Challenges in Heavy Flavor and Quarkonium Production in $p + p$ Collisions at the LHC

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Abstract. I discuss new results and open challenges in open heavy flavor and quarkonium production in $p + p$ collisions at the LHC.

1 Introduction

In this talk, I was asked to cover new results and challenges in the theory of open heavy flavor and quarkonium production at the LHC. The talk was to cover 'cold' matter production, both in $p + p$ collisions and in p+Pb collisions. Given the difficult task of covering both systems in a single presentation, I chose to focus on $p + p$ collisions to emphasize intriguing results in this area, particularly for quarkonium production. Given also the complexities arising from the 'lightest' heavy flavor, charm, I further narrowed the scope to open charm and charmonium. Finally, as an addendum, I mention some interesting new developments on the long discussed possibility of enhanced 'intrinsic' charm in the proton since a fixed-target program at the LHC could potentially directly address this question.

2 Open heavy flavor production

There are currently two approaches to heavy flavor production at colliders: those that employ collinear factorization and those using the low x k_T-factorization approach. We briefly describe each one.

There are two methods of calculating the spectrum of single inclusive open heavy flavor production in perturbative QCD assuming collinear factorization. The underlying idea is similar but the technical approach differs. Both note that large logarithms of p_T/m arise at all orders of the perturbative expansion, spoiling the convergence. The first terms in the expansion are the leading (LL) and next-to-leading logarithmic (NLL) terms, $\alpha_s^2[\alpha_s \log(p_T/m)]^k$ and $\alpha_s^3[\alpha_s \log(p_T/m)]^k$ respectively.

Already at leading order, the heavy flavor p_T distribution is finite at $p_T \to 0$ because of the finite mass scale. A next-to-leading order calculation that assumes production of massive quarks but neglects LL terms, a "massive" formalism, can result in large uncertainties at high p_T even though it may be sufficient at low p_T [1–3]. (This massive formalism is sometimes referred to as a fixed-flavor-number (FFN) scheme.) If, instead, the heavy quark is treated as "massless" and the LL and NLL corrections absorbed into the fragmentation functions, the approach breaks down as p_T approaches m

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even though it improves the result at high p_T. The massless formalism is also sometimes referred to as the zero-mass, variable-flavor-number (ZM-VFN) scheme [4, 5]. There are schemes which interpolate smoothly between the FO/FFN scheme at low p_T and the massless/ZM-VFN scheme.

The fixed-order next-to-leading logarithm (FONLL) approach, is one such scheme. In FONLL, the fixed order and fragmentation function approaches are merged so that the mass effects are included in an exact calculation of the leading (α_s^2) and next-to-leading (α_s^3) order cross section while also including the LL and NLL terms [6]. The NLO fixed order (FO) result is combined with a calculation of the resumed (RS) cross section in the massless limit. These two approaches need to be calculated in the same renormalization scheme. This involves a change in scheme at FO for the two to be consistent. Normally, for charm production at FO, three light flavors are included because the charm quark should not be included in the evolution of α_s. On the other hand, in the RS scheme the charm quark is an active degree of freedom so four light flavors are involved. Thus the FO calculation scheme is translated from three to four light flavors so that the FO calculation matches the order α_s^3 RS result when power-suppressed mass terms are negligible. To subtract the fixed-order terms in the RS result present at FO, an approximation of FO that retains only logarithmic mass terms are retained, the 'massless' limit of FO, FOM0.

The FONLL result is then, schematically,

$$\mathrm{FONLL} = \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})G(m, p_T) \ . \tag{1}$$

The function $G(m, p_T) \sim p_T^2/(p_T^2 + (cm)^2)$ is arbitrary but must approach unity for $m/p_T \to 0$. The form of $G(m, p_T)$ is not known because the NLL power-suppressed terms are not known. Fragmentation functions for D and D^* mesons have been calculated based on a moment-space nonperturbative extraction from e^+e^- data in an approach consistent with an FONLL calculation [7].

A second interpolation scheme is the generalized-mass variable-flavor-number (GM-VFN) scheme. The first implementation was the ACOT scheme in Ref. [8] where they used the fixed order contribution at NLO and including leading log terms in the ZM-VFN part at high p_T. Further calculations in the GM-VFN have been done by Kniehl and collaborators [9]. The large logarithms in charm production for $p_T \gg m$ are absorbed in the charm parton distribution function and can thus be evolved by the evolution equations for the parton distributions. The unsubtracted logarithmic terms can be incorporated into the hard cross section to achieve better accuracy for $p_T \geq m$. There are terms proportional to $\ln m^2$ which are left over and their subtraction does not follow a unique prescription. For example, if the $m = 0$ limit is taken from the beginning, the finite terms are different than if one starts with $m \neq 0$ and takes the limit $m \to 0$ later but in such a way that the ZM-VFN result is obtained for $p_T \to \infty$. Starting from finite quark mass is the usual procedure because the standard parton densities and fragmentation functions are typically defined with charm distributions starting from a mass threshold, not with $m = 0$. By adjusting the subtraction terms, they do not include any interpolating function.

In the k_T-factorization approach, off-shell leading order matrix elements for $g^*g^* \to c\bar{c}$ are used together with unintegrated gluon densities that depend on the transverse momentum of the gluon, k_T, as well as the usual dependence on x and μ_F. The unintegrated gluon distributions are normalized so that $g(x, \mu_F^2) = \int_0^{\mu_F^2} dk_T^2 f_g(x, k_T^2, \mu_F^2)$. There are a variety of unintegrated gluon densities on the market but only a few give shapes consistent with the D^0 distributions at $\sqrt{s} = 7$ TeV. There is only a small enhancement in the charm quark p_T distribution at low p_T over *e.g.* the FONLL result [21].

The motivation for choosing the k_T-factorized approach is the idea that, at sufficiently low x, collinear factorization no longer holds. The fact that calculations assuming collinear factorization compare well with the LHC data, even the forward rapidity data of LHCb [10], covering the lowest x range of charm meson production data, challenge this assumption.

Indeed, new data from the ALICE Collaboration [11], at $0 < p_T < 2$ GeV and consistent with their previous data at central rapidity [12], also support collinear factorization. Their new D^0 result at 7 TeV, at central rapidity, $|y| < 0.5$, are compared to FONLL, GV-VFNS and leading order k_T-factorization calculations. The data/theory ratio is determined for each calculation and only the k_T-factorized calculation is inconsistent with the shape of the data. In the ratio, the data for $2 < p_T < 6$ GeV lie outside the uncertainty band of the calculation [11], precisely the p_T range where the calculation should best apply. The FONLL uncertainty band is large at low p_T when the standard set of masses and scales are employed, using fitted values can reduce the uncertainty [13].

Note that the k_T-factorized result is only at leading order. Perhaps a next-to-leading order calculation could lead to improved agreement. It is worth mentioning, however, that the K factor between leading and next-to-leading order calculations of single inclusive heavy flavor p_T distributions have only a weak dependence on p_T [14]. There is still room for improvement of the unintegrated gluon distributions. Since these distributions have a strong influence on the shape of the calculated kinematic observables, it is more likely that better agreement with the single charm data will come from this direction. The 13 TeV charm production data will be crucial for settling this question since the higher energy reduces the values of the momentum fractions, x, by nearly a factor of two.

A further challenge to theory may arise from measurements of correlated production, specifically of the azimuthal opening angle of heavy flavors, either by direct reconstruction of both D mesons or of a D meson and the decay product of its partner, either a light hadron or a lepton [15]. Naively, at leading order $Q\overline{Q}$ pairs are produced in a back-to-back configuration with a peak at $\Delta\phi = \pi$. Higher order production will, however, result in a more isotropic distribution in $\Delta\phi$ although this result depends on the momenta of the measured particles.

As previously mentioned, the FONLL and GM-VFNS approaches are for single inclusive production and thus cannot be used to study heavy flavor correlations. There are NLO heavy flavor codes that, in addition to inclusive heavy flavor production, also calculate exclusive $Q\overline{Q}$ pair production which allows for studies of correlated observables. The first, `HVQMNR` [16], also includes single Q and total $Q\overline{Q}$ production, as well as fragmentation and decays to leptons. `HVQMNR` uses negative weight events used to cancel divergences numerically and which often tend to give a negative value for the cross section in the lowest pair p_T bin. This negative value can be removed by adding smearing the parton momentum through the introduction of intrinsic transverse momenta, k_T. In addition, it is not an event generator and does not include any resummation. The `POWHEG − hvq` [17] code can include complete events. It is a positive weight generator that includes leading-log resummation. The entire event is available since `PYTHIA` [18] and `HERWIG` [19] are employed to produce the complete event. In addition to NLO codes, correlations can also be accessed through LO event generators like `PYTHIA`.

The k_T-factorization approach has also been used to calculate correlated $c\overline{c}$ production. Since the unintegrated parton densities in this approach have a transverse component, the p_T and $\Delta\phi$ distributions are not delta functions but finite, even at leading order.

LHCb measured a number of charm correlations in $p+p$ collisions at 7 TeV for D meson rapidities $2 < y_D < 4$. Indeed, in Ref. [20], the LHCb collaboration presented correlated J/ψ–charm, charm–charm and charm–anti-charm correlations in a number of final-state charm hadron channels for each. Only the charm–anti-charm hadron pairs may be expected to be produced in a single hard scattering. The J/ψ–charm and charm–charm events seem to be more consistent with production through double parton scattering. The azimuthal and rapidity correlations for these final states are consistent with isotropic emission [20], as one might expect from two hard scatterings. In my presentation, only the $D^0\overline{D}^0$ pair results were discussed since production of these pairs should be dominated by a single hard scattering. Consistent with NLO contributions such as $gg \to c\overline{c}g$, there is not a strong peak at $\Delta\phi = 180°$ but at $\Delta\phi = 0$ which could arise if the final-state gluon is hard [20].

In Ref. [21], the k_T-factorization approach was employed to calculate the $\Delta\phi$ distribution to compare to the LHCb $D^0\overline{D}^0$ pair results. Several unintegrated gluon densities were used in the comparison. None of them could successfully describe the shape of the data over the entire $\Delta\phi$ range. All of them produced a peak at 180° at least as high or higher than that at $\Delta\phi = 0$.

More recently the ALICE collaboration presented an analysis of azimuthal correlations between reconstructed D mesons and a light hadron trigger particle. The light hadrons were primary particles, emitted from the collision points. These particles include those from heavy flavor decays, such as the unreconstructed partner D meson. The data were binned according to the transverse momentum of both the D meson and the light hadron. The minimum light hadron p_T was soft, $p_T > 0.3$ GeV. These data were further subdivided into two p_T ranges, $0.3 < p_T < 1$ GeV and $p_T > 1$ GeV. The D meson p_T was considerably higher: $3 < p_T < 5$ GeV, $5 < p_T < 8$ GeV, and $8 < p_T < 16$ GeV. To improve statistics, the "D meson" is an average over the D^0, D^+ and $D*^+$. Also, instead of covering the forward region, as in LHCb, the ALICE measurements cover the central region, $|y| < 0.5$ for the D and $|\Delta\eta| < 1$ for the light hadron. The general behavior is, however, the same as the LHCb $D^0\overline{D}^0$ pairs, a peak at $\Delta\phi = 0$ and a smaller enhancement at $\Delta\phi = \pi$. The peak at $\Delta\phi = 0$ increases with increasing trigger particle p_T and also with increasing D meson p_T. The data were compared to simulations with various `PYTHIA` tunes and also `POWHEG` + `PYTHIA`. All simulations reproduced the trends of the data [22].

Thus it appears that collinear approaches also perform well for the correlation studies so far available.

3 Quarkonium Production

So far, open heavy flavor single meson and meson pair production in $p + p$ collisions has been discussed. Now I turn to heavy quark-antiquark bound states, referred to as quarkonium. The bound states can be described by a potential model, with energy levels, similar to positronium.

The $J^{PC} = 1^{--}$ bound states below the $D\overline{D}$ and $B\overline{B}$ mass thresholds, J/ψ and ψ', the $\psi(1S)$ and $\psi(2S)$ charmonium ($c\overline{c}$) states respectively, and the $\Upsilon(nS)$ bottomonium ($b\overline{b}$) states with $1 \leq n \leq 4$, have important decays to lepton pairs and can thus be observed as peaks in the lepton pair invariant mass distributions. These states have narrow widths because they cannot decay directly to open heavy flavor hadrons. Indeed the J/ψ and $\Upsilon(1S)$ can only decay to lepton pairs or light hadrons. The higher mass 1^{--} states can decay to the lower states, by *e.g.* $\psi' \to J/\psi\pi\pi$ or via radiative decays to the χ_{cJ} P states, $J^{PC} = J^{++}$ with $J = 0$, 1 and 2: $\psi' \to \chi_c\gamma$ followed by $\chi_c \to J/\psi\gamma$. There are also η_c states, $J^{PC} = 0^{-+}$, with masses somewhat below those of the J/ψ and ψ' that do not decay to lepton pairs, only hadrons. A similar but more complex hierarchy exists for the bottomonium states.

While potential models do a good job of describing quarkonium spectroscopy, perturbative QCD is employed to calculate the production characteristics. A number of models have been used since the discovery of the J/ψ just over 40 years ago, coupled with the advent of perturbative QCD. One of the first was the color evaporation model (CEM) [23–25] which does not specify the color or spin state of the produced $Q\overline{Q}$ pairs. The next was the color singlet model (CSM) [26] which only considered quarkonium states produced as color neutral objects. The p_T distributions of the color singlet states were too soft to explain the first collider data. Higher order corrections, including an approximate next-to-next-to-leading order approach have also been studied. None of these corrections lead to a good description of the data. Since the beginning of the collider era of quarkonium measurements, another path, which has become the currently most successful production approach, non-relativistic QCD (NRQCD) was also pursued [27–29]. This approach is based on an expansion of the cross section in both the strong coupling constant and $Q\overline{Q}$ velocity with a separation of the hard and soft scales so that contributions to the cross section are divided into different color states with different weight factors, long distance matrix elements (LDMEs) which are adjusted to data. All of the above calculations have been done in the collinear factorization approach. Some work has also been done in k_T-factorization for quarkonium production. These calculations typically use either color singlet or color octet/NRQCD matrix elements with unintegrated gluon distributions. In the remainder of this section, I will discuss only recent results in NRQCD and the CEM.

According to the NRQCD factorization theorem for J/ψ production, the production cross section can be written as

$$\sigma_{J/\psi} = \Sigma_n \sigma_{c\overline{c}[n]} \langle O^{J/\psi}[n] \rangle \tag{2}$$

where the sum over n includes all Fock states, including color octet states [27]. The cross section $\sigma_{c\overline{c}[n]}$ is the production rate of a $c\overline{c}$ pair in the color and spin state n, calculated in perturbative QCD. Finally, $\langle O^{J/\psi}[n] \rangle$ represents the LDMEs which describe the conversion of the $c\overline{c}[n]$ state into a final state J/ψ, assuming that the hadronization does not change the spin or momentum. The full cross section includes convolution with the parton densities appropriate to the process in question.

The LDMEs are assumed to be universal, similar to the proton parton densities. Since NRQCD is a double expansion in powers of α_s and v, the LDMEs scale with specific powers of v. The leading term in v in the J/ψ cross section is the color singlet state, $n =^3 S_1^{[1]}$. The normalization of this contribution is fixed by the value of the J/ψ wavefunction at the origin, determined from the lepton pair decay width. The color octet states, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$, are subleading in the velocity expansion. Heavy quark spin symmetry [30–32] can be used to assign LDMEs to other quarkonium states once they have been fit to a specific set of data so that, *e.g.* $\langle O^{\eta_c}[{}^3S_1^{[8]}] \rangle = \langle O^{J/\psi}[{}^1S_0^{[8]}] \rangle$. The cross sections are determined up to next-to-leading order in the p_T distribution.

If the p_T distribution is characterized in powers of $1/p_T^m$, the value of m is smaller for the octet states than the singlet state so that, at high p_T, the octet states dominate production. The relative mix of the color octet contributions is determined by fitting the LDMEs to J/ψ production data, typically above some p_T cut. (Even though one can also calculate the p_T integrated cross section, the p_T distribution is not finite at low p_T, as in all other quarkonium production models, some resummation or k_T smearing is required for this region. The NRQCD approach generally predicts large transverse polarization of quarkonium production at high p_T. In most cases then the LDMEs are fitted to other data and tested against the polarization data.

In comparisons of the NRQCD calculations to data, the color octet LDMEs are determined, along with some uncertainty on their values due to the quality of the data used in the fits. The resulting calculations are usually compared to data with an uncertainty band that is determined by fixing the LDMEs to their central fit values and varying the renormalization and factorization scales by a factor of two relative to the central values of $\mu_R = \mu_F = m_T = (p_T^2 + 4m_c^2)^{1/2}$ with $m_c = 1.5$ GeV. No variation of the charm quark mass or the LDMEs are included in the uncertainty band [29].

One of the most important questions for NRQCD to resolve is whether or not the color octet LDMEs are indeed universal. Their universality has been called into increasing question recently. Different groups arrive at different values of the LDMEs depending on the chosen p_T cut, the data sets included in the analysis, and whether or not the polarization data are included in the fits, see Ref. [29] and references therein. In addition, the LDMEs fit to collider data do not agree with the integrated cross sections, $d\sigma/dy|_{y=0}$ [33]. Recent measurements of the η_c at forward rapidity in LHCb [34] strongly disagree with the p_T distributions calculated assuming heavy quark spin symmetry [35]. These recent results will be briefly discussed here.

In the review presented in Ref. [29], cross sections calculated using the LDMEs extracted by three different groups, using different data sets, are shown side by side. The results include $\sigma(e^+e^- \to J/\psi)$

at $\sqrt{s} = 10.6$ GeV by Belle [36], $d\sigma(ep \to J/\psi)/dp_T)$ at $\sqrt{s} = 319$ GeV by H1 [37, 38]; $d\sigma(pp \to J/\psi)/dp_T$ at $\sqrt{s} = 1.96$ TeV by CDF [39] and $d\sigma(pp \to J/\psi)/dp_T$ at $\sqrt{s} = 7$ TeV by ATLAS [40]; and the polarization$\lambda(p_T)$ measured by CDF at $\sqrt{s} = 1.96$ TeV [41]. The lowest p_T cut, $p_T > 3$ GeV, was chosen by Butenschön and Kniehl [42] and the LDMEs were determined from a global fit to all J/ψ production data in $\gamma\gamma$, e^+e^-, ep, $p\overline{p}$ and pp collisions, excluding the polarization data. Their results agreed well with the cross sections but were far off from the polarization data. Gong *et al.* [43] fit their LDMEs only the $p\overline{p}$ and pp data for $\sqrt{s} > 1$ TeV with $p_T > 5$ GeV. Their calculation came closer to the polarization data but failed to come close to the e^+e^- and ep data. Chao *et al.* [44] took a different approach and fit their LDMEs to the $p\overline{p}$ and pp data as well as the polarization data with $p_T > 7$ GeV. Even when the polarization data were included in the fit, the description was not ideal. The other, lower energy J/ψ data, were not reproduced well at all, with the calculated cross sections far above these measurements. A later calculation by Bodwin *et al.* [45], not included in Ref. [29], used an even higher p_T cut, $p_T > 10$ GeV, to obtain good agreement with the p_T distributions and the polarization data.

Clearly there are large discrepancies in these results. The fitted LDMEs can differ by an order of magnitude and some fit values are negative. (Note that the fitted LDMEs may be negative as long as the weighted sum, convoluted with the parton densities, are positive.) The same LDMEs when used to calculate the total cross section at $y = 0$, are often far from these data [33]. Only the fit by Butenschön and Kniehl, with the lowest p_T cut, comes close. As noted in Ref. [33], the leading order CSM does significantly better in describing the energy dependence of this cross section.

Finally, the same fitted J/ψ LDMEs were used in Ref. [35] to compare the results of all four fits to the η_c p_T distributions measured by LHCb at forward rapidity at $\sqrt{s} = 7$ and 8 TeV. The NRQCD calculations all lie above the η_c data at high p_T. Indeed, the best description of these data is achieved with the color singlet model [35]. A different approach was taken in Ref. [46] where they found, for the J/ψ, $\langle O^{J/\psi}({}^1S_0^{[8]})\rangle$ can be negligibly small so that $\langle O^{\eta_c}({}^3S_1^{[8]})\rangle$ does not play a role in η_c production, allowing the color singlet contributions to dominate η_c production, offering a way out of the problem. However, it does not provide a very good fit to the J/ψ polarization data. Another calculation of η_c production [47] fits the η_c color singlet LDME, a value normally left untouched because it may cause conflict with potential model calculations or J/ψ decay rates (due to HQSS). Hopefully, more data will eventually resolve these differences.

An alternative approach was proposed by Faccioli *et al* [48] that concentrated on fitting the polarization data for the ψ' and $\Upsilon(3S)$, the measured states least subject to feed down contributions. Instead of attempting to describe other data, they looked for the optimal p_T cut for the unpolarized LDME contribution to dominate production, in the region $p_T/m > 3$ in both cases. The remaining dominant LDME can describe the p_T distribution in the same p_T/m region. While this approach may provide an attractive solution for the polarization problem, it is not particularly appealing where other data are concerned.

All these discrepant results suggest that the LDMEs in NRQCD cannot be universal, at least in the present incarnation of the model. The result that comes the closest is the global analysis by Butenschön and Kniehl but it is furthest from the polarization data. Either the $c\overline{c}$ system is too light for NRQCD factorization or heavy quark spin symmetry to apply, some subset of the data are faulty or the LDMEs are process dependent.

If one goes beyond current next-to-leading order (NLO) calculations, it is not clear that the p_T dependence would substantially change, the p_T distributions already include the powers giving the hardest p_T distributions. Adopting more fit parameters, including the mass and scales, will not have a strong effect on the p_T distributions. Changing the factorization scale has the largest effect on the shape but modifying the renormalization scale or the quark mass has a larger effect on the magnitude than the shape of the distributions.

Does NRQCD factorization hold for polarization? If factorization holds for the p_T distributions, then, under the assumption that the spin and momentum are unaffected by hadronization, the initial polarization should survive as well. Another possibility is that some of the data are faulty but this is not for modelers to decide.

There are two k_T-factorization-like approaches that address the low p_T behavior of quarkonium production. Baranov *et al.* [49] have calculated ψ' production in k_T factorization using the color singlet and color octet matrix elements in NRQCD but with unintegrated gluon distributions to probe lower ψ' p_T without resummation. The fits to the color octet LDMEs give smaller values than in NRQCD with collinear factorization, also resulting in better agreement with the ψ' polarization data for $p_T < 16$ GeV. It is not clear, however, how well these results agree with higher p_T measurements or calculations. Other calculations in this approach can reach different conclusions, depending on the unintegrated gluon distributions and the choice of LDMEs.

A similar approach was taken in the color glass condensate (CGC) model of Ma and Venugopalan [50] where they used the same NRQCD LDMEs at $p_T > 7$ GeV as in Ref. [44] also at $p_T < 7$ GeV where the CGC calculation with an unintegrated gluon density related to the color dipole forward scattering amplitude at a fixed saturation scale. There is a reasonable match between the low and high p_T saturation and collinear factorization regions respectively. The polarization was not calculated but should agree with the results of Ref. [44].

The difficulties that have plagued the NRQCD description of J/ψ production are reduced for Υ production. The larger mass, higher scale and lower velocity could make the Υ a better candidate for an accurate description by NRQCD. In addition, since there are more Υ states, there are more color octet LDMEs available for fitting, allowing a description of the p_T-dependent yields and the polarization simultaneously [33, 51–54].

If NRQCD is insufficient, other models should be given a second look. Some new developments to the color evaporation model makes this model worth a second look. In the CEM, heavy flavor and quarkonium production is treated on an equal footing. In the CEM, the quarkonium production cross section is some fraction of all $Q\overline{Q}$ pairs below the $H\overline{H}$ threshold where H is the lowest mass heavy-flavor hadron. Thus the CEM cross section is simply the $Q\overline{Q}$ production cross section with a cut on the pair mass. The color and spin have been integrated over so that the color of the state is said to have been 'evaporated' away without changing the kinematics of the pair. The additional energy needed to produce heavy-flavored hadrons when the partonic center of mass energy, $\sqrt{\hat{s}}$, is less than $2m_H$, the $H\overline{H}$ threshold energy, is nonperturbatively obtained from the color field in the interaction region.

At leading order, the production cross section of quarkonium state C in a $p + p$ collision is

$$\sigma_C^{\rm CEM}(s_{_{NN}}) = F_C \sum_{i,j} \int_{4m^2}^{4m_H^2} d\hat{s} \int dx_1\, dx_2\, f_i^p(x_1, \mu_F^2)\, f_j^p(x_2, \mu_F^2)\, \mathcal{J}(\hat{s}) \hat{\sigma}_{ij}(\hat{s}, \mu_F^2, \mu_R^2) \;, \tag{3}$$

where $ij = q\overline{q}$ or gg and $\hat{\sigma}_{ij}(\hat{s})$ is the $ij \to Q\overline{Q}$ subprocess cross section. Here $\mathcal{J}(\hat{s})$ is a kinematics-dependent Jacobian. At LO $\mathcal{J}(\hat{s}) = \delta(\hat{s} - x_1 x_2 s)/s$, at NLO and for differential cross sections, the expressions are more complex.

The fraction F_C must be universal so that, once it is fixed by data, the quarkonium production ratios should be constant as a function of $\sqrt{s}$, y and p_T. The actual value of F_C depends on the heavy quark mass, m, the scale parameters, the parton densities and the order of the calculation. The same values of the charm quark mass and scale parameters as found in Ref. [13] are employed to obtain the normalization F_C for the J/ψ, Two recent updates of the CEM are worth noting. The first is a calculation of the ψ'/ψ ratio as a function of p_T in an "improved" CEM calculation (ICEM) [55]. In this calculation, the p_T of the quarkonium state is modified by the ratio of the pair invariant mass to the quarkonium mass. Similarly, the range of integration is not from $2m_c$ to $2m_D$ for charmonium but, *e.g.* from the J/ψ or ψ' mass to $2m_D$. Thus a rise in the ratio with p_T consistent with measurement is found [55]. The second is a calculation of the J/ψ polarization in the leading order CEM. If one assumes that the spin of the quarks are either aligned or anti-aligned with the momentum, $J_z = -1, 0$ or 1, and this alignment, like the momentum of the quarks, survives hadronization, the polarization can be obtained analytically. While at leading order one learns only the dominant polarization as a function of energy and rapidity, it is a starting point for further calculation. This work is done in collaboration with UC Davis graduate student V. Cheung [56].

Much has been learned about the quarkonium production mechanism in recent years. However, some of it is contradictory and the final answer remains to be determined.

4 Heavy Flavors in the Proton

An intrinsic heavy flavor component in the proton has long been predicted [57]. While the perturbative, 'extrinsic', contributions to the heavy quark parton distribution functions (PDFs) are most important at low x and depend logarithmically on the heavy quark mass m_Q, the intrinsic heavy quark contributions are dominant at high x and depend on $1/m_Q^2$. Because the extrinsic heavy quarks are generated by gluon splitting, their PDFs are always softer than those of the parent gluon by a factor of $(1-x)$. In contrast, the high x intrinsic heavy quark contributions are kinematically dominated by the regime where the $|uudQ\overline{Q}\rangle$ state is minimally off shell, corresponding to equal rapidities of the constituent quarks, and are thus naturally at high x.

While the first experimental evidence of intrinsic heavy quarks came from the EMC measurement of the large x charm structure function [58], a variety of other charm hadron and charmonium measurements are consistent with the existence of intrinsic charm. Open charm observables in hadroproduction include forward Λ_c production at the ISR [59] and asymmetries between leading and non-leading charm ($\overline{D}$ mesons which share valence quarks with the projectile and D mesons which do not, respectively) measured as functions of x_F and p_T in fixed-target experiments, WA89 and WA82 at CERN; E791 and SELEX at Fermilab, see Refs. [60–62] and references therein. In addition, the NA3 collaboration measured double J/ψ production at forward x_F in πA interactions, difficult to explain without an intrinsic charm mechanism [63]. Finally, a recent calculation of the astrophysical neutrino rate with neutrinos from intrinsic charm improves agreement with the IceCube data [64].

The QCD wavefunction of a hadron can be represented as a superposition of quark and gluon Fock states. For example, at fixed light-cone time, a hadron wavefunction can be expanded as a sum over the complete basis of free quark and gluon states: $|\Psi_h\rangle = \sum_m |m\rangle\, \psi_{m/h}(x_i, k_{T,i})$ where the color-singlet states, $|m\rangle$, represent the fluctuations in the hadron wavefunction with the Fock components $|q_1q_2q_3\rangle$, $|q_1q_2q_3g\rangle$, $|q_1q_2q_3c\overline{c}\rangle$, *etc.* As predicted by Brodsky *et al.*, in the BHPS model intrinsic charm fluctuations [57, 65] can be liberated by a soft interaction which breaks the coherence of the Fock state [66], provided the system is probed during the lifetime of the fluctuations.

This contribution has typically not been included in the standard approach employed by almost all global analyses of PDFs. Instead the heavy quark distributions are generated radiatively, starting from a perturbative boundary condition at a scale of the order of the heavy quark mass. Thus, there are no free fit parameters associated to the heavy quark distribution since it is entirely related to the gluon distribution function at the scale of the boundary condition. However, analyses of the charm distribution in the proton going beyond the common assumption of purely radiatively generated charm date back almost as far as the BHPS predictions themselves. If radiatively generated charm is denoted by $c_0(x, Q)$ and intrinsic charm by $c_1(x, Q)$, the full charm parton distribution is then $c(x, Q) = c_0(x, Q) + c_1(x, Q)$, defined at the initial scale $Q_0 \simeq m_c$.

The BHPS model of the $|uudc\overline{c}\rangle$ Fock state predicts a simple form for $F_{2c}^{\rm IC}(x) = (8/9)xc_1(x)$,

$$F_{2c}^{\rm IC}(x) = (\frac{8}{9}x)\frac{1}{2}N_5 x^2[\frac{1}{3}(1-x)(1+10x+x^2) + 2x(1+x)\ln x] \ .$$

If there is a 1% intrinsic charm contribution to the proton PDF, $N_5 = 36$. Hoffman and Moore incorporated mass effects and introduced next-to-leading order corrections as well as scale evolution [67]. They compared their result to the EMC F_{2c} data from muon scattering on iron at $\overline{\nu} = \overline{Q^2}/2m_p\overline{x} = 53, 95,$ and 168 GeV with the intrinsic charm contribution added to the leading order perturbative calculation of F_{2c}. A complete next-to-leading order analysis of both the 'extrinsic' radiatively-generated charm component and the intrinsic component was later carried out by Harris *et al.* [68]. Given the quality of the data, no statement could be made about the intrinsic charm content of the proton when $\overline{\nu} = 53$ and 95 GeV. However, with $\overline{\nu} = 168$ GeV an intrinsic charm contribution of $(0.86 \pm 0.60)\%$ was indicated, consistent with those of Hoffman and Moore [67].

These early results only employed the EMC data to extract the IC contribution. They also all assumed that this contribution had the BHPS shape and also assumed that $c_1(x) = \overline{c}_1(x)$. Later global analyses incorporating IC also studied other shapes such as meson-cloud models where the 5-quark state could be decomposed into pairs of charm meson-baryon combinations such as $\overline{D}^0\Lambda_c^+$ [69]. In this case $c_1(x) \neq \overline{c}_1(x)$. Pumplin *et al.* also proposed a 'sea-like' IC with the same shape as the perturbative light quark sea. This type of IC would not result in enhanced charm production at high x, contrary to the BHPS and meson-cloud models. Several global analyses that include all data and all partons, not just charm, have been made. These analyses have mostly assumed either the BHPS distribution or sea-like IC [70, 71].

They characterized the magnitude of the IC component ($c_1(x, Q^2)$) by the first moment of the charm distribution at the input scale $Q_0 = m_c = 1.3$ GeV,

$$c_1(N = 1, Q_0^2) = \int_0^1 dx\, c_1(x, Q_0^2) \ . \tag{4}$$

(Note that at $Q_0 = m_c$ the radiatively generated charm component ($c_0(x, Q^2)$) vanishes at NLO in the $\overline{\rm MS}$ scheme so that $c(x, Q_0^2) = c_1(x, Q_0^2)$.) This translates into a momentum fraction carried by IC,

$$\langle x\rangle_{c_1+\overline{c}_1} = \int_0^1 dx\, x[c_1(x, Q_0^2) + \overline{c}_1(x, Q_0^2)] \ . \tag{5}$$

There are three recent updates to the global analyses, reaching different conclusions about the importance of intrinsic charm. The first, by Dulat *et al.* [72], follows the previous work in the context of the CTEQ collaboration [70, 71]. The second, by Jimenez-Delgado *et al.* [73], included more lower energy data than the previous global analyses but with a more restrictive fit tolerance. The third, by the NNPDF Collaboration [74], took a more novel approach to the shape of the charm distribution.

The result of Dulat *et al.* [72] was based on the CT10 NNLO parton densities. Here the strong coupling, $\alpha_S(Q^2)$, the evolution equations and the matrix elements are calculated at NNLO. Only the inclusive jet data still required NLO expressions. Their analysis included DIS data from BCDMS, NMC, CDHSW, and CCFR; SIDIS data from NuTeV and CCFR; the combined DIS and F_{2c} data from HERA; Drell-Yan production; the W charge asymmetry and Z^0 rapidity from CDF and D0; and the inclusive jet measurements from CDF and D0, see Ref. [72] for a complete list. Two models of IC were considered: the BHPS light cone model and the sea-like IC introduced in Ref. [70]. They found $\langle x \rangle_{\rm IC} = \langle x \rangle_{c_1+\bar{c}_1}(Q_0^2) \lesssim 0.025$ for the BHPS shape and $\langle x \rangle_{\rm IC} \lesssim 0.015$ for the sea-like IC. They also tested the sensitivity of their result to individual experiments by introducing a penalty factor for each experiment, designed to determine which data sets used in the global analyses are most sensitive to IC. They found that the upper limit on the BHPS value of $\langle x \rangle_{\rm IC}$ comes from the CCFR structure function data while the HERA combined charm data sets the upper limit on IC from the sea-like model [72].

The study by Jimenez-Delgado *et al.* [73] employed looser kinematic cuts, $Q^2 \geq 1$ GeV^2 and $W^2 \geq 3.5$ GeV^2. to include the full range of high energy scattering data. In particular, they included the lower energy SLAC fixed-target data which did not pass the more stringent standard DIS cuts on the (Q^2, W^2) plane applied in Refs. [70–72]. The EMC F_{2c} data were used as a consistency check. The low energy, high-x, fixed target data lie precisely in the region where IC is expected to be most important. Thus including these data could enhance the sensitivity of the global fit to IC. However, some of these data are on more massive nuclear targets than the deuteron and thus target mass corrections, nuclear corrections for $A > 2$, and higher-twist effects need to be taken into account [73]. They used all three intrinsic charm models previously considered: BHPS, the meson-cloud model (including pseudoscalar and vector mesons as well as spin 1/2 and spin 3/2 charm baryons), and the sea-like component [73]. The IC contribution was evolved up to NLO. They found that the total χ^2 is minimized for $\langle x \rangle_{\rm IC} = 0$ with $\langle x \rangle_{\rm IC} < 0.1\%$ at the 5σ level. The SLAC F_2 (large x), NMC cross sections (medium x) and HERA F_{2c} (low x) display the greatest sensitivity to IC. However, fits without the SLAC data still give a low IC contribution [73], in part due to the different tolerance criteria, $\Delta\chi^2 = 1$ for their fit and $\Delta\chi^2 = 100$ for Dulat *et al.* [72]. Increasing the tolerance to $\Delta\chi^2 = 100$ would accommodate $\langle x \rangle_{\rm IC} = 1\%$ at the 1σ level [73], consistent with Dulat *et al.* [72].

More recently, the NNPDF collaboration completed a global analysis that compared fits with only "perturbative charm" to what they termed "fitted charm", akin to intrinsic charm [74]. Contrary to other analyses, they made no particular assumption about the shape of the fitted charm. Instead, they used a neural network approach to find the best shape of this contribution. While the perturbative charm made no contribution at large x and had a narrow uncertainty band at low x, the fitted charm gave a finite contribution at large x, with a peak around $x \sim 0.5$. The uncertainty at $x < 0.01$ was very large, on the order of 100% at the initial scale of $Q = 1.65$ GeV. The effect of the fitted charm on the other parton densities was negligible. They included the same data as in Ref. [73]. They found that the fits to the inclusive HERA data, the Drell-Yan and fixed-target DIS data were sensitive to the fitted charm component. Indeed, including fitted charm improved the χ^2 of these data. Not surprisingly, the fits to the Tevatron and LHC data are neutral to the fitted charm contribution. Finally, they noted that the EMC F_{2c} data could only be described by the fitted charm contribution.

Given that there is still some controversy on this topic, it is important to collect further large-x data, particularly on F_{2c} to try and place greater confidence on the limits of IC in the nucleon. This would be an important measurement at the future electron-ion collider. The large x part of the charm distribution could be accessed by measuring D mesons at fixed-target energies at the LHC [75–78].

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