 degree. However, one can conclude that effect of the $f_2(1640)$, $f_2(1910)$, $f_2(1950)$, $f_2(2010)$ and $f_2(2150)$ phase shifts is almost negligible. Note that the scarce data above ∼1600 MeV indicate almost no resonant structure.

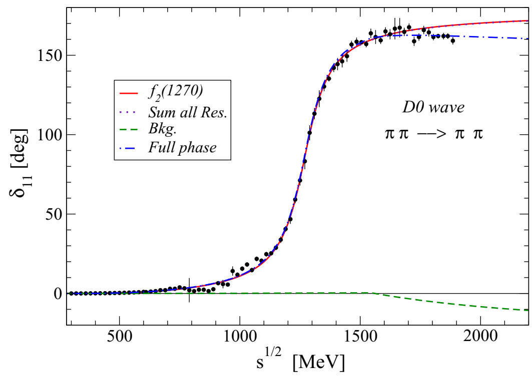

Figure 1. Illustrative comparison of the phase shifts of the resonances in the *D*0 wave. The $f_2(1270)$ resonance is overlapped by sum of all resonances (Sum of all Res.) when background part (Bkg.) is omitted. Except $f_2(1270)$ contributions of other resonances are relatively very small (less than 1 degree).

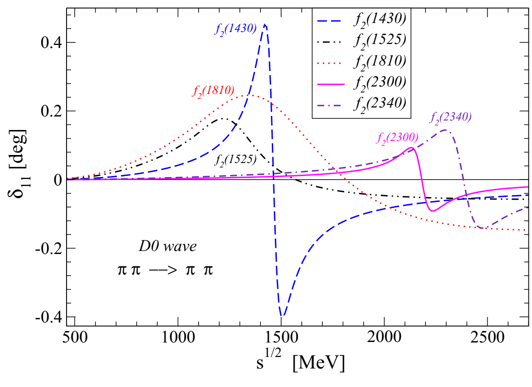

Figure 2. We show contributions to the elastic phase shift from other less important *D*0 wave resonances. Phase shifts of the $f_2(1640)$, $f_2(1910)$, $f_2(1950)$, $f_2(2010)$ and $f_2(2150)$ resonances are almost close to zero.

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Finally for the $F1$ wave which has only one resonance ($\rho_3(1690)$), description of the phase shift is much simpler than $D0$ wave. Figure 3 shows the influence of the resonance and background parts on the full phase shift. Due to the scattered data below ~1700 MeV one can omit the background part, since only the $\rho_3(1690)$ phase shift is enough to describe the data. Effect of the background part is seen above ~1700 MeV to describe the data.

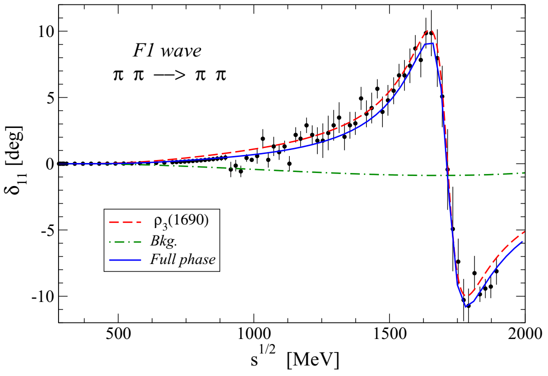

Figure 3. Illustrative comparison of the resonance and background (Bkg.) with the full phase shift of the $F1$ wave (Full phase) and experimental data [9, 10].

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