Hadronic deuteron polarizability contribution to the hyperfine structure in muonic deuterium

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Abstract. The calculation of the contribution to the polarizability of the nucleus to hyperfine structure of muonic hydrogen is carried out within the unitary isobar model and on the basis of experimental data on the structure functions of deep inelastic lepton-proton and lepton-deuteron scattering. The calculation of virtual absorption cross sections of transversely and longitudinally polarized photons by nucleons in the resonance region is performed in the framework of the program MAID.

Hadronic deuteron-polarizability contribution of the order $(Z\alpha)^5$ to the Lamb shift and hyperfine structure is determine of the virtual forward Compton scattering $\gamma^* + d \to \gamma^* + d$ (see Fig.1). This amplitude contain deuteron tensor [1-3]:

$$M_{\mu\nu}^{(d)} = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) C_1(\nu, k^2) + \frac{1}{m_d^2}\left(p_{2\mu} - \frac{m_d \nu}{k^2} k_\mu\right)\left(p_{2\nu} - \frac{m_d \nu}{k^2} k_\nu\right) C_2(\nu, k^2) +$$
$$i\varepsilon_{\mu\nu\alpha\beta} k^\alpha \left\{m_d S^\beta H_1(\nu, k^2) + \left[(P \cdot k)S^\beta - (S \cdot k)P^\beta\right]\frac{H_2(\nu, k^2)}{m_d}\right\}, \tag{1}$$

where k is the virtual photon 4-momentum, $\nu = k_0$ is the virtual-photon energy. The symmetric part of the tensor in (1) contributes to the Lamb shift [4-5] (structure functions $C_{1,2}(\nu, k^2)$), while its antisymmetric part contributes to the hyperfine structure (structure functions $H_{1,2}(\nu, k^2)$) [2-3].

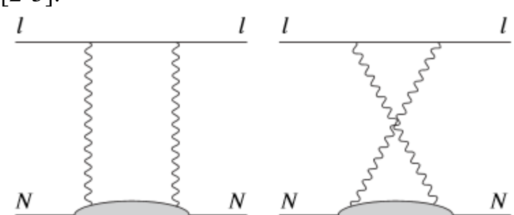

Fig. 1. Two-photon Feynman amplitudes determining the correction on nucleus polarizability

The polarized structure functions of proton and neutron $G_1^{p,n}(\nu, Q^2)$ and $G_2^{p,n}(\nu, Q^2)$ determine a correction of the hadronic deuteron polarizability to hyperfine structure as [2-3]:

$$\Delta E_{HFS}^d = \frac{4}{3}(Z\alpha)^5 \frac{\mu^3}{\pi\, m_N^2}(\Delta_1^d + \Delta_2^d), \tag{2}$$

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where we consider the deuteron as incoherent sum of the proton and neutron, so that $\Delta^d_{1,2}=\Delta^p_{1,2}+\Delta^n_{1,2}$. Corrections Δ_1 and Δ_2 for the proton and neutron are determined by the expressions with corresponding polarized structure functions in the form:

$$\Delta_1=\int_0^\infty \frac{dQ^2}{Q^2}\left\{\frac{9}{4}F_2^2(Q^2)-4m_N^3\int_{\nu_{th}}^\infty \frac{d\nu}{\nu}\beta_1\left(\frac{\nu^2}{Q^2}\right)G_1(\nu,Q^2)\right\}, \tag{3}$$

$$\Delta_2=-12m_N^2\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{th}}^\infty d\nu\,\beta_2\left(\frac{\nu^2}{Q^2}\right)G_2(\nu,Q^2). \tag{4}$$

The polarized structure functions of the proton and deuteron can be measured in inelastic scattering of polarized electrons by polarized protons and deuterons. The progress achieved in the preparation and carrying out of experiments with polarized beams of leptons, have made it possible to carry out precise measurements of polarized structure functions $G_{1,2}$ at SLAC, CERN and DESY. Spin-dependent structure functions can also be expressed in terms of cross-sections of the virtual photo-absorption on nucleons.

$$g_1(\nu,Q^2)=\frac{m_N\,K}{8\pi^2\alpha\left(1+\frac{Q^2}{\nu^2}\right)}\left[\sigma_{\frac{1}{2}}(\nu,Q^2)-\sigma_{\frac{3}{2}}(\nu,Q^2)+2\frac{\sqrt{Q^2}}{\nu}\sigma_{TL}(\nu,Q^2)\right], \tag{5}$$

$$g_2(\nu,Q^2)=\frac{m_N\,K}{8\pi^2\alpha\left(1+\frac{Q^2}{\nu^2}\right)}\left[\sigma_{\frac{3}{2}}(\nu,Q^2)-\sigma_{\frac{1}{2}}(\nu,Q^2)+2\frac{\nu}{\sqrt{Q^2}}\sigma_{TL}(\nu,Q^2)\right], \tag{6}$$

where $g_1(\nu,Q^2)=m_N^2\nu\,G_1(\nu,Q^2)$ and $g_2(\nu,Q^2)=\nu^2 m_N\,G_2(\nu,Q^2)$. To calculate the latter in this work we use interactive program MAID, which is based on a unitary isobaric model [6-7]. As a result of numerical integration (2) in the resonant and non-resonant regions, we obtain the following value of hadronic deuteron polarizability contribution: $\Delta E^d_{HFS}=0.120\,meV$. Theoretical uncertainty of this result is determined by the errors in the measurement of polarized structure functions. The GDH sum rule for the deuteron is not satisfied with good accuracy, it also leads to significant errors. We estimate approximately total theoretical error in 50 %.

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