

Molecular partners of the X(3872) from heavy-quark spin symmetry: a fresh look

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Abstract. The heavy-quark spin symmetry (HQSS) partners of the X(3872) molecule are investigated in a chiral effective field theory (EFT) approach which incorporates contact and one-pion exchange interactions. The integral equations of the Lippmann-Schwinger type are formulated and solved for the coupled-channel problem for the $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ systems with the quantum numbers $J^{PC} = 1^{++}, 1^{+-}, 0^{++}$, and 2^{++} . We confirm that, if the X(3872) is a $1^{++} D\bar{D}^*$ molecular state then, in the strict heavy-quark limit, there exist three partner states, with the quantum numbers $1^{+-}, 0^{++}$, and 2^{++} , which are degenerate in mass. At first glance, this result looks natural only for the purely contact pionless theory since pions contribute differently to different transition potentials and, therefore, may lift the above degeneracy. Nevertheless, it is shown that, by an appropriate unitary transformation, the Lippmann-Schwinger equation in each channel still can be brought to a block-diagonal form, with the same blocks for all quantum numbers, so that the degeneracy of the bound states in different channels is preserved. We stress that neglecting some of the coupled-channel transitions in an inconsistent manner leads to a severe violation of HQSS and yields regulator-dependent results for the partner states. The effect of HQSS violation in combination with nonperturbative pion dynamics on the pole positions of the partner states is discussed.

1 Introduction

The nonperturbative dynamics of light quarks in QCD is associated with the intrinsic scale $\Lambda_{\text{QCD}} \sim 250$ MeV. This scale is significantly smaller than the mass (m_Q) of a heavy (c or b) quark which makes it possible to describe heavy-hadron processes perturbatively by classifying the operators according to the powers of the ratio Λ_{QCD}/m_Q . Then, in the limit $m_Q \rightarrow \infty$, which is the leading order (LO) of this expansion, the terms in the Hamiltonian which depend on the heavy-quark spin vanish, resulting this way in the phenomenon known as Heavy-Quark Spin Symmetry (HQSS)—see, e.g., Ref. [1]. As a consequence, for an experimentally observed state with the given spin, its spin partners should exist as

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well. In a recent study [2], it was emphasized that the implications of HQSS for the X(3872), such as the number of its HQSS partners, their location and decay properties, depend drastically on the nature of this state. In this contribution, we analyze the implications of HQSS under the assumption that the X(3872) is a weakly bound $\bar{D}D^*$ molecule.¹ In the molecular picture a relation between the poles of the molecular partner states of the isovector resonances $Z_b^+(10610)$ and $Z_b^+(10650)$ was derived in Refs. [4–6]. Similarly, it was argued in Refs. [7, 8] that one should expect a shallow molecular partner state for the X(3872) in the $D^*\bar{D}^*$ channel with the quantum numbers $J^{PC} = 2^{++}$ while, in Ref. [9], the width of this state was estimated to be as small as a few MeV using an effective field theory with perturbative pions (X-EFT).

In the current study we take a fresh look at the predictions for the partner states for the X(3872) using an EFT consisting of the contact and one-pion exchange (OPE) interactions iterated to all orders, see Ref. [10] for further details. In particular, we demonstrate that non-perturbative pion dynamics together with the leading HQSS breaking corrections result in a significant shift of the 2^{++} partner binding energy and make it as broad as 50 MeV.

The contribution is organized as follows. Following Refs. [6, 8], in Sec. 2 we start from a contact theory, where analytic predictions for the molecular partner states are possible. We consider first the strict HQSS limit ($m = m_*$) to demonstrate analytically that in this case the X(3872) has three degenerate molecular partners. Then we investigate the impact of the leading HQSS breaking corrections on the results. In Sec. 3 we include in addition the leading-order OPE potential in the strict HQSS limit and beyond to study whether the conclusions drawn in Sec. 2 survive. The contribution is ended with a brief summary.

2 Pionless (contact) theory

2.1 Strict heavy-quark limit

At leading order (LO) the contact Hamiltonian describing the interactions between the D and D^* mesons contains derivativeless interactions only [11]. Hence, only S -wave interactions survive at this order and the relevant basis states composed of D and D^* mesons read

$$\begin{aligned} 0^{++} : & \quad \{D\bar{D}(^1S_0), D^*\bar{D}^*(^1S_0)\}, & 1^{+-} : & \quad \{D\bar{D}^*(^3S_1, -), D^*\bar{D}(^3S_1)\}, \\ 1^{++} : & \quad \{D\bar{D}^*(^3S_1, +)\}, & 2^{++} : & \quad \{D^*\bar{D}^*(^5S_2)\}, \end{aligned} \quad (1)$$

where the individual partial waves are labelled as $^{2S+1}L_J$ with S , L , and J denoting the total spin, the angular momentum, and the total momentum of the two-meson system, respectively. Furthermore, the C-parity eigenstates are defined as $D\bar{D}^*(\pm) = \frac{1}{\sqrt{2}}(D\bar{D}^* \pm D^*\bar{D})$. In this basis and for a given set of quantum numbers $\{JPC\}$, the leading-order EFT potentials $V_{\text{LO}}^{(JPC)}$, which respect heavy-quark spin symmetry, read [7, 11, 12]

$$V_{\text{LO}}^{(0^{++})} = \begin{pmatrix} C_{0a} & -\sqrt{3}C_{0b} \\ -\sqrt{3}C_{0b} & C_{0a} - 2C_{0b} \end{pmatrix}, \quad V_{\text{LO}}^{(1^{+-})} = \begin{pmatrix} C_{0a} - C_{0b} & 2C_{0b} \\ 2C_{0b} & C_{0a} - C_{0b} \end{pmatrix}, \quad (2)$$

$$V_{\text{LO}}^{(1^{++})} = V_{\text{LO}}^{(2^{++})} = C_{0a} + C_{0b}, \quad (3)$$

where C_{0a} and C_{0b} are two independent low-energy constants.

The integral equations for the scattering amplitude can be written as

$$a_{ik}^{(JPC)}(p, p') = V_{ik}^{(JPC)}(p, p') - \sum_j \int dk k^2 V_{ij}^{(JPC)}(p, k) G_j(k) a_{jk}^{(JPC)}(k, p'), \quad (4)$$

¹The possible impact of genuine quarkonium states on the formation of the molecular spin multiplets is addressed in Ref. [3].

where i, j and k label the basis vectors in the order they appear in Eq. (1). Here $G_j(k)$ denotes the matrix of the propagators of the heavy meson-antimeson pair in the intermediate state. For coupled channels (see Eq. (2)) $G(k)$ is a 2×2 diagonal matrix with both nonzero elements given by $G(k) = (k^2/\bar{m} - E - i0)^{-1}$ while in the single-channel case (see Eq. (3)) $G_j = G(k)$. Here we used that in the strict HQSS limit the D^* - and D -meson masses m_* and m , respectively, coincide, $\bar{m} = m_* = m$. The poles appear at the energies where the inverse amplitude, $a_{ik}^{(JPC)}(p, p')^{-1}$, vanishes. For the quantum numbers 1^{++} and 2^{++} , the pole position of the 2^{++} state coincides with that of the $X(3872)$, since the corresponding potentials are the same.

As shown in Refs. [4–6, 10, 13, 14], in the heavy-quark limit, one can predict more states with the same binding energy. To this end, for both quantum numbers 0^{++} and 1^{+-} , one can apply a unitary transformation to the coupled-channel equations (4) to arrive at the diagonalised potential

$$\tilde{V}^{(JPC)} = UV^{(JPC)}U^\dagger = \begin{pmatrix} C_0 & 0 \\ 0 & C'_0 \end{pmatrix}, \quad U = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}, \quad (5)$$

where $C_0 = C_{0a} + C_{0b}$, $C'_0 = C_{0a} - 3C_{0b}$, and $\phi = -\pi/6$ ($\phi = \pi/4$) for the 0^{++} (1^{+-}) potential. Therefore, the poles in both channels are now determined from the equation $\det \left[1 + \int dk k^2 \tilde{V}^{(JPC)} G(k) \right] = 0$, where the propagator matrix is unchanged under rotations, $UG(k)U^\dagger = G(k)$. This equation has two solutions corresponding to the two different linear combinations of the low-energy constants; one of them, C_0 , is the same for all quantum numbers. Therefore the coupled-channel problem defined by Eqs. (1)-(4) in the strict HQSS limit splits into disentangled equations which possess two decoupled solutions,

$$E_X^{(0)} = E_{X_2}^{(0)} = E_{X_1}^{(0)} = E_{X_0}^{(0)} \quad \text{and} \quad E_{X'_0}^{(0)} = E_{X'_1}^{(0)}, \quad (6)$$

where $E_{X_0}^{(0)}$, $E_{X_1}^{(0)}$, and $E_{X_2}^{(0)}$ stand for the binding energies of the spin-0, spin-1, and spin-2 partners of the $X(3872)$ in the strict heavy-quark limit, respectively, defined by the combination of the low-energy constants C_0 while $E_{X'_0}^{(0)}$ and $E_{X'_1}^{(0)}$ label the binding energies of the two additional partner states defined by the low-energy constant C'_0 —see Eq. (5). These findings are in line with those in Ref. [13].

2.2 Partner states beyond strict HQSS limit

Corrections to the HQSS limit at leading order in Λ_{QCD}/m_c give rise to the D^*-D mass splitting $\delta = m_* - m = 141$ MeV which is to be compared to the average mass in the multiplet $\bar{m} = (3m_* + m)/4 = 1973$ MeV.

Since the contact interactions in the 1^{++} and 2^{++} channels coincide exactly (see Eq. (3)), it can be shown (see Ref. [10] for details) that the binding momenta for these states are related as

$$\gamma_{X_2} = \left(1 - \frac{\delta}{2\bar{m}} \right) \gamma_X + \frac{\delta}{\pi\bar{m}} \Lambda + \dots, \quad (7)$$

where the sharp cutoff Λ was introduced to regularise the divergent integrals and the ellipsis stands for the small corrections of higher order. Rewriting this in terms of the masses of these states, M_X and M_{X_2} , one gets

$$M_{X_2} = 2m_* + \frac{\gamma_{X_2}^2}{m_*} = M_X + \delta + \frac{2\gamma_X \Lambda}{\pi\bar{m}^2} \delta + \dots \quad (8)$$

We therefore see that at order $O(\delta)$ an additional counter term is necessary to render the result for the mass of the $J = 2$ partner of the $X(3872)$ well defined. Nevertheless, the leading effect from HQSS

breaking reveals itself in bringing each spin partner state close to the corresponding heavy meson-antimeson threshold ($D\bar{D}$, $D\bar{D}^*$, or $D^*\bar{D}^*$). Indeed, the second term on the r.h.s. of Eq. (8) lifts the degeneracy between the 1^{++} and 2^{++} states by shifting the latter state close to the $D^*\bar{D}^*$ threshold, while the third (cutoff-dependent) contribution is strongly suppressed, at least as γ_X/\bar{m} . As will be discussed later, the coupled-channel effects governed by OPE introduce new soft scales much larger than γ_X thus leading to a larger cutoff dependence of the results and in a more important role of the HQSS breaking contact interaction.

Since the contact terms C_0 and C'_0 are mixed in the integral equations beyond the HQSS limit, predictions for the other partner states with $J^{PC} = 1^{+-}$ and 0^{++} are possible only if both contact terms are extracted from experimental data, which requires two experimental quantities as input. Assuming that the mass of the 1^{+-} partner state near the $D\bar{D}^*$ threshold, hereinafter referred to as X_1 , is known, for the binding momentum of its heavier ($D^*\bar{D}^*$) partner (X'_1) one gets

$$\gamma_{X'_1} = \left(1 - \frac{\delta}{2\bar{m}}\right)\gamma_{X_1} + \frac{\Lambda\delta}{\pi\bar{m}} - \frac{(\gamma_{X_1} - \gamma_X)^2}{\sqrt{\bar{m}\delta}} + i\frac{(\gamma_{X_1} - \gamma_X)^2}{\sqrt{\bar{m}\delta}} + \dots \quad (9)$$

Due to the mixing of two contact terms, the binding energy of the 1^{+-} $D^*\bar{D}^*$ state X'_1 depends now on both input parameters γ_X and γ_{X_1} . It should be noted that the binding momentum $\gamma_{X'_1}$ acquires an imaginary part and so does the binding momentum $\gamma_{X'_0}$ of the X'_0 state (the spin-0 state near the $D^*\bar{D}^*$ threshold). This is a reflection of the fact that beyond the strict HQSS limit in the systems with the quantum numbers 0^{++} and 1^{+-} transitions $D^*\bar{D}^* \rightarrow D\bar{D}^{(*)}$ are possible due to coupled channels already in the pionless theory.

3 Contact + OPE interactions

3.1 Strict HQSS limit

Unlike the leading-order contact interactions, as long as OPE is included, transitions to the heavy-meson states in higher partial waves emerge in addition to the S -wave contributions. The extended set of the basis states therefore reads

$$\begin{aligned} 0^{++} &: \{D\bar{D}(^1S_0), D^*\bar{D}^*(^1S_0), D^*\bar{D}^*(^5D_0)\}, \\ 1^{+-} &: \{D\bar{D}^*(^3S_1, -), D\bar{D}^*(^3D_1, -), D^*\bar{D}^*(^3S_1), D^*\bar{D}^*(^3D_1)\}, \\ 1^{++} &: \{D\bar{D}^*(^3S_1, +), D\bar{D}^*(^3D_1, +), D^*\bar{D}^*(^5D_1)\}, \\ 2^{++} &: \{D\bar{D}(^1D_2), D\bar{D}^*(^3D_2), D^*\bar{D}^*(^5D_2), D^*\bar{D}^*(^1D_2), D^*\bar{D}^*(^5D_2), D^*\bar{D}^*(^5G_2)\}, \end{aligned} \quad (10)$$

where, as before, the C-parity of the state is indicated explicitly in parentheses whenever necessary.

The leading-order OPE potentials connecting the isospin-0 $D^*D^{(*)}$ states in the initial and final states read

$$V_{D\bar{D} \rightarrow D^*\bar{D}^*}(\mathbf{p}, \mathbf{p}') = -C \frac{(\boldsymbol{\epsilon}'_1 \cdot \mathbf{q})(\boldsymbol{\epsilon}'_2 \cdot \mathbf{q})}{q^2 + m_\pi^2}; \quad V_{D\bar{D}^* \rightarrow D\bar{D}^*}(\mathbf{p}, \mathbf{p}') = -C \frac{(\boldsymbol{\epsilon}_1 \cdot \mathbf{q})(\boldsymbol{\epsilon}'_2 \cdot \mathbf{q})}{q^2 + m_\pi^2} \quad (11)$$

$$V_{D\bar{D}^* \rightarrow D^*\bar{D}^*}(\mathbf{p}, \mathbf{p}') = \sqrt{2}C \frac{(\mathbf{A}_1 \cdot \mathbf{q})(\boldsymbol{\epsilon}_2 \cdot \mathbf{q})}{q^2 + m_\pi^2}; \quad V_{D^*\bar{D}^* \rightarrow D^*\bar{D}^*}(\mathbf{p}, \mathbf{p}') = -2C \frac{(\mathbf{A}_1 \cdot \mathbf{q})(\mathbf{A}_2 \cdot \mathbf{q})}{q^2 + m_\pi^2} \quad (12)$$

where $C = 6g_c^2/(4\pi f_\pi)^2$, $\mathbf{q} = \mathbf{p} + \mathbf{p}'$, $\boldsymbol{\epsilon}_j$ and $\boldsymbol{\epsilon}'_j$ stand for the polarization vectors of the initial and final D^* particles, and $\mathbf{A}_j = \frac{1}{\sqrt{2}}[\boldsymbol{\epsilon}_j \times \boldsymbol{\epsilon}'_j]$ with $j = 1, 2$. To arrive at these results, we used that $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ gives

a factors of 3 for isospin-0 and that the $D^*D\pi$ and $D^*D^*\pi$ vertices are

$$v^a(D^* \rightarrow D\pi) = \frac{g_c}{2f_\pi} \tau^a(\boldsymbol{\epsilon} \cdot \mathbf{q}), \quad v^a(D^* \rightarrow D^*\pi) = -\frac{\sqrt{2}g_c}{2f_\pi} \tau^a(\mathbf{A} \cdot \mathbf{q}), \quad (13)$$

with $g_c = 0.57$ and $f_\pi = 92.2$ MeV. The partial-wave-projected OPE potentials entering the integral equations (4) for the given (JPC) are obtained as

$$V_{ik}(p, p') = \frac{1}{2J+1} \int \frac{d\Omega_p}{4\pi} \frac{d\Omega_{p'}}{4\pi} \text{Tr}\{P^\dagger(JLS; \mathbf{n})V_{ik}(\mathbf{p}, \mathbf{p}')P(JL'S'; \mathbf{n}')\}, \quad (14)$$

where i, k label the basis vectors in the order they appear in Eq. (10), $\mathbf{n} = \mathbf{p}/p$ ($\mathbf{n}' = \mathbf{p}'/p'$), and the list of relevant projection operators is given in the Appendix. When solving the integral equations the partial-wave-projected OPE potentials are supplemented by the leading-order S -wave contact interactions considered in the previous section.

As already discussed above, OPE allows for transitions to heavy-meson states in D and even in G waves which have therefore to be included into the extended set of basis states, see Ref. [10] for more details. Nevertheless, similarly to the contact case, with the help of unitary transformations, the corresponding potentials can be brought to the block-diagonal form,

$$\begin{aligned} \tilde{V}^{(0^{++})}(3 \times 3) &= A(2 \times 2) \oplus B(1 \times 1), \\ \tilde{V}^{(1^{+-})}(4 \times 4) &= A(2 \times 2) \oplus B(1 \times 1) \oplus C(1 \times 1), \\ \tilde{V}^{(1^{++})}(3 \times 3) &= A(2 \times 2) \oplus D(1 \times 1), \\ \tilde{V}^{(2^{++})}(6 \times 6) &= A(2 \times 2) \oplus D(1 \times 1) \oplus E(3 \times 3). \end{aligned} \quad (15)$$

The OPE contributes to all five submatrices, A , B , C , D , and E , while the contact interaction contributes only to matrix A (in the form of the linear combination $C_0 = C_{0a} + C_{0b}$) and to matrix B (as the linear combination $C'_0 = C_{0a} - 3C_{0b}$). Since matrix A enters all four potentials in Eq. (15) simultaneously, the degenerate bound states controlled by the contact potential C_0 appear in all four channels as before and, again as before, two additional degenerate bound states from the contact interaction C'_0 may exist in the channels 0^{++} and 1^{+-} .

It should be stressed that the irreducible decomposition discussed above leads to degenerate states only if the basis vectors are included consistently. Specifically, neglecting D waves can only be done altogether in all particle channels, while any inconsistent inclusion of the D waves or neglecting the particle coupled-channel dynamics, as advocated e.g. in Ref. [7], leads to cutoff-dependent results and therefore to a HQSS violation already in the strict HQ limit. This message is illustrated in Fig. 1, where using the $X(3872)$ as input, the binding energy of its 2^{++} spin partner is shown to be strongly cutoff dependent, as long as the coupled-channel transitions in the potentials $DD\bar{D}^* \rightarrow DD\bar{D}^*$ and $D^*\bar{D}^* \rightarrow D^*\bar{D}^*$ driven by the OPE interaction are neglected.

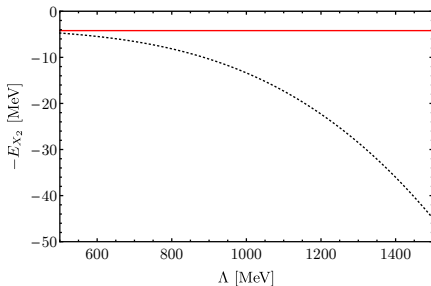


Figure 1. The binding energy of the 2^{++} spin partner of the $X(3872)$ in the strict heavy-quark limit: Red solid line—all coupled-channel transitions are included, $E_{X_2} = E_X = 4.2$ MeV; black dashed line—particle coupled-channel transitions in the potentials $DD\bar{D}^* \rightarrow DD\bar{D}^*$ and $D^*\bar{D}^* \rightarrow D^*\bar{D}^*$ driven by the OPE interaction are neglected.

3.2 Beyond HQSS limit

As already discussed in Sec. 2.2, due to HQSS breaking the degeneracy of the states found in the strict HQSS limit is lifted and the molecular states reside in the vicinity of their respective thresholds. Furthermore, using the $X(3872)$ as the single input quantity, beyond the HQSS limit predictions are possible only for the $2^{++} D^* \bar{D}^*$ molecular partner while the partner states in the other channels require one more experimental input [7, 10].

The probably most spectacular new effect that comes into the system when both OPE and the leading corrections in δ are taken into account simultaneously is the finite width of the $2^{++} D^* \bar{D}^*$ state that can now decay into the $D\bar{D}$ and $D\bar{D}^*$ pairs in D waves. Within a theory with perturbative pions, in Ref. [9], the width of the 2^{++} state was estimated to lie in the range from just a few units to about a dozen MeV, depending on a particular model used for the pion form factor—see Table I of Ref. [9]. However, we expect the nonperturbative pion dynamics to be especially relevant for the transitions at hand since the momenta of the $D\bar{D}$ and $D\bar{D}^*$ pairs, q_1 and q_2 , are much larger than the other soft scales in the problem: $q_1 = \sqrt{2\mu(2\delta)} \approx 700$ MeV and $q_2 = \sqrt{2\mu_*\delta} \approx 500$ MeV. A direct consequence of this is that D waves fed by the OPE are not subject to a kinematic suppression relative to the S waves and therefore also require resummation.

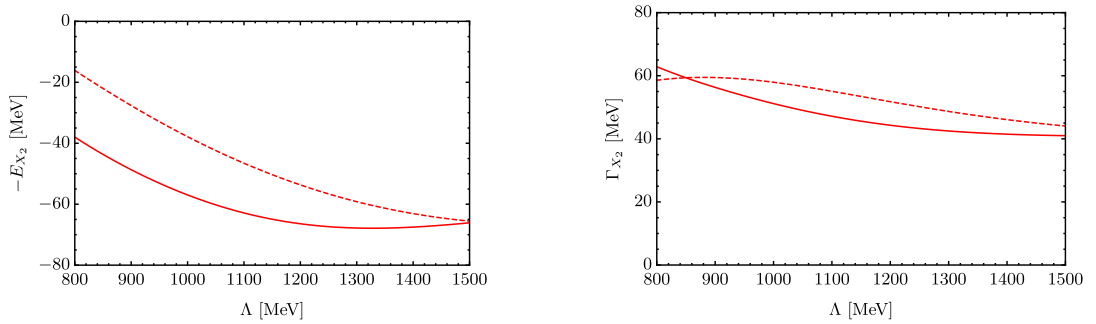


Figure 2. The energy and the width of the 2^{++} bound $D^* \bar{D}^*$ state extracted from the nonperturbative solutions of the integral equations with the contact and OPE interactions as functions of the cutoff Λ using two different regularisation schemes: i) sharp cutoff (solid lines), ii) the exponential regularisation of the form $f(p) = \exp(-p^6/\Lambda^6)$ (dashed lines).

To verify this claim, in Ref. [10], we extracted the binding energy and the width of the 2^{++} state from the nonperturbative solutions of the integral equation (4) calculated with the contact and (static) OPE interactions. These quantities are shown in Fig. 2 as functions of the cutoff used to regularise the Lippmann-Schwinger equations. From the results presented in Fig. 2 for two different regularisation schemes one is led to conclude that the nonperturbative treatment of the pions generates a significant shift of the 2^{++} spin partner of the $X(3872)$ and makes it as broad as 50 ± 10 MeV, which is much larger than in the perturbative treatment of Ref. [9]. The width is largely controlled by unitarity and coupled-channel dynamics revealing only a rather mild cutoff dependence. On the contrary, the cutoff dependence of the binding energy is quite substantial which calls for the presence of an additional HQSS breaking counter term in order to absorb the dependence of the results on the regulator. It should be stressed, however, that the largest difference corresponds to the region of smaller cutoffs where the separation of the soft and hard scales is less pronounced. Given a relatively small separation of scales in the problem, further efforts should be devoted to studying the convergence pattern of the

approach and especially to estimating the role of higher-order contact interactions. Also the role of ultrasoft scales from the three-body effects in the OPE potential, neglected in the current study, needs to be examined along the lines of Refs. [15–17].

4 Summary

Assuming the X(3872) with the quantum numbers $J^{PC} = 1^{++}$ to be an isosinglet molecule state, in this contribution, we discussed the implications of heavy-quark spin symmetry (HQSS) for the molecular partners of the X(3872). Using an EFT approach with contact interactions in the $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ channels in the strict HQSS limit the X(3872) is shown to have three degenerate partner states with the 1^{+-} , 0^{++} and 2^{++} quantum numbers. This conclusion holds also when one-pion exchange (OPE) interactions are included but only if all allowed coupled-channel transitions between the $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ channels governed by the one-pion exchange potential are taken into account. In particular, neglecting some of the coupled-channel transitions leads to a severe violation of HQSS as well as to regulator-dependent results for the partner states. Using the binding energy of the X(3872) as the single input quantity, beyond the HQSS limit predictions are possible only for the 2^{++} $D^*\bar{D}^*$ molecular partner while the partner states in the other channels require one more experimental input. As long as the HQSS breaking terms and the OPE interactions are included simultaneously, the pole of the $D^*\bar{D}^*$ 2^{++} state in the scattering amplitudes experiences a sizeable shift from the vicinity of the $D^*\bar{D}^*$ threshold deeper to the complex plain. The main effect of OPE in this problem is that it allows for new coupled-channel transitions to on-mass-shell states, thus resulting in the finite width of the 2^{++} partner. Our estimate based on the nonperturbative solutions of the integral equations yields the width which is as large as 50 MeV. We stress that the coupled-channel effects induced by the resummation of OPE to all orders lead to a significant increase of this width, as compared to the approach with the pions treated perturbatively.

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5 Appendix

In order to arrive at partial wave projections of the potentials derived above, we introduce the following set of normalized projectors:

$$P(D\bar{D}(^1D_2))_{ij} = -\sqrt{\frac{15}{2}}v_{ij}, \quad (16)$$

$$P(D\bar{D}^*(^3S_1))_i = \varepsilon_i, \quad (17)$$

$$P(D\bar{D}^*(^3D_1))_i = -\frac{3}{\sqrt{2}}\varepsilon_j v_{ij}, \quad (18)$$

$$P(D\bar{D}^*(^3D_2))_{ij} = -\frac{\sqrt{5}}{2}\varepsilon_k(i\varepsilon_{ikl}v_{lj} + i\varepsilon_{jkl}v_{li}), \quad (19)$$

$$(20)$$

$$P(D^* D^* ({}^3S_1))_i = A_i, \quad (21)$$

$$P(D^* \bar{D}^* ({}^3D_1))_i = -\frac{3}{\sqrt{2}} A_j v_{ij}, \quad (22)$$

$$P(D^* \bar{D}^* ({}^5S_2))_{ij} = \frac{1}{2} S_{ij}, \quad (23)$$

$$P(D^* \bar{D}^* ({}^1D_2))_{ij} = -\sqrt{\frac{5}{2}} (\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2) v_{ij}, \quad (24)$$

$$P(D^* \bar{D}^* ({}^5D_1))_i = -\frac{\sqrt{3}}{2} i \varepsilon_{ijk} S_{jm} v_{km}, \quad (25)$$

$$P(D^* \bar{D}^* ({}^5D_2))_{ij} = -\sqrt{\frac{45}{56}} (S_{ik} v_{kj} + S_{jk} v_{ki} - \frac{2}{3} \delta_{ij} S_{kl} v_{kl}), \quad (26)$$

$$P(D^* \bar{D}^* ({}^5G_2))_{ij} = \sqrt{\frac{175}{32}} S_{kl} v_{ijkl} \quad (27)$$

where

$$v_{ij} = n_i n_j - \frac{1}{3} \delta_{ij}, \quad (28)$$

$$v_{ijkl} = n_i n_j n_k n_l - \frac{1}{7} (n_i n_j \delta_{kl} + n_i n_k \delta_{jl} + n_i n_l \delta_{jk} + n_j n_k \delta_{il} + n_j n_l \delta_{ik} + n_k n_l \delta_{ij}) + \frac{1}{35} (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \quad (29)$$

$$A_i = \frac{i}{\sqrt{2}} \varepsilon_{ijk} \varepsilon_{1j} \varepsilon_{2k}, \quad (30)$$

$$S_{ij} = \varepsilon_{1i} \varepsilon_{2j} + \varepsilon_{1j} \varepsilon_{2i} - \frac{2}{3} \delta_{ij} (\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2). \quad (31)$$

The normalization condition for the projector operators reads

$$P_{ijk..nm}^{JLS}(n) P_{ijk..nm}^{\dagger JLS}(n) = 2J + 1. \quad (32)$$

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