Explanation of the $X(4260)$ and $X(4360)$ as Molecular States

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Abstract. We study the $X(4260)$ and $X(4360)$ solving Faddeev Equation under the Fixed Center Approximation. We find a state of $I = 1$ with mass around 4320 MeV and a width bout 25 MeV for the case of ρ meson scattering from $X(3700)(D\bar{D})$ and 4256 MeV and a width about 30 MeV of $\bar{D}$ scattering from $D_1(2420)(\rho D)$. The results obtained in present work are in good agreement with experimental results.

1 Introduction

The question of "What the hadron is made of?" is a permanent question more than 50 years. This question was answered by Murray Gell-Man and George Zweig introducing the quark model for the first time. In this model hadrons are made of $q\bar{q}$ (meson) or qqq (baryons). However all the hadrons cannot be explained within the quark model and some complex structure such as glueballs, hybrids and molecules are needed.

Quantum Chromodynamics (QCD) is the theory of strong interactions which describe the nature and internal structure of hadrons. However, because of confinement problem of QCD in the low energy region, the perturbation theory does not work. Hence one needs nonperturbative methods such as Chiral Perturbation Theory [1–4], lattice QCD [5, 6] and the QCD sum rules [7, 8] to investigate the hadrons in the low and medium energy region.

The Chiral Perturbation Theory is one of the most powerful method to deal with hadrons, at low energy region. To extended the Chiral Perturbation Theory at medium energy region unitary extensions of chiral perturbation theory was introduced [9–11].

In order to investigate the three body systems one needs to solve the Faddeev Equations. The Fixed Center Approximation (FCA) was formulated to calculate the three body systems since the Faddeev Equations is quite long and complicated to deal with the three body systems. In this method a pair of particles bound together which is called a cluster and a third particle scatters from that cluster. The method has proved to be rather reliable for cases like light (scattering particle)-heavy (cluster) systems. Hence the cluster is not modified by the third particle and one can safely use the FCA to Faddeev Equations to calculate the three or many body systems. The limits on this model has been done in the work of [12, 13] and the authors have explained $\phi(2170)$ meson and $\bar{K}NN$ system properly. The $\Delta_{5/2^+}(2000)$ puzzle was investigated using the FCA to Faddeev Equations [14]. In this work the authors gave a plausible explanation of the $\Delta_{5/2^+}(2000)$ puzzle. The three body $N\bar{K}K$ scattering amplitude was calculated by using the FCA to Faddeev Equations, taking the $\bar{K}(N)$ as a scattering particle and $KN(\bar{K}K)$ as a cluster [15]. A peak appeared in the modulus squared of the

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three body scattering amplitude with a mass at around 1920 MeV with spin-parity $J^p = 1/2^+$. The authors suggested that the $N\bar{K}K$ three body systems is the $N^*(1920)(1/2^+)$ state.

More recently this method is used for the charm and bottom mesons with total spin-3 successfully. The $\rho D^*\bar{D}^*$ three body systems has been studied within the FCA to Faddeev Equations [16] and they made predictions on three body states for spin $J = 3$ case. Similarly, $\rho B^*\bar{B}^*$ systems with total spin $J = 3$ has been investigated by using the Faddeev Equations under the FCA [17]. In this work, $B\bar{B}^*$ system with J=2 forms a cluster and ρ meson scattering from that cluster. The two-body ρB^* and $\rho\bar{B}^*$ scattering have been calculated in the Local Hidden Gauge Theory [18]. In [17] the authors found a $I(J^{PC}) = 1(3^{--})$ state with a mass at around 10987 ± 40 MeV and width 40 ± 15 MeV. Besides, $\rho\bar{K}K$ and $\eta\bar{K}K$ three body systems was investigated within FCA to Faddeev Equation [19, 20]. In these works $\rho(1700)$ and $\eta(1475)$ mesons have been explained as $\rho K\bar{K}$ and $\eta K\bar{K}$ molecular states respectively.

In recent years, $c\bar{c}$ mesons has been of great interest to both theorists and experimentalists. For instance, the internal structure of the $X(3872)$ [21], $X(4140)$ [22], $X(4260)$ [23–25] and $X(4360)$ [26–28] mesons are not clarified using the quark model. These mesons have been investigated theoretically and experimentally and supported the existence of new types of hadronic states.

The BABAR Collaboration discovered a structure with a mass of 4324 ± 24 MeV/c^2 and a width 172 ± 33 MeV/c^2 [23]. In the same experiment, they also searched the compatibility of this structure width the $X(4260)$. The Belle Collaboration measured the $e^+e^- \to \pi\pi^+(2S)$ cross section threshold 5500 MeV. They observed two states; one has mass of $4361 \pm 9 \pm 9$ MeV/c^2 with a width of $74 \pm 15 \pm 10$ MeV/c^2 and the other state has mass around $4664 \pm 11 \pm 5$ MeV/c^2 with a width of $48 \pm 15 \pm 3$ MeV/c^2, in the same year [23]. Next, the Cleo Collaboration researched for the $X(4260)$ meson [24]. They observed a state $m = 4284^{+17}_{-16}(stat) \pm 4(syst)$ MeV/c^2 and $\Gamma = 73^{+39}_{-25}(stat) \pm 5(syst)$ for the new resonance $X(4260)$. The BABAR Collaboration searched $e^+e^- \to \pi\pi J/\psi$ in the c.m. energy range $3740 - 5500$ MeV using initial state radiation events. They obtained $m = 4245 \pm 5(stat) \pm 4(syst)$ MeV/c^2 and $\Gamma = 114^{+16}_{-15}(stat) \pm 7(syst)$ MeV/c^2 for the $X(4260)$ state. Very recently, the process $e^+e^- \to \pi\pi J\psi$ was investigated by BESS III Collaboration [29]. They have measured the cross section of the $e^+e^- \to \pi\pi^+ J/\psi$ process at center of mass energies from 3770 to 4600 MeV using $9fb^{-1}$ from the data collected with the BESIII detector operating at the BEPCII storage ring. They have observed two resonances: the first state has a mass of $4222.0 \pm 3.1(stat) \pm 1.4(syst)$ MeV/c^2 and a width of $44.1 \pm 4.3(stat) \pm 2.0(syst)$ MeV correspond to $X(4260)$ and the second one has a mass of $4320.0 \pm 10.4(stat) \pm 7.0(syst)$ MeV/c^2 and a width of $101.4^{+25.3}_{-19.7} \pm 10.2$ MeV observed in $e^+e^- \to \pi\pi^+ J/\psi$ for the first time.

In the present paper, we study $\rho D\bar{D}$ three body system solving the Faddeev Equations under the FCA. In order to calculate the amplitude of the $\rho D\bar{D}$ three body system, we take $D\bar{D}$ and ρD as clusters and ρ and $\bar{D}$ mesons scattering from these clusters respectively. In the case of ρ scattering from X(3700), dynamically generated states of $D\bar{D}$, we have only one state with total isospin I=1. But for the case of $\bar{D}$ scattering from $D_1(2420)$, obtained dynamically ρD and its coupled channels, we have two states with I=0 and I=1 for the total three body system.

2 Formalism for $\rho D\bar{D}$ Three Body System

We study $\rho D\bar{D}$ three body system by using the FCA to Faddeev Equation. In this method we are taking two particles as a cluster and a third particle scattering from that cluster. In the case of the $\rho D\bar{D}$ three body system, there is two options for the cluster, one is the $D\bar{D}$ and the other one is the ρD. The ρD was studied [30, 31] in the open and hidden charm sector and $D\bar{D}$ in [32, 33] within the framework of the chiral unitary approach.

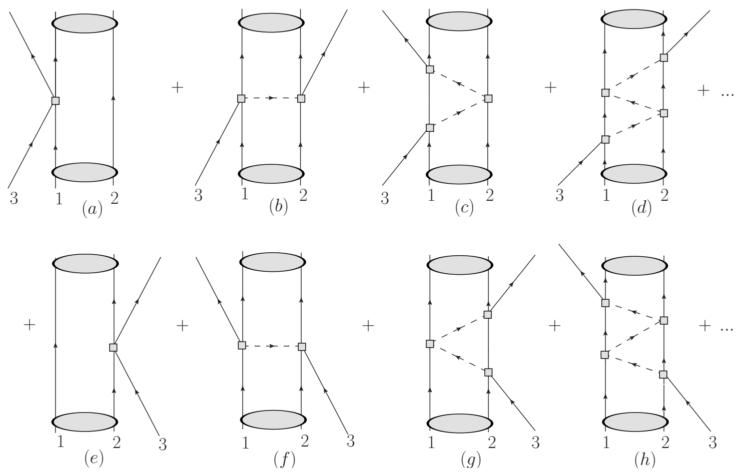

Figure 1. Diagrammatic representation of the fixed center approximation to Faddeev equations.

In order to calculate the three body scattering amplitude T by solving Faddeev Equations, one needs to calculate two partition functions T_1 and T_2. The diagrammatic sketches are shown in Fig.1. $T_1(T_2)$ sums all diagrams of the series of Fig.1 which begin with the interaction of particle 3 with particle 1(2) of the cluster.

Then T can read a summation of the two partition functions T_1 and T_2,

$$\begin{aligned} T_1 &= t_1 + t_1 G_0 T_2, \\ T_2 &= t_2 + t_2 G_0 T_1 \\ T &= T_1 + T_2 \end{aligned} \tag{1}$$

Because we follow the normalization of Mandl and Shaw [34], we must determine the field normalization. We find for the single scattering,

$$\begin{aligned} S_1^{(1)} &= -it_1 \frac{1}{\mathcal{V}^2}(2\pi)^4 \delta^4(k_3 + k_{cls} - k_3^{'} - k_{cls}^{'}) \\ &\times \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega_3^{'}}} \frac{1}{\sqrt{2\omega_1}} \frac{1}{\sqrt{2\omega_1^{'}}}, \end{aligned} \tag{2}$$

$$\begin{aligned} S_2^{(1)} &= -it_2 \frac{1}{\mathcal{V}^2}(2\pi)^4 \delta^4(k_3 + k_{cls} - k_3^{'} - k_{cls}^{'}) \\ &\times \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega_3^{'}}} \frac{1}{\sqrt{2\omega_2}} \frac{1}{\sqrt{2\omega_2^{'}}}, \end{aligned} \tag{3}$$

where $k, k'(k_{cls}k'_{cls})$ refer to the momentum of initial, final scattering particle (cls for the cluster), $\omega(\omega')$ is the on-shell energy initial(final) particle and $\mathcal{V}$ is the volume of the box to normalize the external fields to unity.

The double-scattering diagram is given by

$$\begin{aligned} S^{(2)} &= -i(2\pi)^4 \frac{1}{\mathcal{V}^2} \delta^4(k_3 + k_{cls} - k_3^{'} - k_{cls}^{'}) \\ &\times \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega_3^{'}}} \frac{1}{\sqrt{2\omega_1}} \frac{1}{\sqrt{2\omega_1^{'}}} \frac{1}{\sqrt{2\omega_2}} \frac{1}{\sqrt{2\omega_2^{'}}} \\ &\times \int \frac{d^3q}{(2\pi)^3} F_{cls}(q) \frac{1}{q^{02} - \vec{q}^2 - m_3^2 + i\epsilon} t_1 t_2, \end{aligned} \tag{4}$$

where $F_{cls}(q)$ is the form factor of the cluster that we shall discuss below. The full S-matrix for scattering of particle 3 with the cluster will be given by

$$\begin{aligned} S &= -iT \frac{1}{\mathcal{V}^2} (2\pi)^4 \delta^4(k_3 + k_{cls} - k_3^{'} - k_{cls}^{'}) \\ &= \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega_3^{'}}} \frac{1}{\sqrt{2\omega_{cls}}} \frac{1}{\sqrt{2\omega_{cls}^{'}}}. \end{aligned} \tag{5}$$

As we can see the field normalization factors that appear in the amplitude of the different terms are different. However, if we combine Eqs. (2),(3),(4),(5) and approximate $\omega_i = m_i$ where i corresponds to particle 1-3, then we can introduce suitable factors in the elementary amplitudes,

$$\tilde{t}_1 = \frac{m_{cls}}{m_1} t_1, \quad \tilde{t}_2 = \frac{m_{cls}}{m_2} t_2. \tag{6}$$

One can sum the partition functions T_1 and T_2 and obtain,

$$T = T_1 + T_2 = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2 G_0}{1 - \tilde{t}_1\tilde{t}_2 G_0^2} \tag{7}$$

where the function G_0 is the propagator of the particle 3 inside the cluster and given by

$$G_0 = \frac{1}{2m_{cls}} \int \frac{d^3q}{(2\pi)^3} F_{cls}(q) \frac{1}{q^{02} - \vec{q}^{\,2} - m_3^2 + i\epsilon}. \tag{8}$$

The function $F_{cls}(q)$ in the above equation and also in Eq. (4) is the form factor of the cluster and given by

$$\begin{aligned} F_{cls}(q) &= \frac{1}{\mathcal{N}} \int_{\substack{p<k_{max} \\ |\vec{p}-\vec{q}|<k_{max}}} d^3p \, \frac{1}{2\omega_1(\vec{p}\,)} \, \frac{1}{2\omega_2(\vec{p}\,)} \, \frac{1}{m_{cls} - \omega_1(\vec{p}\,) - \omega_2(\vec{p}\,)} \\ &\times \left(\frac{1}{2\omega_1(\vec{p} - \vec{q}\,)}\right)\left(\frac{1}{2\omega_2(\vec{p} - \vec{q}\,)}\right) \frac{1}{m_{cls} - \omega_1(\vec{p} - \vec{q}\,) - \omega_2(\vec{p} - \vec{q}\,)}, \end{aligned} \tag{9}$$

with the normalization factor,

$$\mathcal{N} = \int_{p<k_{max}} d^3p \left[\frac{1}{2\omega_1(\vec{p}\,)} \, \frac{1}{2\omega_2(\vec{p}\,)} \, \frac{1}{m_{cls} - \omega_1(\vec{p}\,) - \omega_2(\vec{p}\,)}\right]^2. \tag{10}$$

Table 1. The C coefficients in Eq. (12) for the $I = 1/2$ case.

Channels	πD^*	$D\rho$	$\bar{K}D_s^*$	$D_s\bar{K}^*$	ηD^*	$D\omega$	$\eta_c D^*$	DJ/ψ
πD^*	-2	$\frac{\gamma}{2}$	$\sqrt{\frac{3}{2}}$	0	0	$-\frac{\gamma}{2}$	0	$-\sqrt{2}\gamma$
$D\rho$	$\frac{\gamma}{2}$	-2	0	$-\sqrt{\frac{3}{2}}$	$\frac{\gamma}{2}$	0	$\sqrt{2}\gamma$	0
$\bar{K}D_s^*$	$\sqrt{\frac{3}{2}}$	0	-1	0	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{2}{3}}\gamma$	0	$\frac{2\gamma}{\sqrt{3}}$
$D_s\bar{K}^*$	0	$-\sqrt{\frac{3}{2}}$	0	-1	$-\sqrt{\frac{2}{3}}\gamma$	$-\sqrt{\frac{3}{2}}$	$\frac{2\gamma}{\sqrt{3}}$	0
ηD^*	0	$\frac{\gamma}{2}$	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{2}{3}}\gamma$	0	$\frac{\gamma}{6}$	0	$\frac{\sqrt{2}\gamma}{3}$
$D\omega$	$-\frac{\gamma}{2}$	0	$-\sqrt{\frac{2}{3}}\gamma$	$-\sqrt{\frac{3}{2}}$	$\frac{\gamma}{6}$	0	$\frac{\sqrt{2}\gamma}{3}$	0
$\eta_c D^*$	0	$\sqrt{2}\gamma$	0	$\frac{2\gamma}{\sqrt{3}}$	0	$\frac{\sqrt{2}\gamma}{3}$	0	$\frac{4\gamma}{3}$
DJ/ψ	$-\sqrt{2}\gamma$	0	$\frac{2\gamma}{\sqrt{3}}$	0	$\frac{\sqrt{2}\gamma}{3}$	0	$\frac{4\gamma}{3}$	0

Table 2. The C^I coefficients in Eq. (12) for the $I = 3/2$ case.

Channels	πD^*	$D\rho$
πD^*	1	1
$D\rho$	1	1

In present work we calculate $\rho - D\bar{D}$ and $\bar{D} - \rho D$ three body scattering amplitude. Let us start from $\rho - D\bar{D}$ scattering. Here $D\bar{D}$ is the cluster and ρ is orbiting around that cluster. To investigate this we need the two body ρD and $\rho\bar{D}$ scattering amplitude which is done in [30, 31]. We are going to follow the same procedure. There are eight coupled channels πD^*, $D\rho$, KD_s^*, D_sK^* ,ηD^*, $D\omega$, $\eta_c D^*$, DJ/ψ in $I = 1/2$, and two coupled channels, πD^* and $D\rho$, in the $I = 3/2$ case.

We solve the Bethe-Salpeter equation to calculate the two body scattering amplitude with coupled channels unitary approach:

$$T = (\hat{1} + V\widehat{G})^{-1}(-V)\vec{\epsilon}.\vec{\epsilon}' \tag{11}$$

where $\vec{\epsilon}(\vec{\epsilon}')$ is the polarization of the incoming(outgoing) vectors mesons and V is the potentials and given by

$$\tilde{V}^I_{ij}(s,t,u) = -\frac{C^I_{ij}}{4f^2}(s-u)\epsilon.\epsilon'. \tag{12}$$

The coefficients C^I in an isospin bases are given in Table 1 and Table 2 for the $I = 1/2$ and the $I = 3/2$ respectively. In Table 1, $\gamma = (m_l/m_H)^2$, m_l and m_H are scales in the order of magnitude of the light and heavy vector mesons masses, respectively.

In the Bethe-Salpeter equations, G is given as below for the l channel,

$$\begin{aligned}\widehat{G_l} &= i\left(1+\frac{p^2}{3M^2}\right)\int\frac{dq^4}{(2\pi)^4}\frac{1}{q^2-m_l^2+i\epsilon}\frac{1}{(P-q)^2-M_l^2+i\epsilon}\\ &= \frac{1}{16\pi^2}\left(1+\frac{p^2}{3M^2}\right)\Bigg(\alpha_i+Log\frac{m_l^2}{\mu^2}+\frac{M_l^2-m_l^2+s}{2s}Log\frac{M_l^2}{m_l^2}+\\ &\frac{p}{\sqrt{s}}\Big(Log\frac{s-M_l^2+m_l^2+2p\sqrt{s}}{-s+M_l^2-m_l^2+2p\sqrt{s}}+Log\frac{s+M_l^2-m_l^2+2p\sqrt{s}}{-s-M_l^2+m_l^2+2p\sqrt{s}}\Big)\Bigg)\end{aligned} \tag{13}$$

where α_i is the subtraction constant, μ is a cutoff scale and M_l and m_l are masses of the vector and pseudoscalar mesons in the l channel, respectively.

The other option in the three body $\rho D\bar{D}$ scattering is $\bar{D}-\rho D$. In this case ρD is the cluster and $\bar{D}$ is orbiting around this cluster. Hence, we need two body $D\bar{D}$ and ρD scattering amplitude with coupled channels. There are six coupled channels $D\bar{D}$, $K\bar{K}$, $\pi\bar{\pi}$, $\eta\eta$, $\eta_c\eta$, $D_s\bar{D}_s$ and five coupled channels $D\bar{D}$, $K\bar{K}$, $\pi\bar{\pi}$, $\pi\eta$, $\eta_c\pi$ in the $I=0$ and $I=1$ case respectively. Next we solve the Bethe-Salpeter equation,

$$T=(\hat{1}-VG)^{-1}V \tag{14}$$

where G is the loop function of pseudoscalar-pseudoscalar mesons which is given in Eq. (13) changing the vector meson masses (M_l) with the pseudoscalar meson masses (m_l) and removing the factor $\left(1+p^2/3M^2\right)$.

3 Results and Discussion

We calculate the modulus squared of the scattering amplitude of $\rho D\bar{D}$ within the FCA to the Faddeev Equations. In Eq. (12), as it appears the meson decay constant that we take $f_\pi = 93$ MeV for the light mesons, $f_D = 1.77 f_\pi$ and $f_{D_s} = 2.24 f_\pi$ for the heavy mesons. As we stated before, in the loop function Eq. (13) there is the loop parameter μ. We set this parameter to $\mu = 1500\ MeV$ both $\rho-(D\bar{D})$ and $\bar{D}-(\rho D)$ three body scatterings. In the same formula there is also subtraction constant (α_i) in the loop function which is only free parameter of the theory. To obtain $D_1(2420)$, the bound state of ρD with coupled channels, we use $\alpha_i = -1.55$ as in [31] and also $X(3700)$, the bound state of $D\bar{D}$ with coupled channels with $I=0$, we take $\alpha_i = -1.3$ as in Refs. [32, 33].

In Fig. 2 we depict $|T^2|$ for $\rho-(D\bar{D})_{X(3700)}$ with total isospin $I=1$. As we can see in this figure there is a clear peak at 4320 MeV with a width about 25 MeV. This state could correspond to the $X(4360)$ with quantum numbers $I^G(J^{PC)} = ?^?(1^{--})$ [35].

The modulus squared of the $\bar{D}(\rho D)_{D_1(2420)}$ three body scattering amplitude with total isospin I=1 is shown Fig 3. There is a peak around 4256 MeV with about $25-30$ MeV. This state could be associated with $X(4260)$ with the quantum number $I^G(J^{PC)} = ?^?(1^{--})$ with a width of 120 MeV [35]. Our numerical result is in good agreement for the mass but not for the width. In [29], the cross section for the process $e^+e^- \rightarrow \pi\pi^+ J\psi$ is measured at center of mass energies from 3770 to 4600 MeV. They have observed two resonance states (see Fig.1 of Ref. [29]). The first one has a mass $m = 4222.0\pm3.1(stat)\pm1.4(syst)\ MeV/c^2$ and a width $44.1\pm4.3(stat)\pm2.0(syst)\ MeV$, while the second one has a mass $4320.0\pm10.4(stat)\pm7.0(syst)\ MeV/c^2$ and a width of $101.4^{+25.3}_{-19.7}\pm10.2\ MeV$. The first resonance near 4222 MeV/c^2 is associated to the $X(4260)$. Hence newly BESIII result for the width of $X(4260)$ is in good agreement with our result. In conclusion the results from BESIII Collaboration are in good agreement with our results.

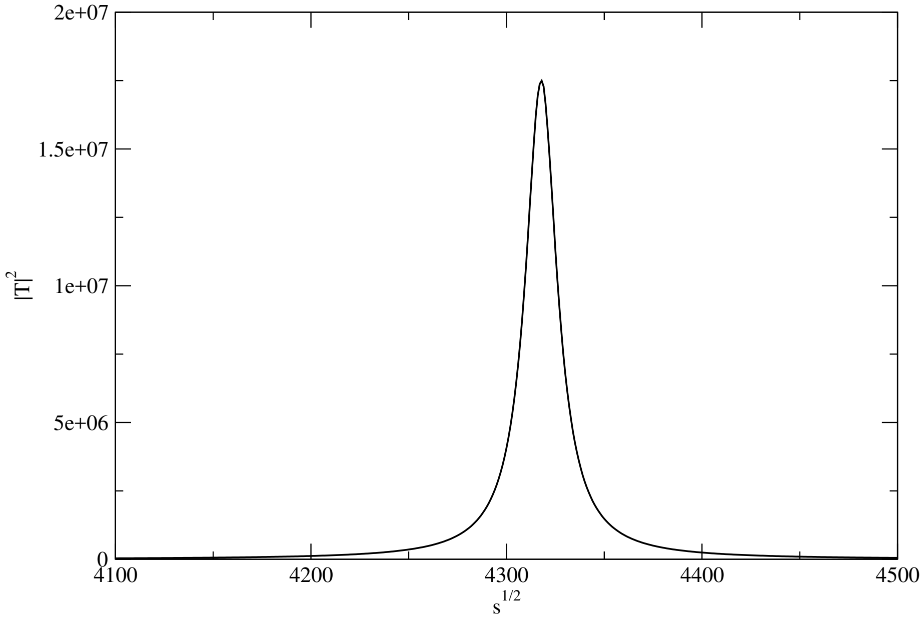

Figure 2. Modulus squared of the $\rho(D\bar{D})_{X(3700)}$ scattering amplitude with total isospin $I = 1$.

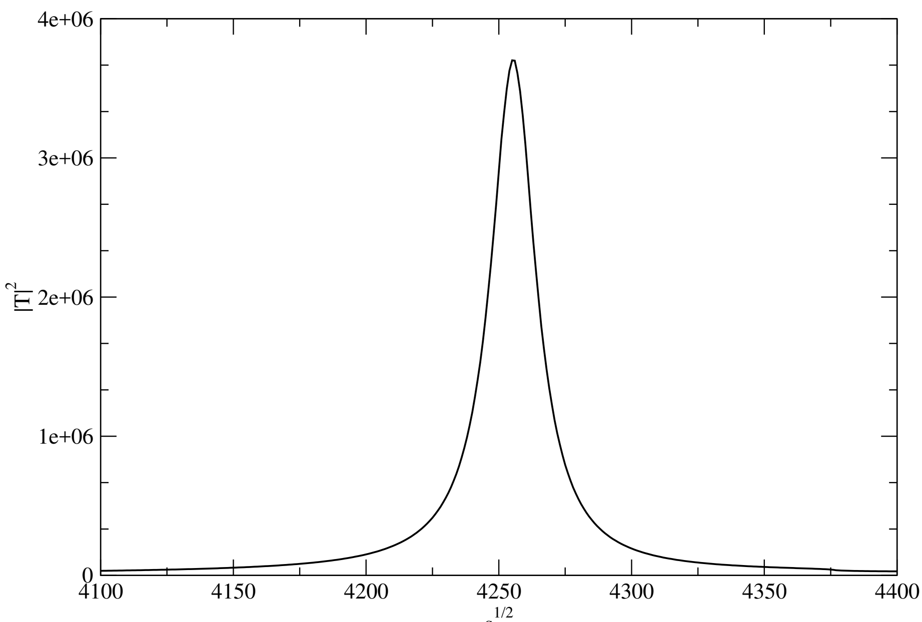

Figure 3. Modulus squared of the $\bar{D}(\rho D)_{D_1(2420)}$ scattering amplitude with total isospin $I = 1$.

Acknowledgments

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