Relativistic corrections to electromagnetic heavy quarkonium production

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Abstract. We report on the calculation [1] of the relativistic $O(\alpha_s^0 v^2)$ corrections to the quarkonium production process $e^+e^- \rightarrow \chi_{cJ} + \gamma$ in non-relativistic QCD (NRQCD). In our work we incorporate effects from operators that contribute through the sub-leading Fock state $|Q\bar{Q}g\rangle$, that were not taken into account by previous studies. We determine the corresponding matching coefficients that should be included into theoretical predictions for the electromagnetic production cross-section of $\chi_{cJ}$. This process could be, in principle, measured by the Belle II experiment.

1 Introduction

Heavy quarkonia provide a privileged window to study the interplay of perturbative and non-perturbative effects in QCD. The non-relativistic nature of the system and the presence of a hierarchy of well-separated scales makes the application of the effective field theory (EFT) approach advantageous, where NRQCD [2] allows to factorize cross-sections and decay rates of heavy quarkonia into the short-distance matching coefficients and long-distance matrix elements (LDME). The assumed universality of LDMEs gives the theory its predictive power. While LDMEs are non-perturbative and must be therefore fitted from experiment or (if possible) computed on the lattice, the short-distance coefficients are determined from perturbative matching between QCD and NRQCD.

A quarkonium production cross-section in the NRQCD factorization picture is a double series in $\alpha_s$ and the relative heavy quark velocity $v$. Higher order corrections in $\alpha_s$ arise from loop effects, while higher order corrections in $v$ are generated by the inclusion of higher-dimensional operators. As both quantities are not independent (e.g. $\alpha_s(m) \approx 0.24$, $v^2 \approx 0.3$ in charmonium, with $m$ being the charm quark mass), a consistent computation of higher order corrections requires both loop corrections to matching coefficients of lower dimensional operators and relativistic corrections from higher dimensional operators.

A distinctive feature of NRQCD in comparison to many other approaches (e.g. Color Singlet Model) is the color octet mechanism. This property of the theory tells us that a $Q\bar{Q}$-pair evolving into a heavy quarkonium does not necessarily has to be in the color singlet configuration. Schematically, we can write the Fock state expansion of a heavy quarkonium as

$$|H\rangle \sim a_0 |Q\bar{Q}\rangle + a_1 |Q\bar{Q}g\rangle + a_2 |Q\bar{Q}gg\rangle + \ldots.$$ (1)

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Sub-leading Fock states that contain gluons are usually suppressed by power of \( v \) as compared to the dominant state \(|Q\bar{Q}\rangle\). Nevertheless, they must be taken into account when higher order relativistic or loop corrections are computed. Studying the importance of the higher Fock state contributions for phenomenology is therefore an important test for the validity of NRQCD.

One of the processes, where such contributions might be of phenomenological relevance, is the exclusive electromagnetic production of \( \chi_{cJ} \) and a hard (\( |k_p| \sim m \)) photon. This reaction should be observable at B-factories with a sufficiently high center of mass (CM) energy, for example at Belle II. Unfortunately, at the present time no published experimental measurements are available.

Nevertheless, in the last decade this process has received much attention from the theory side: corrections of order \( O(\alpha_s^3 v^0) \) [3], \( O(\alpha_s v^0) \) [4, 5], \( O(\alpha_s^2 v^2) \) [6, 7] and \( O(\alpha_s v^2) \) [8] are known. One of the reasons for that is that some of the exotic XYZ states with even charge conjugation (c.f. [9]) may also be produced in the channel \( e^+e^- \to H + \gamma \). Precise predictions for the production of \( \chi_{cJ} \) are therefore very important to understand the true nature of the quarkonium exotics.

All of the above-mentioned studies were concerned only with NRQCD operators that contribute through the dominant Fock state \(|Q\bar{Q}\rangle\). However, at \( O(\alpha_s^2 v^2) \) operators that contribute through the sub-leading Fock state \(|Q\bar{Q}g\rangle\) must be also taken into account. This prompts us to consider relativistic corrections generated by these operators. To this purpose, in the perturbative matching between QCD and NRQCD we include diagrams that contain an ultrasoft (\(|k_{j}| \sim m v^2\)) final state gluon and treat this gluon as a part of the quarkonium system. The QCD amplitudes are therefore expanded not only in the soft momenta of the heavy quarks but also in the ultrasoft momentum of the gluon. By decomposing Cartesian tensors up to rank five into irreducible spherical tensors we project out contributions that belong to the total angular momenta \( J = 0, 1, 2 \) and extract short-distance coefficients multiplying NRQCD production operators. We determine new relativistic corrections induced by operators that contain one chromoelectric field and hence give no contribution through the dominant Fock state.

This note is organized in the following way. In Sec. 2 we describe electromagnetic production of \( \chi_{cJ} \) in NRQCD and explain the outline of the calculation. Matching between QCD and NRQCD is carried out in Sec. 3. In Sec. 4 we present our new results, while in Sec. 5 we summarize our work and discuss further developments.

## 2 Electromagnetic \( \chi_{cJ} \) production in NRQCD

Using NRQCD factorization conjecture, the cross-sections for the electromagnetic production of \( \chi_{cJ} \) up to relative order \( v^2 \) can be written as

\[
\sigma_{\chi_{c0}} = \frac{F_1(3P_0)}{2m^2} \langle 0| \chi^\uparrow(-\frac{i}{2}\mathbf{D} \cdot \sigma)\psi|\chi_{c0}\rangle \langle \chi_{c0}|\psi^\uparrow(-\frac{i}{2}\mathbf{D} \cdot \sigma)\chi|0\rangle \\
+ \frac{G_1(3P_0)}{6m^4} \langle 0| \chi^\uparrow(-\frac{i}{2}\mathbf{D} \cdot \sigma)\psi|\chi_{c0}\rangle \langle \chi_{c0}|\psi^\uparrow(-\frac{i}{2}\mathbf{D} \cdot \sigma)(-\frac{i}{2}\mathbf{D})^2\chi|0\rangle + h.c.\rangle \\
+ \frac{iT_8(3P_0)}{3m^3} \langle 0| \chi^\uparrow(-\frac{i}{2}\mathbf{D} \cdot \sigma)\psi|\chi_{c0}\rangle \langle \chi_{c0}|\psi^\uparrow(g\mathbf{E} \cdot \sigma)\chi|0\rangle + h.c.\rangle, \tag{2}
\]

\[
\sigma_{\chi_{c1}} = \frac{F_1(5P_1)}{2m^2} \langle 0| \chi^\uparrow(-\frac{i}{2}\mathbf{D} \times \sigma)\psi|\chi_{c1}\rangle \langle \chi_{c1}|\psi^\uparrow(-\frac{i}{2}\mathbf{D} \times \sigma)\chi|0\rangle \\
+ \frac{G_1(5P_1)}{4m^4} \langle 0| \chi^\uparrow(-\frac{i}{2}\mathbf{D} \times \sigma)\psi|\chi_{c1}\rangle \langle \chi_{c1}|\psi^\uparrow(-\frac{i}{2}\mathbf{D} \times \sigma)(-\frac{i}{2}\mathbf{D})^2\chi|0\rangle + h.c.\rangle \\
+ \frac{iT_8(5P_1)}{2m^3} \langle 0| \chi^\uparrow(-\frac{i}{2}\mathbf{D} \times \sigma)\psi|\chi_{c1}\rangle \langle \chi_{c1}|\psi^\uparrow(g\mathbf{E} \times \sigma)\chi|0\rangle + h.c.\rangle, \tag{3}
\]
\[
\sigma_{\chi_c} = \frac{F_1(3P_2)}{m^2} \langle 0| \chi^{|(-iD^i\sigma^j)\psi|\chi_{c\gamma}(\chi_c \gamma^\dagger(-\frac{i}{2}D^i\sigma^j)\chi|0)} + \frac{G_1(3P_2)}{2m^4} \left( \langle 0| \chi^{|(-iD^i\sigma^j)\psi|\chi_{c\gamma}(\chi_c \gamma^\dagger(-\frac{i}{2}D^i\sigma^j)(-\frac{i}{2}D^i\sigma^j)|0}) + h.c. \right) + \frac{iG_7(3P_2)}{3m^5} \left( \langle 0| \chi^{|(-iD^i\sigma^j)\psi|\chi_{c\gamma}(\chi_c \gamma^\dagger(gE^i\sigma^j)\chi)|0}) + h.c. \right),
\]

with \( D = \nabla - igA \), \( \psi \dagger D^i \chi \equiv \psi \dagger (D^i\chi) - (D^i\psi) \dagger \chi \) and \( d^i\beta^j = \frac{d^i\beta^j + d^j\beta^i}{2} - \frac{1}{2} \delta^i\beta^j (a \cdot b) \), where \( \psi (\chi) \) is a Pauli field that annihilates (creates) a heavy quark (antiquark), \( \sigma \) is the Pauli vector, \( A \) is the gluon field and \( E \) is the chromoelectric field.

Our goal is to determine the matching coefficients \( T_{8}(3P_J) \). As the corresponding operators contain chromoelectric fields, they contribute only through the sub-leading Fock state \(|Q\bar{Q}\rangle\). This property distinguishes them from the operators multiplying \( F_1(3P_J) \) and \( G_1(3P_J) \), that can be accessed already from the dominant Fock state \(|Q\bar{Q}\rangle\).

Since we are working with an exclusive electromagnetic process, we can employ NRQCD factorization at the amplitude level [10]. This approach greatly simplifies the calculation and avoids the necessity to perform non-relativistic expansion of the phase-space measure. The corresponding NRQCD amplitudes up to the relative order \( \nu^2 \) read

\[
\mathcal{R}_{\text{NRQCD}}^{J=0} = \frac{c_{J=0}^{\gamma=0}}{m^2} \langle \chi_{c\gamma}| \psi^\dagger \left( -\frac{i}{2}D^i \cdot \sigma \right) \chi|0) + \frac{d_{J=0}^{\gamma=0}}{m^4} \langle \chi_{c\gamma}| \psi^\dagger gE \cdot \sigma \chi|0),
\]

\[
\mathcal{R}_{\text{NRQCD}}^{J=1} = \frac{(c_{J=1}^{\gamma=1} \gamma^i)}{m^2} \langle \chi_{c\gamma}| \psi^\dagger \left( -\frac{i}{2}D^i \times \sigma \right) \chi|0) + \frac{(d_{J=1}^{\gamma=1} \gamma^i)}{m^4} \langle \chi_{c\gamma}| \psi^\dagger gE \times \sigma \chi|0),
\]

\[
\mathcal{R}_{\text{NRQCD}}^{J=2} = \frac{(c_{J=2}^{\gamma=2} \gamma^{ij})}{m^2} \langle \chi_{c\gamma}| \psi^\dagger \left( -\frac{i}{2}D^i \sigma^j \right) \chi|0) + \frac{(d_{J=2}^{\gamma=2} \gamma^{ij})}{m^4} \langle \chi_{c\gamma}| \psi^\dagger gE^i \sigma^j \chi|0),
\]

where we are interested in the short distance coefficients \( d_{J=0}^{\gamma}, (d_{J=1}^{\gamma=1} \gamma^i) \) and \( (d_{J=2}^{\gamma=2} \gamma^{ij}) \). The final cross-section is obtained by squaring the non-perturbative NRQCD amplitude with the so determined coefficients and integrating over the phase space of the physical quarkonium.

In the modern NRQCD matching calculations it is customary to apply the covariant projector technique [11]. The usage of projectors allows to avoid many complications related to the non-relativistic expansion of the QCD amplitudes. Manifest Lorentz covariance of this approach greatly facilitates automation via suitable software tools (e.g. FeynCalc [12, 13] or FDC [14]). However, we are not aware of any calculation where the projector technique was used to access operators that give no contribution through the dominant Fock state \(|Q\bar{Q}\rangle\). This is why for this work we chose to use the non-covariant matching in the spirit of the original NRQCD publication [2].
3 Matching between QCD and NRQCD

Non-covariant matching implies that the QCD amplitude is not only expanded in suitable small quantities but also brought into the non-relativistic form where it resembles the NRQCD amplitude, such that one can directly read off the short-distance coefficients. This can be achieved in the following way. First of all, since we are working at tree-level, every closed spinor chain can be decomposed into scalar, pseudoscalar, vector, axial-vector and tensor components. It turns out that our amplitudes contain only vector and axial-vector contributions, for which we need to work out the non-relativistic expansions. This is done by using an explicit representation of Dirac spinors (with non-relativistic normalization) and by rewriting Dirac matrices in terms of Pauli matrices. To project out short-distance coefficients in the NRQCD production amplitude to the CM frame. The boost introduces a dependence of short-distance coefficients on the physical heavy quarkonium mass, which can be, however, eliminated using Gremm-Kapustin relations [19]. In our work we applied the known [22] Gremm-Kapustin relations, which can be, however, eliminated using Gremm-Kapustin relations [19]. In our work we applied both approaches and arrived to the same results.

When working in the rest frame, the matching calculation becomes very similar to that of the decay $\chi_{cJ} \to 2\gamma$ [20, 21]. Indeed, we can recover the corresponding short distance coefficients from our results by taking the limit, where the CM-energy goes to zero. To remove the quarkonium mass $M_H$ from the short distance coefficients, we apply the known [22] Gremm-Kapustin relations, which are derived from the NRQCD equations of motion, e.g.

$$\langle \chi_{cJ}|\bar{\psi}_{t} \left( -\frac{i}{2} \vec{D} \cdot \sigma \right) \left( -\frac{i}{2} \vec{D} \right)^2 \chi |0\rangle = mE_B \langle \chi_{cJ}|\bar{\psi}_{t} \left( -\frac{i}{2} \vec{D} \cdot \sigma \right) \chi |0\rangle + im \langle \chi_{cJ}|\bar{\psi}_{t} g\vec{E} \cdot \sigma \chi |0\rangle$$

with $E_B = M_H - 2m$.

The calculation in the CM frame requires somewhat more effort, mainly because the energies and 3-momenta of $Q, \bar{Q}$ and $g$ in a moving quarkonium can be very large, such that one cannot naively expand the QCD amplitudes in these quantities. However, using suitable Lorentz boosts, we can rewrite all those energies and 3-momenta in terms of the soft or ultrasoft rest frame momenta and thus carry out the necessary expansions. This method was first introduced in [23] and applied to perturbative quarkonium systems that consist of two heavy quarks ($\bar{Q}Q$, 2-body system). In this work we extended the Braaten-Chen formalism to a 3-body system that contains two heavy quarks and a gluon, which is our process of interest.
The starting point here is to consider a moving perturbative quarkonium composed of two heavy quarks and a gluon with the on-shell momenta \( p_1, p_2 \) and \( k_g \) respectively. They can be rewritten in terms of the Jacobi momenta for a 3-body system, such that

\[
p_1 = \frac{1}{3} P + Q_1 - Q_2, \quad p_2 = \frac{1}{3} P - Q_1 - Q_2, \quad k_g = \frac{1}{3} P + 2 Q_2
\]  

(11)

In the rest frame, where \( p_{1,R} + p_{2,R} + k_{g,R} = P_R = 0 \), we have

\[
p_{1,R} = \frac{1}{3} P_R + q_1 - q_2, \quad p_{2,R} = \frac{1}{3} P_R - q_1 - q_2, \quad k_{g,R} = \frac{1}{3} P_R + 2 q_2
\]  

(12)

or

\[
q_1 = \frac{1}{2} (p_{1,R} - p_{2,R}), \quad q_2 = \frac{1}{6} (2k_{g,R} - p_{1,R} - p_{2,R})
\]  

(13)

and in particular

\[
p_{1,R} = q_1 - q_2, \quad p_{2,R} = -q_1 - q_2, \quad k_{g,R} = 2q_2
\]  

(14)

where \( q_1 \) and \( q_2 \) are soft or ultrasoft and hence can be used as expansion parameters. Furthermore,

\[
q_1^0 = \frac{1}{2} \left( \sqrt{(q_1 - q_2)^2 + m^2} - \sqrt{(q_1 + q_2)^2 + m^2} \right),
\]  

(15)

\[
q_2^0 = \frac{1}{6} \left( 4|q_2| - \sqrt{(q_1 - q_2)^2 + m^2} - \sqrt{(q_1 + q_2)^2 + m^2} \right),
\]  

(16)

\[
P_{R}^0 = \left( 2|q_2| + \sqrt{(q_1 - q_2)^2 + m^2} + \sqrt{(q_1 + q_2)^2 + m^2} \right).
\]  

(17)

Since scalar products of 4-vectors are Lorentz invariant, we also have

\[
Q_{1/2}^i = (q_{1/2}^0)^2 - q_{1/2}^2, \quad P^2 = (P_R^0)^2, \quad P^0 = \sqrt{(P_R^0)^2 + P^2}.
\]  

(18)

To rewrite the lab frame 4-momenta \( Q_1 \) and \( Q_2 \) in terms of the soft or ultrasoft rest frame 3-momenta \( q_1 \) and \( q_2 \), we use that

\[
Q_{1/2}^i = \Lambda_{1/2}^i q_{1/2}^i,
\]  

(19)

where the components of the boost matrix are given by

\[
\Lambda_0^i = \sqrt{1 + \frac{P^2}{(P_R^0)^2}}, \quad \Lambda_0^0 = \frac{P^i}{P_R^0}, \quad \Lambda_j^i = \delta^{ij} + \left( 1 + \frac{P^2}{(P_R^0)^2} - 1 \right) \frac{p^i p^j}{P^2}.
\]  

(20)

As far as the Dirac spinors are concerned, we can begin with the rest frame, where

\[
u_R(p_{1,R}) = N_1 \left( \frac{\xi_{p_{1,R} \sigma}}{E_{1,R} + m} \right), \quad \nu_R(p_{2,R}) = N_2 \left( \frac{p_{2,R} \sigma}{E_{2,R} + m} \right),
\]  

(21)

with \( E_{i,R} = \sqrt{p_{i,R}^2 + m^2} \) and \( N_i = \sqrt{E_{i,R} + m} / 2 E_{i,R} \). The boost matrix for these spinors reads

\[
S(\Lambda) = \sqrt{\frac{(P_R^0)^2 + P^2 + P_R^0}{2 P_R^0}} \left( \begin{array}{c} 1 \\ \frac{\sigma \cdot P}{\sqrt{(P_R^0)^2 + P^2 + P_R^0}} \end{array} \right),
\]  

(22)
so that we arrive to

\[ u(p_1) = \frac{N_1}{\sqrt{2P_R^0(\sqrt{(P_R^0)^2 + P^2 + P_R^0})}}(P_R^0 + P_R^{-1})(\xi_{E,\ell + m E}^0), \tag{23} \]

\[ v(p_2) = \frac{N_2}{\sqrt{2P_R^0(\sqrt{(P_R^0)^2 + P^2 + P_R^0})}}(P_R^0 + P_R^{-1})(\xi_{E,\ell + m E}^0). \tag{24} \]

Expanding the QCD amplitude for the process (c.f. Fig. 1)

\[ e^-(p_{e^-}) + e^+(p_{e^+}) \rightarrow Q(p_1) + \bar{Q}(p_2) + g(k_g) + \gamma(k_p), \tag{25} \]

in \( q_1 \) and \( q_2 \) and matching to the perturbative NRQCD amplitudes, we obtain the following short-distance coefficients for the chromoelectric 2-quark operators

\[ d_0^{J=0} = -\frac{i e^3 Q^2 \mathbf{t} \cdot (\mathbf{k}_p \times \mathbf{e}^r(k_p))}{s - 4m^2}, \tag{26} \]

\[ d_2^{J=0} = -\frac{i e^3 Q^2 (12m^2 + s)\mathbf{e}^r(k_p) \cdot \mathbf{t}}{6s(4m^2 - s)}, \tag{27} \]

\[ (d_0^{J=1})^j = \frac{i e^3 Q^2 \left( (1 - M/\sqrt{s}) (\mathbf{k}_p \cdot \mathbf{t}) (\mathbf{k}_p \times \mathbf{e}^r(k_p))^\dagger (\mathbf{k}_p \cdot \mathbf{t}) \right)}{s - 4m^2}, \tag{28} \]

\[ (d_2^{J=1})^j = -\frac{i e^3 Q^2 \left( (1 - M/\sqrt{s}) (\mathbf{k}_p \cdot \mathbf{t}) (\mathbf{k}_p \times \mathbf{e}^r(k_p))^\dagger (\mathbf{e}^r(k_p) \times \mathbf{t})^\dagger \right)}{s - 4m^2}, \tag{29} \]

\[ (d_0^{J=2})^j = 0, \tag{30} \]

\[ (d_2^{J=2})^j = \frac{i e^3 Q^2 \left( (1 - M/\sqrt{s}) (\mathbf{k}_p \cdot \mathbf{t}) (\mathbf{k}_p \times \mathbf{e}^r(k_p))^\dagger (\mathbf{k}_p \cdot \mathbf{t}) - (1 - M/\sqrt{s}) (\mathbf{k}_p \cdot \mathbf{t}) (\mathbf{k}_p \times \mathbf{e}^r(k_p))^\dagger (\mathbf{k}_p \cdot \mathbf{t}) + i \leftrightarrow j \right)}{s - 4m^2}. \tag{31} \]

Here, \( Q \) is the heavy quark charge, \( s \) is the CM energy, \( \mathbf{e}^r(k_p) \) is the polarization of the external photon, \( \mathbf{k}_p \) is the direction of the photon momentum and \( \mathbf{t} = \vec{\ell}(l_2)\gamma\mu(l_1) \) is the leptonic polarization vector.

One should mention that on the QCD of the matching we also encountered terms that are singular in the limit \( \mathbf{k}_g \rightarrow 0 \), i.e. proportional to \( 1/||\mathbf{k}_g|| \). Such IR singularities arise when the ultrasoft gluon is emitted from an external heavy quark line and should cancel in the matching. Indeed, when 2-quark operators with Lagrangian insertions are included on the NRQCD side of the matching, they precisely reproduce the IR-behavior of the QCD amplitude. The cancellation of IR singularities on both sides of the matching was explicitly checked in both frames.
4 Results

By comparing the production cross-sections obtained from the NRQCD amplitudes Eqs. (5)-(7) to Eqs. (2)-(4) we can read off the values of the matching coefficients

\[
F_1(3\, P_0) = \frac{8\pi^2 \alpha^3 Q^4(s - M_{K_{0}^0}^2)(s - 12m^2)^2}{9s^3m^2(s - 4m^2)^2},
\]
\[
G_1(3\, P_0) = \frac{8\pi^2 \alpha^3 Q^4(s - M_{K_{0}^0}^2)(s - 12m^2)(96sm^2 - 560m^4 - 9s^2)}{45s^3m^2(s - 4m^2)^3},
\]
\[
T_8(3\, P_0) = \frac{4\pi^2 \alpha^3 Q^4(s - M_{K_{0}^0}^2)(s - 144m^4)}{9s^3m^2(s - 4m^2)^2},
\]
\[
F_1(3\, P_1) = \frac{16\pi^2 \alpha^3 Q^4(s - M_{K_{+}}^2)(s + 4m^2)}{9s^2m^2(s - 4m^2)^2},
\]
\[
G_1(3\, P_1) = \frac{64\pi^2 \alpha^3 Q^4(s - M_{K_{+}}^2)(15sm^2 + 52m^4 - 2s^2)}{45s^2m^2(s - 4m^2)^3},
\]
\[
T_8(3\, P_1) = \frac{16\pi^2 \alpha^3 Q^4(s - M_{K_{+}}^2)(s + 2m^2)}{9s^2m^2(s - 4m^2)^2},
\]
\[
F_1(3\, P_2) = \frac{16\pi^2 \alpha^3 Q^4(s - M_{K_{+}}^2)(12sm^2 + 96m^4 + s^2)}{45s^3m^2(s - 4m^2)^2},
\]
\[
G_1(3\, P_2) = \frac{16\pi^2 \alpha^3 Q^4(s - M_{K_{+}}^2)(s + 6m^2)(12sm^2 + 320m^4 - 3s^2)}{225s^3m^2(s - 4m^2)^3},
\]
\[
T_8(3\, P_2) = \frac{16\pi^2 \alpha^3 Q^4(s - M_{K_{+}}^2)(s + 6m^2)}{45s^2m^2(s - 4m^2)^2},
\]

where the determined values of \(T_8(3\, P_j)\) constitute the main result of this work. Some explanations are in order. The reason for the appearance of the physical quarkonium mass \(M_{K_{+}}^2\) is the phases-space integration where we use the kinematics of the physical quarkonium, i.e. \(p^2 = M_{K_{+}}^2\). We choose to keep this dependence here, although it, of course, could be removed with the aid of Gremm-Kapustin relations.

Since no experimental measurements for this process are available yet, it is not possible to say how the inclusion of the corrections from \(|Q\bar{Q}g\rangle\) Fock states influences the agreement with the real data. A numerical study that should incorporate new corrections into the theoretical prediction for the production cross-section is in preparation.

5 Discussion and outlook

In summary, using NRQCD factorization we computed a particular type of \(O(\alpha_s^3 v^2)\) corrections to the quarkonium production process \(e^+e^- \rightarrow \chi_{cj} + \gamma\), that were not considered in the previous studies of this reaction. To our knowledge, this is the first production calculation which takes into account operators that contribute through the sub-leading Fock state \(|Q\bar{Q}g\rangle\), such that the ultrasoft final state gluon is treated as a part of the quarkonium system. The non-covariant matching between QCD and NRQCD was carried out both in the rest and in the lab frame of the quarkonium and the matching coefficients agree. The publication and the numerical analysis of the obtained results are in preparation.
With the increasing precision of NRQCD predictions for quarkonium production and decays, we
expect to see further studies dealing with the effects of higher order relativistic corrections in the near
future. Such corrections are particularly important for charmonia, where the comparably large value
of $v$ leads to a slow convergence of the non-relativistic expansion. In this respect we hope that the
present work can inspire future studies in this direction, which will shed more light on the role of the
higher Fock state contributions in the phenomenology of heavy quarkonia.

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