X, Y and Z States

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Abstract. Many new states in the charmonium mass region were recently discovered by BaBar, Belle, CLEO-c, CDF, DØ, BESIII, LHCb and CMS Collaborations. We use the QCD Sum Rule approach to study the possible structure of some of these states.

1 Introduction

During the last decade several experimental facilities, such as BaBar at SLAC and Belle at KEK, CLEO-III and CLEO-c at CESR, CDF and DØ at Fermilab, BESIII at IHEP and LHCb and CMS at CERN, have increased the available data on new charmonium-like states, called X, Y and Z states. Tables with a list of these states can be found in Refs. [1, 2]. In 2014 there were twenty three of these X, Y, Z states, many not confirmed. They do not seem to have a simple $c\bar{c}$ structure. Although the masses of these states are above the corresponding thresholds of decays into a pair of open charm mesons, they decay into J/ψ or ψ' plus pions, which is unusual for $c\bar{c}$ states. Besides, their masses and decay modes are not in agreement with the predictions of potential models, which, in general, describe very well $c\bar{c}$ states. For these reasons they are considered as being good candidates of exotic hadrons. We call exotic states hadrons having other structure than the ordinary mesons and baryons, containing constituent quark-antiquark and three quarks respectively. The idea of unconventional quark structures is quite old and the light scalar mesons were the first candidates for tetraquark exotic states. These states are allowed by the strong interactions, both at the fundamental level and at the effective level, and their absence in the experimentally measured spectrum has always been a mystery.

Among these new charmonium states, the charged ones are definitely exotics, since they can not be simple $c\bar{c}$ states. The $Z^+(4430)$, found by the Belle Collaboration in 2007, was the first observed one [3–5]. The BaBar Collaboration searched for the $Z^-(4430)$ signature in four decay modes and concluded that there is no significant evidence for a signal peak in any of these processes [6]. However, a few years ago Belle and LHCb collaborations have confirmed the $Z^+(4430)$ observation and have determined the preferred assignment of the quantum numbers to be $J^P = 1^+$ [5, 7]. The LHCb Collaboration also did the first attempt to demonstrate the resonant behavior of the $Z^+(4430)$ state [7]. They have performed a fit in which the Breit-Wigner amplitude was replaced by a combination of independent complex amplitudes at six equally spaced points in $m_{\psi(2S)\pi}$ range covering the $Z^+(4430)$ peak region. The resulting Argand diagram is consistent with a rapid phase transition at the peak

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of the amplitude, just as expected for a resonance. Therefore, the confirmation of the observation of the $Z^+(4430)$ by the LHCb Collaboration with the demonstration of its resonant behavior can be considered as the first experimental proof of the existence of the exotic states.

The other very interesting state, which is the most well studied among the new charmonium states, is the $X(3872)$. It was first observed in 2003 by the Belle Collaboration [8, 9], and has been confirmed by five collaborations: BaBar [10], CDF [11–13], DØ [14], LHCb [15, 16] and CMS [17]. The LHCb collaboration has determined the $X(3872)$ quantum numbers to be $J^{PC} = 1^{++}$, with more than 8σ significance [16]. Calculations using constituent quark models give masses for possible charmonium states, with $J^{PC} = 1^{++}$ quantum numbers, which are much bigger than the observed $X(3872)$ mass: $2\,^3P_1(3990)$ and $3\,^3P_1(4290)$ [18]. These results, together with the coincidence between the X mass and the $D^{*0}D^0$ threshold: $M(D^{*0}D^0) = (3871.81 \pm 0.36)$ MeV [19], inspired the proposal that the $X(3872)$ could be a molecular $(D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$ bound state with a small binding energy [20]. Other interesting possible interpretation of the $X(3872)$, first proposed in Ref. [21], is that it could be a tetraquark state resulting from the binding of a diquark and an antidiquark. The difference between these two interpretations is only the way that the 4-quarks are organized inside the state. A molecular $(D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$ bound state with small binding energy would be bigger than a compact tetraquark state. In any case, there is little doubt in the community that the $X(3872)$ structure is more complex than just a $c\bar{c}$ state.

In the following we discuss some of these new states using the QCD sum rule (QCDSR) approach.

2 QCD Sum Rules

The method of the QCDSR, was introduced by Shifman, Vainshtein and Zakharov [22] for the study of the mesons. They demonstrated that, for the determination of the mass of the state using the method, the non-perturbative power corrections are more important than the strong coupling, α_s, corrections. The non-perturbative power corrections were introduced through a series expansion of operators. As the dimension of the operators increase, the power of the momentum transfer, Q^2, in the denominator of the terms also increases, giving a series in $1/Q^2$ which can be truncated for large values of Q^2. The sum rule method was latter extended to baryons by Ioffe [23] and Chung *et al.* [24]. Since then the QCDSR technique has been applied to study numerous hadronic properties with various flavor content and has been discussed in many reviews [25–30] emphasizing different aspects of the method.

Table 1. Charmonium states observed in the last years.

state	Production mode	Ref.
$X(3872)$	$B \to K(\pi^+\pi^- J/\psi)$	[8]
$Y(4260)$	$e^+e^- \to \gamma_{ISR}(J/\psi\pi^+\pi^-)$	[31]
$Z_c^+(3900)$	$e^+\ e^- \to \pi^-(\pi^+ J/\psi)$	[32]
$Z_c^+(4025)$	$e^+\ e^- \to \pi^-(D^*\bar{D}^*)^+$	[33]
$X^\pm(5568)$	$p\ \bar{p} \to (B_s^0\pi^\pm) + \cdots$	[34]

The method is based in the evaluation of the correlation function:

$$\Pi(q) \equiv i \int d^4x\, e^{iq\cdot x}\langle 0|T[j(x)j^\dagger(0)]|0\rangle\ , \tag{1}$$

in two different ways. At the quark level in terms of quark and gluon fields, and at the hadronic level by introducing hadron parameters. In Eq. (1) $j(x)$ is a current which has the quantum numbers of the hadron we want to study.

In what follows we present some results of the QCDSR calculations on the X, Y, Z states presented in Table 1. We assume these states to have a more complicated structure than simple quark-antiquark states. For more details we refer the reader to our recent reviews on the subject [29, 30].

3 X, Y and Z states

3.1 $X(3872)$

As discussed in Sec. 1, the $X(3872)$ was the first observed non-conventional charmonium, i.e., it has a mass significantly smaller than the one predicted by the standard quark model with these quantum numbers, which are $J^{PC} = 1^{++}$ [16]. Moreover, the X decays with comparable strength into J/ψ plus two and J/ψ plus three pions [19], showing a strong isospin violation which is not compatible with a $c - \bar{c}$ state. Finally, this state has a decay width of less than 1.2 MeV [19], which is too small to be easily accounted for.

If we assume the X to be described by a $J^{PC} = 1^{++}$ four-quark current either in a diquark-antidiquark configuration:

$$j_\mu^{(q,di)} = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(q_a^T C\gamma_5 c_b)(\bar{q}_d\gamma_\mu C\bar{c}_e^T) + (q_a^T C\gamma_\mu c_b)(\bar{q}_d\gamma_5 C\bar{c}_e^T)] \; , \tag{2}$$

or in a molecular $D\bar{D}^*$ configuration:

$$j_\mu^{(q,mol)}(x) = \frac{1}{\sqrt{2}}\Big[\Big(\bar{q}_a(x)\gamma_5 c_a(x)\bar{c}_b(x)\gamma_\mu q_b(x)\Big) - \Big(\bar{q}_a(x)\gamma_\mu c_a(x)\bar{c}_b(x)\gamma_5 q_b(x)\Big)\Big], \tag{3}$$

we find that it is possible to describe its mass [35, 36], but it is not possible to describe its width [37]. The mass obtained using the current in Eq. (2) was $M_X = (3.92 \pm 0.13)$ GeV [35] . In the case of the current in Eq. (3), the result for the mass obtained in Ref. [36] was $M_X = (3.87 \pm 0.07)$ GeV, both in good agreement with the experimental mass. However, the decay width obtained in Ref. [37] for the decay mode $X \to J/\psi\pi\pi$ was $\Gamma_{X\to J/\psi\pi\pi} = (50 \pm 15)$ MeV, much bigger than the experimental upper limit. Therefore, from a QCDSR calculation it is not possible to explain the small width of the $X(3872)$ if it is a pure four-quark state. In Ref. [38] the $X(3872)$ was treated as a mixture of a $c\bar{c}$ state with a four-quark state:

$$J_\mu^q(x) = \sin(\alpha)\, j_\mu^{(q,4)}(x) + \cos(\alpha)\, j_\mu^{(q,2)}(x), \tag{4}$$

with $j_\mu^{(q,4)}(x)$ given in Eq. (2) or Eq. (3) and

$$j_\mu^{(q,2)}(x) = \frac{1}{6\sqrt{2}}\langle\bar{q}q\rangle[\bar{c}_a(x)\gamma_\mu\gamma_5 c_a(x)]. \tag{5}$$

The necessity of mixing a $c\bar{c}$ component with a molecule was already pointed out in some works [39–42]. In particular, in Ref. [43], a simulation of the production of a bound $D^0\bar{D}^{*0}$ state with binding energy as small as 0.25 MeV, obtained a cross section of about two orders of magnitude smaller than the prompt production cross section of the $X(3872)$ observed by the CDF Collaboration. The authors of Ref. [43] concluded that S-wave resonant scattering is unlikely to allow the formation of a loosely bound $D^0\bar{D}^{*0}$ molecule in high energy hadron collision. On the other hand, the CDF data on $X(3872)$ production were well explained in [44] where the authors assumed that it is a mixture with a two-quark (χ'_{c1}) and a four-quark ($D\bar{D}^*$) component.

From the results presented in Ref. [38] one can conclude that it is possible to reproduce the experimental mass of the $X(3872)$ for a wide range of mixing angles, α, but, as observed in [38], it is not so easy to reproduce the experimental decay width. In Ref. [38] it was shown that with a mixing angle $\alpha = 9^0 \pm 4^0$ in Eq. (4) it is possible to describe the experimental mass of the $X(3872)$ with a decay width of $\Gamma(X \to J/\psi\,(n\pi)) = (9.3 \pm 6.9)$ MeV, which is compatible with the experimental upper limit.

To summarize, we could say that, in a QCDSR calculation, the $X(3872)$ can be well described basically by a $c\bar{c}$ current with a small, but fundamental, admixture of molecular ($D\bar{D}^*$) or tetraquark ($[cq][\bar{c}\bar{q}]$) components [38].

3.2 $Y(4260)$

The $Y(4260)$ was first observed by the BaBar collaboration in the e^+e^- annihilation through initial state radiation [31], and it was confirmed by the CLEO and Belle collaborations [45]. The $Y(4260)$ was also observed in the $B^- \to Y(4260)K^- \to J/\Psi\pi^+\pi^-K^-$ decay [46], and CLEO reported two additional decay channels: $J/\Psi\pi^0\pi^0$ and $J/\Psi K^+K^-$ [45].

The mass of the $Y(4260)$ is higher than the $D^{(*)}\bar{D}^{(*)}$ threshold, therefore, if it was a normal $c\bar{c}$ charmonium state, it should decay mainly into $D^{(*)}\bar{D}^{(*)}$. However, the observed Y state does not match the peaks in $e^+e^- \to D^{(*)\pm}D^{(*)\mp}$ cross sections measured by Belle [47] and BaBar [48, 49]. Besides, the $\Psi(3S)$, $\Psi(2D)$ and $\Psi(4S)$ $c\bar{c}$ states have been assigned to the well established $\Psi(4040)$, $\Psi(4160)$, and $\Psi(4415)$ mesons respectively, and the prediction from quark models for the $\Psi(3D)$ state is 4.52 GeV. Therefore, the mass of the $Y(4260)$ is not consistent with any of the 1^{--} $c\bar{c}$ states [29, 50, 51].

There are many theoretical interpretations for the $Y(4260)$ such as tetraquark state, hadronic molecule of D_1D, D_0D^*, $\chi_{c1}\omega$, $\chi_{c1}\rho$, $J/\psi f_0(980)$, hybrid charmonium, charm baryonium, cusp, etc [1, 2, 29]. However, there are some calculations, within the QCDSR approach that can not explain the mass of the $Y(4260)$ treating it as a tetraquark state [52], as a D_1D, D_0D^* hadronic molecule [52], or a $J/\psi f_0(980)$ molecular state [53]. Therefore, as in the case of the $X(3872)$, in Ref. [54] the $Y(4260)$ was treated as a mixture of a $c\bar{c}$ state with a four-quark state:

$$j_\mu(x) = \sin(\theta)\; j^{(4)}_\mu(x) + \cos(\theta)\; j^{(2)}_\mu(x), \tag{6}$$

where

$$j^{(4)}_\mu(x) = \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}\Big[(q_a^T(x)C\gamma_5 c_b(x))(\bar{q}_d(x)\gamma_\mu\gamma_5 C\bar{c}_e^T(x)) + (q_a^T(x)C\gamma_5\gamma_\mu c_b(x))(\bar{q}_d(x)\gamma_5 C\bar{c}_e^T(x))\Big], \tag{7}$$

and

$$j^{(2)}_\mu = \frac{1}{\sqrt{2}}\langle\bar{q}q\rangle\;\bar{c}_a(x)\gamma_\mu c_a(x). \tag{8}$$

Varying the value of the mixing angle in the range $\theta = (53.0 \pm 0.5)^0$ it was found [54] that $m_Y = (4.26 \pm 0.13)$ GeV, which is in a very good agreement with the experimental mass of the $Y(4260)$.

The width of the decay channel $Y(4260) \to J/\psi\pi\pi$, was also evaluated in Ref. [54] considering the same mixing angle and assuming that the two pions in the final state come from the σ and $f_0(980)$ scalar mesons. The obtained value for the width is $\Gamma_{Y\to J/\psi\pi\pi} \approx (4.1 \pm 0.6)$ MeV, which is much smaller than the total experimental width: $\Gamma_{exp} \approx (95 \pm 14)$ MeV [19].

To compare the decay width into the $J/\psi\pi\pi$ channel with the total width we have to consider other possible decay channels. With the mixed current, the main decay channel of the $Y(4260)$ should be into D mesons, mostly due to the charmonium part of the current, but also from the tetraquark part through quark rearrangement. Therefore, the total width of the $Y(4260)$ should be given by the sum of the partial widths of all these channels. Unfortunately, the QCDSR approach does not allow the evaluation of the decay channels involving D mesons, since one can only use the QCDSR approach to study properties of the low-lying state. Therefore, the charmonium part of the current can only be used to study the decay of J/ψ.

If one considers the experimental upper limits, from the BaBar [49] and CLEO [55] collaborations, for the branching ratios

$$\frac{\mathcal{B}(Y(4260) \to X)}{\mathcal{B}(Y(4260) \to J/\psi\pi\pi),} \tag{9}$$

where $X = D\bar{D},\ D\bar{D}^*$ and $D^*\bar{D}^*$, one can see that the width obtained in [54], for the $J/\psi\pi\pi$ channel, is consistent with the total experimental width of the $Y(4260)$. Therefore, they concluded that it is possibile to explain the $Y(4260)$ exotic state as a mixed charmonium-tetraquark state.

3.3 $Z_c^+(3900)$

From March to October of 2013 the BESIII collaboration reported the observation of four charmonium charged states. The first one was the $Z_c^+(3900)$ [32], observed almost at the same time by the Belle collaboration [56], in the $M(\pi^\pm J/\psi)$ mass spectrum of the $Y(4260) \to J/\psi\pi^+\pi^-$ decay channel. The existence of this structure was promptly confirmed by the authors of Ref. [57] using CLEO-c data.

Assuming $SU(2)$ symmetry, the mass obtained in QCDSR for the Z_c coincides with the one obtained for the $X(3872)$. However, the $Z_c(3900)$ decay width represents a challenge to theorists. While its mass is very close to the $X(3872)$ mass, which may be considered its isosinglet partner, it has a much larger decay width. Indeed, while the $Z_c(3900)$ decay width is in the range $40-50$ MeV, the $X(3872)$ width is smaller than 1.2 MeV. This difference can be attributed to the fact that the $X(3872)$ may contain a significant $|c\bar{c}\rangle$ component [38], which is absent in the $Z_c(3900)$. As pointed out in Ref. [58], this would also explain why the Z_c has not been observed in B decays.

According to the experimental observations, the $Z_c(3900)$ decays into $J/\psi\,\pi^+$ with a relatively large decay width. This is unexpected for a $D^* - \bar{D}$ molecular state, in which the distance between the D^* and the $\bar{D}$ is large. This decay must involve the exchange of a charmed meson, which is a short range process and hence unlikely to occur in large systems. In Ref. [59] it was shown that, in order to reproduce the measured width, the effective radius must be $\langle r_{eff}\rangle \simeq 0.4$ fm. This size scale is small and pushes the molecular picture to its limit of validity. In another work [60], the new state was treated as a charged $D^* - \bar{D}$ molecule and the authors explored its electromagnetic structure, arriving at the conclusion that its charge radius is of the order of $\langle r^2\rangle \simeq 0.11$ fm^2. Taking this radius as a measure of the spatial size of the state, we conclude that it is more compact than a J/ψ, for which $\langle r^2\rangle \simeq 0.16$ fm^2. In Ref. [61] the combined results of refs. [59] and [60] were taken as an indication that the Z_c is a compact object, which may be better understood as a quark cluster, such as a tetraquark. Moreover, the $Z_c(3900)$ was interpreted as the isospin 1 partner of the $X(3872)$, as the charged state predicted in Ref. [21]. Therefore, the quantum numbers for the neutral state in the isospin multiplet were assumed to be $I^G(J^{PC}) = 1^+(1^{+-})$. The interpolating field for $Z_c^+(3900)$ used in Ref. [61] is given by Eq. (2) with the plus sign changed to a minus sign. The three-point QCDSR was used to evaluate the coupling constants in the vertices $Z_c^+(3900)J/\psi\pi^+$, $Z_c^+(3900)\eta_c\rho^+$, $Z_c^+(3900)D^+\bar{D}^{*0}$ and $Z_c^+(3900)\bar{D}^0D^{*+}$.

In the case of the $Z_c^+ \to J/\psi\pi^+$ decay, the generic decay diagram in terms of quarks has two "petals", one associated with the J/ψ and the other with the π^+. Among the possible diagrams, there are two distinct subsets. Diagrams with no gluon exchange between the petals and, therefore, no color exchange between the two final mesons in the decay. If there is no color exchange, the final state containing two color singlets was already present in the initial state. This happens because, although the initial current, Eq. (2), has a non-trivial color structure, it can be rewritten as a sum of molecular type currents with trivial color configuration through a Fierz transformation. To avoid this problem we consider in the OPE side only the diagrams with non-trivial color structure, as the one shown in

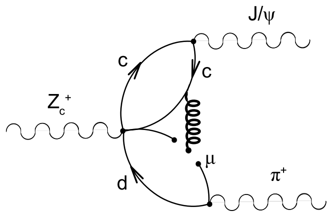

Figure 1. CC diagram which contributes to the OPE side of the sum rule.

Fig. 1. This type of diagram represents the case where the Z_c^+ is a genuine four-quark state with a complicated color structure. These diagrams are called color-conected (CC).

Here only color-connected diagrams were considered, since the $Z_c(3900)$ is expected to be a genuine tetraquark state with a non-trivial color structure. The obtained couplings, with the respective decay widths, are given in Table 2. A total width of $\Gamma = (63.0 \pm 18.1)$ MeV was found for the $Z_c(3900)$, in good agreement with the two experimental values: $\Gamma = (46 \pm 22)$ MeV from BESIII [32], and $\Gamma = (63 \pm 35)$ MeV from BELLE [56].

Table 2. Coupling constants and decay widths in different channels

Vertex	coupling constant (GeV)	decay width (MeV)
$Z_c^+(3900)J/\psi\pi^+$	3.89 ± 0.56	29.1 ± 8.2
$Z_c^+(3900)\eta_c\rho^+$	4.85 ± 0.81	27.5 ± 8.5
$Z_c^+(3900)D^+\bar{D}^{*0}$	2.5 ± 0.3	3.2 ± 0.7
$Z_c^+(3900)\bar{D}^0D^{*+}$	2.5 ± 0.3	3.2 ± 0.7

From the results in Table 2 it is possible to evaluate the ratio

$$\frac{\Gamma(Z_c(3900) \to D\bar{D}^*)}{\Gamma(Z_c(3900) \to \pi J/\psi)} = 0.22 \pm 0.12. \qquad (10)$$

3.4 $Z_c^+(4025)$

Soon after the $Z_c^+(3900)$ observation, the BESIII collaboration reported the observation of other three charged states: $Z_c^+(4025)$ [33], $Z_c^+(4020)$ [62] and $Z_c^+(3885)$ [63]. Up to now it is not clear if the states $Z_c^+(3900)$-$Z_c^+(3885)$ and the states $Z_c^+(4025)$-$Z_c^+(4020)$ are the same states seen in different decay channels, or if they are independent states.

In the case of $Z_c^+(4025)$, a study of the reaction $e^+e^- \to (D^*\bar{D}^*)^\pm\pi^\mp$ was performed by the BESIII Collaboration at $\sqrt{s} = 4.26$ GeV and a peak was seen in the $(D^*\bar{D}^*)^\pm$ invariant mass distribution just about 10 MeV above the threshold. [33]. The authors assume in the paper that the $(D^*\bar{D}^*)^\pm$ pair is created in a S-wave and then the $Z_c^+(4025)$ must have $J^P = 1^+$ to match, together with the pion, the quantum numbers $J^P = 1^-$ of the virtual photon from the e^+e^- pair. However, they also state that the experiment does not exclude other spin-parity assignments.

Many theoretical papers were devoted to understand these new states. In Ref. [64], assuming the $X(3872)$ to be a $D\bar{D}^*$ molecule, the authors found a series of new hadronic molecules, including the $Z_c^+(3900)$ and the $Z_c^+(4025)$. They would correspond to bound states (with uncertainties of about 50 MeV in the binding) of $D\bar{D}^*$ and $D^*\bar{D}^*$ respectively, with quantum numbers $I(J^P) = 1(1^+)$. Remarkably, even with uncertainties, these states always appear in the bound region. In refs. [65, 66], using QCDSR and assuming a structure of $D\ D$, the authors obtained a possible $I(J\ )\quad 1(1\ )$ state compatible with the $Z_c^+(4025)$, but with $\sim$ 250 MeV uncertainty in the energy. In Ref. [67], using a tetraquark current and QCDSR, a state with $I(J^P) = 1(2^+)$ compatible with $Z_c(4025)$ was obtained, once again with a large error in the energy of 190 MeV. In Ref. [68] the new Z_c states were investigated from a different perspective and, using pion exchange, a $D^*\bar{D}^*$ state with $I(J^P) = 1(1^+)$ compatible with the $Z_c(4025)$ was obtained.

A moleculelike picture for $Z_c(4025)$ seems to be quite plausible since its mass is merely 8 MeV away from the $\bar{D}^{*0}D^{*+}$ threshold. In Ref. [69], a study of the $D^*\bar{D}^*$ system has also been done within QCD sum rules, using a interpolating current corresponding to the $\bar{D}^{*0}D^{*+}$ molecule. The idea was to test if the $Z_c(4025)$ could be interpreted as a 1^+ or 2^+ resonance of the $\bar{D}^{*0}D^{*+}$ system. The 0^+ assignment is ruled out for $Z_c(4025)$ by spin-parity conservation for the $e^+e^- \to \left(D^*\bar{D}^*\right)^{\pm}\pi^{\pm}$ process. The tensor interpolating current used in [69] was:

$$j_{\mu\nu}(x) = \left[\bar{c}_a(x)\gamma_\mu u_a(x)\right]\left[\bar{d}_b(x)\gamma_\nu c_b(x)\right], \tag{11}$$

where a, b denote the color indices. The corresponding two-point correlation function is:

$$\Pi_{\mu\nu\alpha\beta}(q^2) = i\int d^4x e^{iqx}\langle 0 \mid T\left[j_{\mu\nu}(x)j^\dagger_{\alpha\beta}(0)\right] \mid 0\rangle. \tag{12}$$

The 0^+, 1^+ and 2^+ components of the correlation function written in Eq. (12) can be obtained by using the following projectors

$$\begin{aligned}
\mathcal{P}^{(0)} &= \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}, \\
\mathcal{P}^{(1)} &= \frac{1}{2}\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} - \Delta^{\mu\beta}\Delta^{\nu\alpha}\right), \\
\mathcal{P}^{(2)} &= \frac{1}{2}\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta},
\end{aligned} \tag{13}$$

where $\Delta_{\mu\nu}$ is defined in terms of the metric tensor, $g^{\mu\nu}$, and the four momentum q of the correlation function as

$$\Delta_{\mu\nu} \equiv -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}. \tag{14}$$

In the three cases a state with mass 3950 ± 100 MeV was found [69]. The central value of the mass of these states is more in line with the results of Refs. [70, 71], although with the error bar, they could as well be related to a resonance.

Bumps close to the threshold of a pair of particles should be treated with caution. Sometimes they are identified as new particles, but they can also be a reflection of a resonance below threshold. Further examples of this phenomenon may be found in Ref. [72], where the theory of $D^*\bar{D}^*$ interactions is reviewed and it is pointed out that a $(D^*\bar{D}^*)$ state with a mass above the threshold is very difficult to support. In particular, in Ref. [71] it was found that there is only one bound state of $(D^*\bar{D}^*)$ in $I^G = 1^-$, with quantum numbers $J^{PC} = 2^{++}$ with a mass around 3990 MeV and a width of about 100 MeV. Both mass and width are compatible with the reanalysis of data carried out in [72]. Therefore, we can conclude that such $J^P = 2^+$ $D^*\bar{D}^*$ bound state provides a natural explanation for the state observed in [33].

An argument against the existence of a new resonance above the threshold is the fact that if the state were a $J^P = 1^+$ produced in S-wave, as assumed in the experimental work, it would easily decay into $J/\psi\pi$ exchanging a D meson in the t-channel. This is also the decay channel of the $Z_c(3900)$, which would then have the same quantum numbers as the state claimed in Ref. [33]. However, while a peak is clearly seen in the $J/\psi\pi$ invariant mass distribution in the case of the $Z_c(3900)$, no trace of a peak is seen around 4025 MeV in spite of using the same reaction and the same e^+e^- energy.

3.5 $X^+(5568)$

This year the D0 Collaboration has announced the observation of a new state in the $B_s^0\pi^\pm$ mass spectrum, the $X^\pm(5568)$ [34].The $X(5568)$ would be a very important addition to the list of undoubtedly exotic mesons, since its wave function consists of four different flavors: u, b, d and s quarks. However, the LHCb Collaboration has not confirmed the observation of the $X(5568)$ [73], since in their analysis no structure is found in the $B_s^0\pi^\pm$ mass spectrum from the $B_s^0\pi^+$ threshold up to $M_{B_s^0\pi^+} \leq 5.7$ GeV.

The announcement of the exotic state $X(5568)$ stimulated the theoretical interest and several theoretical works have been done to investigate the properties of such state. There are studies based on QCDSR, quark models, coupled channel analysis and more general arguments. In some of these studies it was not possible to explain the reported properties of the $X(5568)$ neither as a molecule nor as a tetraquark state [74–79]. However, in many other calculations it was possible to explain the reported properties [80–87].

In Ref. [78] we used a $(I)J^P = (1)\,0^+$ scalar-diquark scalar-antidiquark tetraquark current for the $X^+(5568)$:

$$j_S = \epsilon_{abc}\epsilon_{dec}(u_a^T C\gamma_5 s_b)(\bar{d}_d\gamma_5 C\bar{b}_e^T), \tag{15}$$

where $a,\ b,\ c,\ ...$ are colour indices and C is the charge conjugation matrix. Our study indicates that although it is possible to obtain a stable mass in agreement with the state found by the D0 collaboration, a more restrictive analysis (simultaneous requirement of the OPE convergence and the dominance of the pole on the phenomenological side) leads to a higher mass. In particular, considering condensates up to dimension 8 we get [78]:

$$m_X = (6.39 \pm 0.10)\ \text{GeV}, \tag{16}$$

which is not in agreement with the experimental mass of the $X(5568)$ determined by the D0 Collaboration [34], leading us to conclude that the $X(5568)$ state can not be represented by the scalar tetraquark current.

Clearly, more analysis are required to clarify this situation from the experimental side as well as from the theoretical side.

4 Conclusions

Here we have reported the masses of some of these X, Y and Z states, using the QCDSR approach. In some cases a tetraquark configuration was favored, as for the $Z_c^+(3900)$, and in some other cases a molecular configuration was favored, like the $Z_c^+(4025)$. In the case of the $X(3872)$ (and also $Y(4260)$) we found that it is only possible to explain all the available experimental data if it is a mixed state with charmonium and four-quark components, and in the case of the $X^+(5568)$ it was not possible to describe it as a tetraquark state.

The most important message from the experimental program carried out by the BaBar, Belle, CLEO-c, CDF, DØ, BESIII, LHCb and CMS Collaborations is that definitely there is something really new happening in the charmonium spectroscopy. The program started in 2003 with the measurement of the $X(3872)$. There is no doubt in the community that the $X(3872)$ structure is more complex than a simple $c\bar{c}$ state. However, we can say that the confirmation of the observation of the $Z^+(4430)$ by the LHCb Collaboration together with the measurements of the Z_c (3900), which was measured by BESIII and confirmed by other groups, reinforced our belief that we are observing multiquark states.

In the next years it is important: i) from the experimental side to determine the quantum numbers of all these states and eliminate the suspicion that some of them could be mere threshold effects and not real particles. ii) from the theoretical side to focus on the calculation of the decay widths in all the different approaches, since, as we have discussed, the masses are easily obtained by different methods and they are not sufficient to discriminate between different theoretical models.

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References

[1] N. Brambilla *et al.*, Eur. Phys. J. C **74**, 2981 (2014), arXiv:1404.3723.

[2] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai and S. Yasui, PTEP **2016**, 062C01 (2016), arXiv:1603.09229.

[3] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **100** (2008) 142001, arXiv:0708.1790.

[4] R. Mizuk *et al.* [BELLE Collaboration], Phys. Rev. D **80**, 031104 (2009), arXiv:0905.2869.

[5] K. Chilikin *et al.* [Belle Collaboration], Phys. Rev. D **88**, 074026 (2013), arXiv:1306.4894.

[6] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **79**, 112001 (2009), arXiv:0811.0564.

[7] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **112**, 222002 (2014), arXiv:1404.1903.

[8] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91** (2003) 262001, arXiv:hep-ex/0309032.

[9] I. Adachi *et al.* (Belle Collaboration), arXiv:0809.1224.

[10] B. Aubert *et al.* (BaBar Collaboration), Phys. Rev. D **77** (2008) 111101, arXiv:0803.2838.

[11] D. E. Acosta *et al.* (CDF II Collaboration), Phys. Rev. Lett. **93** (2004) 072001, arXiv:hep-ex/0312021.

[12] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **98** (2007) 132002, arXiv:hep-ex/0612053.

[13] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **103** (2009) 152001, arXiv:0906.5218.

[14] V. M. Abazov *et al.* (DØ Collaboration), Phys. Rev. Lett. **93** (2004) 162002, arXiv:hep-ex/0405004.

[15] R. Aaij *et al.* [LHCb Collaboration], Eur. Phys. J. C **72**, 1972 (2012), arXiv:1112.5310.

[16] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **110**, 222001 (2013), arXiv:1302.6269.

[17] S. Chatrchyan *et al.* [CMS Collaboration], JHEP **1304**, 154 (2013), arXiv:1302.3968.

[18] T. Barnes and S. Godfrey, Phys. Rev. D **69**, 054008 (2004); T. Barnes, S. Godfrey and E.S. Swanson, Phys. Rev. D **72**, 054026 (2005).

[19] C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).

[20] E.S. Swanson, Phys. Rept. **429**, 243 (2006).

[21] L. Maiani, V. Riquer, F. Piccinini and A.D. Polosa, Phy. Rev. D **71**, 014028 (2005).

[22] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, *Nucl. Phys.* B**147**, 385 (1979); **147**, 448; **147**, 519.

[23] B. L. Ioffe, *Nucl. Phys.* B**188**, 317 (1981); **191**, 591(E).

[24] Y. Chung, H. G. Dosch, M. Kremer, D. Schall, *Phys. Lett.* B**102**, 175 (1981); *Nucl. Phys.* B**197**, 55 (1982).

[25] L.J. Reinders, H. Rubinstein and S. Yazaki, *Phys. Rep.* **127**, 1 (1985).

[26] S. Narison, *QCD as a theory of hadrons*, Cambridge Monogr. *Part. Phys. Nucl. Phys. Cosmol.* **17**, 1 (2002); *QCD spectral sum rules*, World Sci. Lect. Notes Phys., **26**, 1 (1989); *Acta Phys. Pol.* **26**, 687 (1995); *Riv. Nuov. Cim.***10N2**, 1 (1987); *Phys. Rept.*, **84**, 263 (1982).

[27] S. Narison, *QCD spectral sum rules, World Sci. Lect. Notes Phys.* **26**, 1 (1989).

[28] P. Colangelo and A. Khodjamirian, Boris Ioffe Festschrift "At the Frontier of Particle Physics / Handbook of QCD", edited by M. Shifman (World Scientific, Singapore, 2001, [arXiv:hep-ph/0010175].

[29] M. Nielsen, F.S. Navarra and S.H. Lee, *Phys. Rept.* **497**, 41 (2010).

[30] M. Nielsen and F. S. Navarra, Mod. Phys. Lett. A **29**, 1430005 (2014).

[31] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **95**, 142001 (2005).

[32] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **110**, 252001 (2013), arXiv:1303.5949.

[33] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **112**, 132001 (2014), arXiv:1308.2760.

[34] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **117**, 022003 (2016), arXiv:1602.07588.

[35] R.D. Matheus *et al.*, *Phys. Rev.* D **75**, 014005 (2007).

[36] S.H. Lee, M. Nielsen and U. Wiedner, *Jour. Korean Phys. Soc.* **55**, 424 (2009).

[37] F.S. Navarra, M. Nielsen, *Phys. Lett.* B **639**, 272 (2006).

[38] R.D. Matheus *et al.*, *Phys. Rev.* D **80**, 056002 (2009).

[39] C. Meng, Y.J. Gao and K.-T. Chao, hep-ph/0506222.

[40] M. Suzuki, *Phys. Rev. D* **72**, 114013 (2005).

[41] Y. Dong *et al. Phys. Rev. D* **77**, 094013 (2008).

[42] B.-Q. Li C. Meng and K.-T. Chao, *Phys. Rev. D* **80**, 104012 (2009).

[43] C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa and C. Sabelli, *Phys. Rev. Lett.* **103**, 162001 (2009).

[44] C. Meng, H. Han and K. T. Chao, arXiv:1304.6710.

[45] Q. He *et al.* [CLEO Collaboration], Phys. Rev. D **74**, 091104(R) (2006); C.Z. Yuan *et al.* [Belle Collaboration], Phys. Rev. Lett. **99**, 182004 (2007).

[46] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **73**, 011101 (2006).

[47] G. Pakhlova *et al.* [Belle Collaboration], Phys. Rev. Lett. **98**, 092001 (2007).

[48] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **77**, 011103 (2008).

[49] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **79**, 092001 (2009), arXiv:0903.1597.

[50] S. L. Zhu, Int. J. Mod. Phys. E **17**, 283 (2008), hep-ph/0703225.

[51] E. Klempt and A. Zaitsev, Phys. Rept. **454**, 1 (2007).

[52] R.M. Albuquerque and M. Nielsen, Nucl. Phys. **A815**, 53 (2009); Erratum-ibid. A857 (2011) 48.

[53] R. M. Albuquerque, M. Nielsen and R. R. da Silva, Phys. Rev. D **84**, 116004 (2011), arXiv:1110.2113.

[54] J. M. Dias, R. M. Albuquerque, M. Nielsen and C. M. Zanetti, Phys. Rev. D **86**, 116012 (2012), arXiv:1209.6592.

[55] D. Cronin-Hennessy *et al.* [CLEO Collaboration], Phys. Rev. D **80**, 072001 (2009), arXiv:0801.3418.

[56] Z.Q. Liu *et al.* [BELLE Collaboration], Phys. Rev. Lett. **110**, 252002 (2013), arXiv:1304.0121.

[57] T. Xiao, S. Dobbs, A. Tomaradze and K.K. Seth, Phys. Lett. B **727**, 366 (2013), arXiv:1304.3036.
[58] Q. Wang, C. Hanhart and Q. Zhao, *Phys. Rev. Lett.* **111**, 132003 (2013).
[59] N. Mahajan, arXiv:1304.1301.
[60] E. Wilbring, H. -W. Hammer and U. -G. Meissner, *Phys. Lett. B* **726**, 326 (2013).
[61] J. M. Dias, F. S. Navarra, M. Nielsen and C. M. Zanetti, *Phys. Rev. D* **88**, 016004 (2013).
[62] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **111**, 242001 (2013), arXiv:1309.1896.
[63] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **112**, 022001 (2014), arXiv:1310.1163.
[64] F. -K. Guo, C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, *Phys. Rev. D* **88**, 054007 (2013).
[65] W. Chen, T. G. Steele, M. -L. Du and S. -L. Zhu, Eur. Phys. J. C **74**, no. 2, 2773 (2014), arXiv:1308.5060.
[66] C. -Y. Cui, Y. -L. Liu and M. -Q. Huang, Eur. Phys. J. C **73**, no. 12, 2661 (2013), arXiv:1308.3625.
[67] C. -F. Qiao and L. Tang, Eur. Phys. J. C **74**, 2810 (2014), arXiv:1308.3439.
[68] J. He, X. Liu, Z. -F. Sun and S. -L. Zhu, Eur. Phys. J. C **73**, 2635 (2013), arXiv:1308.2999.
[69] K. P. Khemchandani, A. Martinez Torres, M. Nielsen and F. S. Navarra, Phys. Rev. D **89**, 014029 (2014), arXiv:1310.0862.
[70] R. Molina and E. Oset, *Phys. Rev. D* **80**, 114013 (2009).
[71] F. Aceti, M. Bayar, J. M. Dias and E. Oset, Eur. Phys. J. A **50**, 103 (2014), arXiv:1401.2076.
[72] A. Martinez Torres, K. P. Khemchandani, F. S. Navarra, M. Nielsen and E. Oset, Phys. Rev. D **89**, 014025 (2014), arXiv:1310.1119.
[73] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **117**, no. 15, 152003 (2016), arXiv:1608.00435.
[74] T. J. Burns and E. S. Swanson, Phys. Lett. B **760**, 627 (2016), arXiv:1603.04366.
[75] F. K. Guo, U. G. Meissner and B. S. Zou, Commun. Theor. Phys. **65**, no. 5, 593 (2016), arXiv:1603.06316.
[76] W. Wang and R. Zhu, Chin. Phys. C **40**, no. 9, 093101 (2016), arXiv:1602.08806.
[77] A. Ali, L. Maiani, A. D. Polosa and V. Riquer, Phys. Rev. D **94**, no. 3, 034036 (2016), arXiv:1604.01731.
[78] C. M. Zanetti, M. Nielsen and K. P. Khemchandani, Phys. Rev. D **93**, no. 9, 096011 (2016), arXiv:1602.09041.
[79] X. Chen and J. Ping, Eur. Phys. J. C **76**, no. 6, 351 (2016), arXiv:1604.05651.
[80] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D **93**, no. 7, 074024 (2016), arXiv:1602.08642.
[81] Z. G. Wang, Commun. Theor. Phys. **66**, no. 3, 335 (2016), arXiv:1602.08711.
[82] L. Tang and C. F. Qiao, Eur. Phys. J. C **76**, no. 10, 558 (2016), arXiv:1603.04761.
[83] X. G. He and P. Ko, Phys. Lett. B **761**, 92 (2016), arXiv:1603.02915.
[84] Y. R. Liu, X. Liu and S. L. Zhu, Phys. Rev. D **93**, no. 7, 074023 (2016), arXiv:1603.01131.
[85] F. Stancu, J. Phys. G **43**, no. 10, 105001 (2016), arXiv:1603.03322
[86] M. Albaladejo, J. Nieves, E. Oset, Z. F. Sun and X. Liu, Phys. Lett. B **757**, 515 (2016), arXiv:1603.09230.
[87] W. Chen, H. X. Chen, X. Liu, T. G. Steele and S. L. Zhu, Phys. Rev. Lett. **117**, no. 2, 022002 (2016), arXiv:1602.08916.