Rare B meson decays on the lattice

Andria Agadjanov[1,a]

[1] *Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*

Abstract. We discuss a framework for the measurement of the $B \to K^*$ transition form factors in lattice simulations, when the K^* eventually decays. The possible mixing of πK and ηK states is considered. We reproduce the two-channel analogue of the Lellouch-Lüscher formula, which allows one to extract the $B \to K^* l^+ l^-$ decay amplitude in the low-recoil region. Since the K^* is a resonance, we provide a procedure to determine the form factors at the complex pole position in a process-independent manner. The infinitely-narrow width approximation of the results is also studied.

1 Introduction

At present, several rare B meson decay modes are very promising in search for physics beyond the Standard Model (BSM). The $B \to K^* l^+ l^-$ is, in particular, regarded as one of the most important processes, since the polarization of the $K^*(892)$ resonance results in many observables. Recently, LCHb and Belle collaboration have observed an interesting pattern of deviations from the SM predictions in this mode [1–3]. Similar discrepancies have been seen concerning the branching ratios of $B_s \to \phi\mu^+\mu^-$ and $B \to K\mu^+\mu^-$ [4]. The possible explanations of these anomalies fall into one of two categories: BSM or the strong sector of SM, described by QCD. In our opinion, it is very important to have a precise knowledge of the *form factors* of the corresponding hadronic matrix elements, which enter the analysis of the experimental data. They contain theoretical uncertainties that should be quantified. Lattice QCD, which is well suited in the low recoil region, is the only first-principle method to tackle this problem. In particular, the calculations, based on the light cone sum rules suffer from relatively large uncertainties in this kinematical region [5, 6]. Hence, there is a strong interest in a direct lattice measurement of the form factors.

The first unquenched lattice QCD results on the $B \to K^*$ form factors have recently appeared [7–9]. The major drawback of these calculations is that values of the pion mass are unphysical. In this case, the K^* resonance is a stable particle and thus the standard lattice techniques can be used for the analysis of the data. When the K^* eventually decays into πK, the final-state meson interaction introduces a dependence of the current matrix elements on the volume. This finite-volume effect cannot be neglected, since it does not fall off exponentially, but only as a power of the volume. It has been first calculated by L. Lellouch and M. Lüscher, who considered the $K \to \pi\pi$ decay [10]. The developed method is a generalization of the Lüscher finite-volume approach [11]. The latter provides a framework to extract the elastic phase shifts and the resonance parameters (the mass and

[a] e-mail: aagadjanov@hiskp.uni-bonn.de

width) from the two-particle discrete energy levels spectrum, measured on the lattice. The Lellouch-Lüscher method has been subsequently generalized to include the baryons, the higher partial waves and multiple strongly-coupled decay channels [12–15].

Further, one should have a proper *definition* of the matrix elements involving resonances, such as K^* or Δ. Assuming the Breit-Wigner form of the resonance, the imaginary part of the transition amplitude is parametrized in terms of the current matrix elements [16, 17]. Such a definition of the form factors, however, yields a process- and model-dependent result, since the background is unknown. Recently, we have proposed an alternative, process-independent definition, in which the resonance matrix element is given as a residue of the respective three-point Green's function, taken at complex resonance pole [18, 19] (see also Ref. [20]). This is basically a generalization of the work by S. Mandelstam on the matrix elements between the bound states [21]. If the resonance width is not very small, using different definitions might have an effect on the extracted observables.

The present paper summarizes the findings of Ref. [22]. We provide, along the lines of the Lellouch-Lüscher method, a framework for the extraction of the $B \to K^*$ form factors on the lattice. The presence of the ηK threshold is taken into account. One could expect that the effect of this threshold might be seen in the lattice data. We also derive a formula to determine the form factors at the K^* pole in the two-channel case. We further show that the results are simplified in the limit of the infinitely narrow K^*. In particular, they have the same form as in the previously studied one-channel problem [19].

2 The $B \to K^*$ form factors on the lattice

The effective theory of the $b \to s$ transition is based on the weak Hamiltonian

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i W_i \,, \tag{1}$$

where G_F denotes the Fermi constant, V_{ts}, V_{tb} are elements of the CKM matrix and the C_i are Wilson coefficients. The seven $B \to K^*$ form factors are contained in the matrix elements of the W_7, W_9 and W_{10} quark bilinear operators (see below). We assume that the contributions of the four-quark operators to the decay amplitude are small in the low recoil region. Accordingly, the amplitude, extracted from lattice data, coincides approximately with the full one. This issue is, however, not settled in the community yet. In particular, the new experimental data should provide more details regarding the charm-resonance effects (see, e.g., Ref. [23]).

The lattice simulations are performed in a finite spatial volume. It is convenient to choose the irreducible representations, in which no partial wave mixing occurs. In the present case, these are S- and P- waves, whereas the neglect of D- and higher partial waves can be justified, if one stays below multi-particle thresholds. To that end, we consider the decay in the K^* rest frame:

$$\mathbf{k} = 0, \quad \mathbf{p} = \mathbf{q} = \frac{2\pi}{L}(0, 0, n), \qquad n \in \mathbb{Z}, \tag{2}$$

where L denotes the side length of the volume, $\mathcal{V} = L^3$. Here, $\mathbf{k}$ and $\mathbf{p}$ are the four-momenta of the K^* and B, respectively, and $\mathbf{q} = \mathbf{p} - \mathbf{k}$ is a momentum transfer to the lepton pair. When the K^* is not at rest, only some of the form factors can be extracted without mixing (for more details, see Ref. [22]).

Using the helicity formalism, we choose the current matrix elements in the form

$$\langle V(+)|J^{(+)}|B(p)\rangle = -\frac{2im_V|\mathbf{q}|V(q^2)}{m_B + m_V}, \tag{3}$$

Table 1. Extraction of matrix elements in the irreps without partial-wave mixing

Little group	Irrep	Form factor
C_{4v}	$\mathbb{E}$	V, A_1, T_1, T_2
	$\mathbb{A}_1$	A_0, A_{12}, T_{23}

$$\langle V(0)|i(E_B - m_V)J_A + |\mathbf{q}|J_A^{(0)}|B(p)\rangle = -2im_V|\mathbf{q}|A_0(q^2), \quad \text{etc.,} \tag{4}$$

where $E_B = \sqrt{m_B^2 + \mathbf{q}^2}$ is energy of the B meson, and $\langle V(+)|, \langle V(0)|$ are the state vectors with a positive circular and longitudinal polarizations, respectively,

$$\langle V(+)| = \frac{\langle V(1)| - i\langle V(2)|}{\sqrt{2}}, \quad \langle V(0)| = \langle V(3)|, \quad V(\lambda) \equiv V(\mathbf{0}, \lambda),\ \lambda = 1, 2, 3. \tag{5}$$

Here, the current operators are given by

$$J^{(\pm)} = \frac{1}{\sqrt{2}}\bar{s}(\gamma_1 \pm i\gamma_2)b, \quad J_A^{(0)} = \bar{s}\gamma_3\gamma_5 b, \quad J_A = \bar{s}\gamma_4\gamma_5 b. \tag{6}$$

The full set of expressions can be found in Ref. [22]. We denote below these current matrix elements shortly as F^M, $M = 1, ..., 7$.

In order to determine the values of the form factors at the K^* resonance pole, it is necessary to consider lattice simulations in asymmetric boxes (see Ref. [19]). These boxes, which are of the type $L \times L \times L'$, have the same symmetry properties as the symmetric ones boosted in the $\mathbf{d} = (0, 0, n)$ direction. In Table 1, the irreps of the corresponding little group, where the matrix elements should be measured, are listed. Also, the states $\langle V(\pm)|, \langle V(0)|$ are created by acting with the following local field operators, transforming according to these irreps, on the vacuum state $\langle 0|$:

$$O_{\mathbb{E}}^{(\pm)}(\mathbf{0}, t) = \frac{1}{\sqrt{2}}\sum_{\mathbf{x}}(O_1(\mathbf{x}, t) \mp iO_2(\mathbf{x}, t)), \quad O_{\mathbb{A}_1}^{(0)}(\mathbf{0}, t) = \sum_{\mathbf{x}} O_3(\mathbf{x}, t), \tag{7}$$

where $O_i(x)$ are spatial components of the vector field (see, e.g., Ref. [24]).

When the K^* becomes a resonance in lattice simulations, the matrix elements can still be measured. But now the mass m_V is replaced by the discrete total CM energy E_n of the n-th eigenstate of the strong Hamiltonian ($n = 0, 1, ...$). The current matrix elements F^M become functions of the E_n and $|\mathbf{q}|$: $F^M = F^M(E_n, |\mathbf{q}|)$.

3 Lellouch-Lüscher formula

Applying the Lellouch-Lüscher method to a given electroweak process can be conveniently carried out in the the non-relativistic effective field theory in a finite volume. We use the covariant version of the theory formulated in Refs. [25, 26]. This approach seems to be algebraically simpler than the one based on the Bethe-Salpeter (BS) equation (see, e.g., Refs. [27, 28]). The crucial point is that the results, obtained with these methods, are the same and do not depend on the form of the potential in the Lippmann-Schwinger equation or BS equation kernel. In other words, the low-energy constants of the effective Lagrangian will drop out in the final expressions.

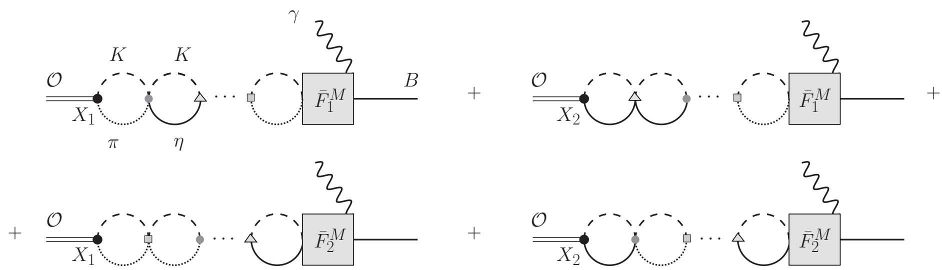

Figure 1. Diagrammatic representation of the $B \to K^*$ transition in a finite volume. The grey circle, square, and triangle depict different couplings in the $\pi K - \eta K$ system. The quantities X_1, X_2 are couplings of the K^* operator to the respective channels. The quantities $\bar{F}^M_\alpha(E, |\mathbf{q}|)$, $\alpha = 1, 2$, are volume-independent up to exponentially suppressed contributions

We start with a two-channel Lippmann-Schwinger equation in a finite volume

$$T_L = V + VG_LT_L. \tag{8}$$

Here, the potential V can be expicitly written in terms of three real parameters: the so-called eigenphases $\delta_1(p_1)$, $\delta_2(p_2)$ and mixing parameter $\varepsilon(E)$,

$$V = 8\pi\sqrt{s}\begin{pmatrix} \frac{1}{p_1}(t_1 + s_\varepsilon^2 t) & -\frac{1}{\sqrt{p_1p_2}}c_\varepsilon s_\varepsilon t \\ -\frac{1}{\sqrt{p_1p_2}}c_\varepsilon s_\varepsilon t & \frac{1}{p_2}(t_2 - s_\varepsilon^2 t)\end{pmatrix}, \tag{9}$$

where $t_\alpha \equiv \tan\delta_\alpha(p_\alpha)$, $\alpha = 1, 2$, $t = t_2 - t_1$, $c_\epsilon \equiv \cos\epsilon(E)$ and $s_\epsilon \equiv \sin\epsilon(E)$. Further, p_1 and p_2 denote the relative 3-momenta in the πK and ηK channels, respectively. They are related to the total energy E through the expressions

$$p_1^2 = \frac{\lambda(m_\pi^2, m_K^2, s)}{4s}, \quad p_2^2 = \frac{\lambda(m_\eta^2, m_K^2, s)}{4s}, \tag{10}$$

where $s = E^2$. Note that the potential V is the same as in the infinite volume up exponentially suppressed contributions. The non-trivial volume dependence is contained in the finite-volume counterpart of the loop integral,

$$G_L = \begin{pmatrix} -\frac{p_1}{8\pi\sqrt{s}}\cot\phi(p_1) & 0 \\ 0 & -\frac{p_2}{8\pi\sqrt{s}}\cot\phi(p_2)\end{pmatrix}, \tag{11}$$

where $\phi(p_\alpha)$, $\alpha = 1, 2$, are known functions that are related to the Lüscher zeta-function (see, e.g., Refs. [22, 24]). Further. the T_L-matrix has simple poles at $E = E_n$, where E_n are eigenvalues of the Hamiltonian in a finite volume,

$$T_L^{\alpha\beta} = \frac{f_\alpha f_\beta}{E_n - E} + \cdots . \tag{12}$$

Here, the quantities f_1, f_2 are given by

$$f_1^2 = \frac{8\pi\sqrt{s}}{p_1}\frac{\tau_1^2(t_2 + \tau_2 - s_\varepsilon^2 t)}{f'(E)}\bigg|_{E=E_n}, \quad f_2^2 = \frac{8\pi\sqrt{s}}{p_2}\frac{\tau_2^2(t_1 + \tau_1 + s_\varepsilon^2 t)}{f'(E)}\bigg|_{E=E_n}, \tag{13}$$

where $\tau_\alpha \equiv \tan\phi(p_\alpha)$, $f'(E) \equiv df(E)/dE$ and $f(E) \equiv (t_1+\tau_1)(t_2+\tau_2) + s_\varepsilon^2(t_2-t_1)(\tau_2-\tau_1)$. We note that the eigenvalues $E = E_n$ also satisfy the two-channel Lüscher equation [12, 29, 30],

$$(t_1+\tau_1)(t_2+\tau_2) + s_\varepsilon^2(t_2-t_1)(\tau_2-\tau_1)\Big|_{E=E_n} = 0, \tag{14}$$

which enables one to extract the scattering parameters $\delta_1(p_1)$, $\delta_2(p_2)$ and $\varepsilon(E)$ from lattice data.

The graphs, which contribute to the $B \to K^*$ transition matrix elements in a finite volume, are shown in Fig. 1. The quantities $\bar{F}^M_\alpha(E, |\mathbf{q}|)$, $\alpha = 1, 2$, denote the sum of all two-particle irreducible graphs in the respective channels. After summing up the bubble graphs, the final result takes a form

$$|F^M(E_n, |\mathbf{q}|)| = \frac{\mathcal{V}^{-1}}{8\pi E}\Big|p_1\tau_1^{-1}f_1\,\bar{F}^M_1 + p_2\tau_2^{-1}f_2\,\bar{F}^M_2\Big|\Big|_{E=E_n}. \tag{15}$$

The Eq. (15) is the two-channel analogue of the Lellouch-Lüscher formula (see also Refs. [12, 14]). It allows one to determine the quantities $\bar{F}^M_\alpha(E, |\mathbf{q}|)$, $\alpha = 1, 2$, which are related to the decay amplitudes $\mathcal{A}^M_1(B \to \pi K l^+ l^-)$ and $\mathcal{A}^M_2(B \to \eta K l^+ l^-)$ through the two-channel Watson theorem,

$$\mathcal{A}^M_1 = \frac{1}{\sqrt{p_1}}(u^M_1 c_\varepsilon e^{i\delta_1} - u^M_2 s_\varepsilon e^{i\delta_2}), \quad \mathcal{A}^M_2 = \frac{1}{\sqrt{p_2}}(u^M_2 c_\varepsilon e^{i\delta_2} + u^M_1 s_\varepsilon e^{i\delta_1}), \tag{16}$$

where

$$u^M_1 = (\sqrt{p_1}c_\varepsilon\bar{F}^M_1 + \sqrt{p_2}s_\varepsilon\bar{F}^M_2)\cos\delta_1, \quad u^M_2 = (\sqrt{p_2}c_\varepsilon\bar{F}^M_2 - \sqrt{p_1}s_\varepsilon\bar{F}^M_1)\cos\delta_2. \tag{17}$$

In comparison with the one-channel case, there is only one equation to determine two unknown variables $\bar{F}^M_1$, $\bar{F}^M_2$ and their relative sign. Accordingly, one needs at least three different measurements at the same energy, which involves the extraction of the excited energy levels (see Ref. [12]). The alternative options are discussed in Ref. [22].

4 Form factors at the K^* pole

The current matrix elements involving resonances have the proper field-theoretical meaning only if they are analytically continued to the resonance pole position. Before this procedure could be carried out, one should first locate the K^* resonance position. Assuming that the pole is located on the second Riemann sheet (II), the T-matrix in the infinite volume is given by

$$T_{II} = \frac{8\pi\sqrt{s}}{h(E)}\begin{pmatrix} \frac{1}{p_1}[t_1(1-it_2)+s_\varepsilon^2 t] & -\frac{1}{\sqrt{p_1p_2}}c_\varepsilon s_\varepsilon t \\ -\frac{1}{\sqrt{p_1p_2}}c_\varepsilon s_\varepsilon t & \frac{1}{p_2}[t_2(1+it_1)-s_\varepsilon^2 t] \end{pmatrix}, \tag{18}$$

where the quantity $h(E)$ is given by

$$h(E) \equiv (t_1 - i)(t_2 + i) + 2is_\varepsilon^2(t_2 - t_1). \tag{19}$$

The resonance pole position $E = E_R \equiv \sqrt{s_R}$ is obtained by solving the equation

$$h(E_R) = 0, \tag{20}$$

where the (modified) effective range expansion should be applied (see Ref. [22]).

Applying the non-relativistic effective field theory, three-point function in the infinite volume is given by

$$i\,\langle 0|T[O(x)J^M(0)]|B(p)\rangle = \int \frac{d^4P}{(2\pi)^4}\,e^{-iPx}\Gamma^M(P, p), \tag{21}$$

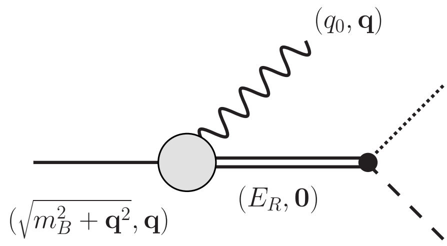

Figure 2. The factorization of the amplitudes at the resonance pole. The photon virtuality, given by Eq. (73), is complex

where $J^M(0)$, $M = 1, ..., 7$ denote the operators in the current matrix elements and $O(x)$ is a local operator with quantum numbers of the K^* that transforms according to the given irrep. Further, the quantity $\Gamma^M(P, p)$ in the frame $P^\mu = (P_0, \mathbf{0})$, $p^\mu = (\sqrt{m_B^2 + \mathbf{q}^2}, \mathbf{q})$ reads

$$\Gamma^M(P, p) = X^T [G_{II}(s) + G_{II}(s)T_{II}(s)G_{II}(s)]\bar{F}^M(P_0, |\mathbf{q}|). \tag{22}$$

Here the loop function $G_{II}(s)$ on the second Riemann sheet takes the form

$$G_{II}(s) = \begin{pmatrix} -\frac{ip_1}{8\pi\sqrt{s}} & 0 \\ 0 & \frac{ip_2}{8\pi\sqrt{s}} \end{pmatrix}. \tag{23}$$

Next, we *define* the current matrix elements at the resonance pole as

$$F_R^M = \lim_{P^2 \to s_R} Z_R^{-1/2}(s_R - P^2)\,\Gamma(P, p). \tag{24}$$

where Z_R is the (complex) wave-function renormalization constant of the resonance,

$$Z_R = -\frac{1}{64\pi^2 E_R^2}\Big[\sum_{\alpha=1}^{2}(-1)^\alpha X_\alpha p_\alpha(E_R)h_\alpha(E_R)\Big]^2. \tag{25}$$

Here, the quantities h_1, h_2 are given by

$$h_1^2 = -\frac{8\pi\sqrt{s}}{p_1}\frac{2E(t_2 + i - s_\varepsilon^2 t)}{h'(E)}\Big|_{E=E_R}, \quad h_2^2 = -\frac{8\pi\sqrt{s}}{p_2}\frac{2E(t_1 - i + s_\varepsilon^2 t)}{h'(E)}\Big|_{E=E_R}, \tag{26}$$

where $h'(E) \equiv dh(E)/dE$. Separating the pole contribution $s = s_R$ in Eqs. (22), one finally obtains

$$F_R^M(E_R, |\mathbf{q}|) = -\frac{i}{8\pi E}(p_1 h_1 \bar{F}_1^M - p_2 h_2 \bar{F}_2^M)\Big|_{E=E_R}. \tag{27}$$

The corresponding form factors can be read off from the expressions for the current matrix elements, in which the kinematic factors are low-energy polynomials.

In practice, the pole extraction of the form factors proceeds as follows. The finite-volume matrix element is measured at different two-particle energies $E_n(L)$ and a fixed value of $|\mathbf{q}|$. After that, an analytic continuation is performed to the complex resonance pole, keeping $|\mathbf{q}|$ fixed. Note that the photon virtuality becomes complex at the pole

$$q^2 = \left(E_R - \sqrt{m_B^2 + \mathbf{q}^2}\right)^2 - \mathbf{q}^2 . \tag{28}$$

In fact, the residue of the full amplitude at the pole should factorize in the product of the resonance form factor and the vertex, describing the transition of a resonance into the final state, see Fig. 2. The background becomes irrelevant, which leads to the determination of the form factor at the pole in a process-independent manner. From this figure it is clear that the photon virtuality, defined through the use of the 4-momentum conservation, coincides with the one given in Eq. (28) and thus must be complex.

5 Infinitely narrow width

The results presented above simplify considerably in the limit case of a K^* resonance with an infinitely narrow width. Here, we have in mind the hypothetical situation, when the pole is located above the ηK threshold. We have previously considered this limit in study of the $\Delta N\gamma^*$ transition [19]. The multi-channel case is, however, more subtle, since the relations between the infinite- and finite-volume matrix elements become obscure. Nevertheless, the final results have exactly the same form as in the one-channel problem.

First, we suppose that the resonance behavior near the Breit-Wigner pole $E = E_{BW}$ emerges in the quantity $t_1 = \tan\delta_1$, whereas the quantity t_2 stays regular in this energy interval:

$$\cot\delta_1(E) = \frac{E_{BW} - E}{\Gamma/2}. \tag{29}$$

where Γ denotes the width of the narrow resonance. The scattering amplitude T on the first Riemann sheet takes the form

$$T_{\alpha\beta} = \frac{b_\alpha b_\beta}{s_{BW} - s - i\sqrt{s_{BW}}\,\Gamma} + \text{regular terms at } E \to E_{BW} , \tag{30}$$

where $s_{BW} = E_{BW}^2$ and the regular terms emerge from the contribution of t_2. Here, the quantities b_1, b_2 are given by

$$b_1 = \sqrt{\frac{8\pi s_{BW}\Gamma}{p_1}}\, c_\varepsilon , \quad b_2 = \sqrt{\frac{8\pi s_{BW}\Gamma}{p_2}}\, s_\varepsilon . \tag{31}$$

Further, it is possible introduce the infinite-volume quantities ("form factors"), which parameterize the imaginary parts of the decays amplitudes $\mathcal{A}_1^M$, $\mathcal{A}_2^M$ in the vicinity of the Breit-Wigner resonance (see Fig. 3). Denoting them as $F_A^M(E, |\mathbf{q}|)$, one obtains

$$\mathcal{A}_\alpha^M(E, |\mathbf{q}|) = \frac{b_\alpha F_A^M(E_{BW}, |\mathbf{q}|)}{E_{BW}^2 - E^2 - iE_{BW}\Gamma} + \cdots , \quad \alpha = 1, 2, \tag{32}$$

where the ellipses stand for the terms emerging from the regular contributions in Eq. (30).

On the real energy axis, taking the limit $\Gamma \to 0$ in the Lellouch-Lüscher formula, Eq.15, leads to a simple result (see Ref. [22])

$$\left|F^M(E_n, |\mathbf{q}|)\right| = \frac{\mathcal{V}^{-1}}{\sqrt{2E_n}} \left|F_A^M(E_n, |\mathbf{q}|)\right| + O(\Gamma^{1/2}), \quad E_n = E_{BW} + O(\Gamma). \tag{33}$$

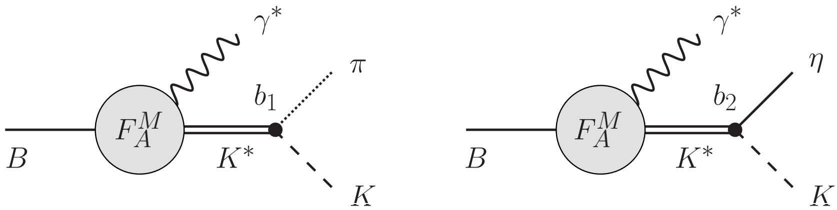

Figure 3. Decay amplitudes $\mathcal{A}_\alpha^M$, $\alpha = 1, 2$, in the vicinity of the infinitely narrow K^*. The quantities b_α, $\alpha = 1, 2$, denote the couplings of the K^* to the respective channels at $E = E_{BW}$.

As seen, in the vicinity of the Breit-Wigner resonance, the infinite-volume quantities $F_A^M(E_{BW}, |\mathbf{q}|)$ are equal to the current matrix elements $F^M(E_{BW}, |\mathbf{q}|)$, measured on the lattice , up to a known normalization factor.

The formula Eq. (27), which determines the values of the form factors at the resonance pole, is simplified in the infinitely narrow width limit as well. One gets

$$F_R^M(E_R, |\mathbf{q}|)\big|_{\Gamma\to 0} = F_A^M(E_{BW}, |\mathbf{q}|) + O(\Gamma^{1/2}). \tag{34}$$

As expected, for infinitely narrow resonance, the form factors $F_A^M(E, |\mathbf{q}|)$ and $F_R^M(E, |\mathbf{q}|)$, defined on the real energy axis and complex plane, respectively, coincide.

6 Summary

The present work summarizes the results of Ref. [22], in which we have formulated a theoretical framework for the extraction of the $B \to K^*$ form factors on the lattice. The calculations have been done conveniently in the non-relativistic effective field theory. By taking into account the possible admixture of the ηK to πK final states, we have reproduced the two-channel analogue of the Lellouch-Lüscher formula. This result enables one to extract the $B \to K^* l^+ l^-$ decay amplitude in the low-recoil region.

Due to the resonance nature of the K^*, we have given a field-theoretical definition of the current matrix elements, which is free of process-dependent ambiguities. It implies an analytic continuation in the complex energy plane to the complex resonance pole position. Accordingly, we have derived the formula for the determination of the $B \to K^*$ form factors at the K^* pole.

Finally, we have showed that the results are considerably simplified in the vicinity of the infinitely narrow resonance. This limit is more involved in the multi-channel case than in the previously considered one-channel problem. Still, even in the multi-channel case, the current matrix elements measured on the lattice are equal to the ones in the infinite volume, up to a normalization factor that does not depend on the dynamics.

Acknowledgements

We thank F. K. Guo and J. A. Oller for useful discussions. We acknowledge the support by the DFG (CRC 16, "Subnuclear Structure of Matter" and CRC 110 "Symmetries and the Emergence of Structure in QCD") and by the Bonn-Cologne Graduate School of Physics and Astronomy. This research is supported in part by Volkswagenstiftung under contract no. 86260.

References

[1] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **111**, 191801 (2013)
[2] R. Aaij *et al.* [LHCb Collaboration], JHEP **1602**, 104 (2016)
[3] A. Abdesselam *et al.* [Belle Collaboration], arXiv:1604.04042 [hep-ex].
[4] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **113**, 151601 (2014)
[5] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014029 (2005)
[6] A. Bharucha, D. M. Straub and R. Zwicky, JHEP **1608**, 098 (2016)
[7] Z. Liu, S. Meinel, A. Hart, R. R. Horgan, E. H. Müller and M. Wingate, arXiv:1101.2726 [hep-ph]
[8] R. R. Horgan, Z. Liu, S. Meinel and M. Wingate, Phys. Rev. D **89**, 094501 (2014)
[9] R. R. Horgan, Z. Liu, S. Meinel and M. Wingate, PoS LATTICE **2014**, 372 (2015)
[10] L. Lellouch and M. Lüscher, Commun. Math. Phys. **219**, 31 (2001)
[11] M. Lüscher, Nucl. Phys. B **354**, 531 (1991)
[12] M. T. Hansen and S. R. Sharpe, Phys. Rev. D **86**, 016007 (2012)
[13] R. A. Briceño and Z. Davoudi, Phys. Rev. D **88**, 094507 (2013)
[14] R. A. Briceño, M. T. Hansen and A. Walker-Loud, Phys. Rev. D **91**, 034501 (2015)
[15] R. A. Briceño and M. T. Hansen, Phys. Rev. D **92**, 074509 (2015)
[16] I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, arXiv:0810.0997 [nucl-th]
[17] D. Drechsel, O. Hanstein, S. S. Kamalov and L. Tiator, Nucl. Phys. A **645**, 145 (1999)
[18] V. Bernard, D. Hoja, U.-G. Meißner and A. Rusetsky, JHEP **1209**, 023 (2012)
[19] A. Agadjanov, V. Bernard, U.-G. Meißner and A. Rusetsky, Nucl. Phys. B **886**, 1199 (2014)
[20] M. Albaladejo and J. A. Oller, Phys. Rev. D **86**, 034003 (2012)
[21] S. Mandelstam, Proc. Roy. Soc. Lond. A **233**, 248 (1955)
[22] A. Agadjanov, V. Bernard, U.-G. Meißner and A. Rusetsky, Nucl. Phys. B **910**, 387 (2016)
[23] J. Lyon and R. Zwicky, arXiv:1406.0566 [hep-ph]
[24] M. Göckeler, R. Horsley, M. Lage, U.-G. Meißner, P. E. L. Rakow, A. Rusetsky, G. Schierholz and J. M. Zanotti, Phys. Rev. D **86**, 094513 (2012)
[25] G. Colangelo, J. Gasser, B. Kubis and A. Rusetsky, Phys. Lett. B **638**, 187 (2006)
[26] J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B **850**, 96 (2011)
[27] C. H. Kim, C. T. Sachrajda and S. R. Sharpe, Nucl. Phys. B **727**, 218 (2005)
[28] N. H. Christ, C. H. Kim and T. Yamazaki, Phys. Rev. D **72**, 114506 (2005)
[29] S. He, X. Feng and C. Liu, JHEP **0507**, 011 (2005)
[30] C. Liu, X. Feng and S. He, Int. J. Mod. Phys. A **21**, 847 (2006)