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XIIth Quark Confinement & the Hadron Spectrum

The Parameters of The Tsallis Distribution at the LHC

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Abstract. This talk focuses on fits, using the Tsallis distribution, extending to very large values of the transverse momentum of charged particles in p-p collisions and on the energy dependence of the parameters. A thermodynamically consistent form of the distribution is used for fitting the transverse momentum spectra at mid-rapidity. The fits based on the proposed distribution provide a very good description over 14 orders of magnitude. The parameters obtained show a smooth increase in the value of q and a corresponding smooth decrease in the value of T with increasing beam energy.

1 Introduction

It was shown in recent publications that fits based on the Tsallis distribution [1] give a good description of transverse momentum distributions of charged particles measured in p-p collisions at the LHC [2–10]. Some of these fits extend to values of p_T up to 200 GeV/c [11–15] and provide an excellent description over 14 orders of magnitude in the transverse momentum spectrum. The reason for such a good description was conjectured in [10] by suggesting that Tsallis statistics is a sort of effective theory for QCD description in the region where (approximate) scale invariance holds. The relation of Tsallis statistics and QCD was later explored in detail in [12].

Of the many forms proposed, one in particular [16, 17] leads to a consistent version of thermodynamics for the particle number, energy density and pressure.

The Tsallis thermodynamic quantities, for a system of massive particles, can be written as integrals over certain combinations of the Tsallis distribution f, the modulus of the momentum $p \equiv |\boldsymbol{p}|$ and the energy $E_{\rm p} \equiv \sqrt{p^2 + m^2}$. The distribution is defined for any $q \geq 1$ by

$$f \equiv \left[1 + (q-1)\,\frac{E_p - \mu}{T}\right]^{-\frac{1}{q-1}}, \tag{1}$$

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which, aside from q and T contains also the chemical potential μ as a parameter. It is now obvious that the above reduces to the Boltzmann-Gibbs exponential distribution, in the limit $q \to 1$.

It can be shown (see [17] for more details) that the total particle number N, the total energy E, and the pressure P are given by

$$N = nV \equiv gV \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} f^q, \tag{2}$$

$$E = \epsilon V \equiv gV \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} E_p f^q, \tag{3}$$

$$P \equiv g \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{p^2}{3 E_p} f^q, \tag{4}$$

where we note that the lower case letters stand for the corresponding densities, V being the volume and g the degeneracy factor.

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor. This introduces only one new parameter q which for transverse momentum spectra is close to 1. The advantages are that all thermodynamic consistency conditions are satisfied:

$$n = \left.\frac{\partial P}{\partial \mu}\right|_T, \quad s = \left.\frac{\partial P}{\partial T}\right|_\mu, \quad T = \left.\frac{\partial \epsilon}{\partial s}\right|_n, \quad \mu = \left.\frac{\partial \epsilon}{\partial n}\right|_s.$$

are satisfied. and the parameter T truly deserves its name since

$$T = \left.\frac{\partial E}{\partial S}\right|_{V,N}.$$

2 Transverse Momentum Distributions

In terms of the rapidity and transverse mass variables, $E_p = m_T \cosh y$, Eq. (1) becomes (at mid-rapidity $y = 0$ and for $\mu = 0$)

$$\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = gV \frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}}. \tag{5}$$

Integrating the above expression over the rapidity variable leads to the equivalent form first derived in [18]

$$\begin{aligned}\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = \left.\frac{dN}{dy}\right|_{y=0} & \frac{p_T m_T}{T} \left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)} \\ & \times \frac{(2-q)(3-2q)}{(2-q)m^2 + 2mT + 2T^2} \\ & \times \left[1 + (q-1)\frac{m}{T}\right]^{1/(q-1)}\end{aligned} \tag{6}$$

or, showing only the dependence on the transverse variables:

$$\begin{aligned}\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = \left.\frac{dN}{dy}\right|_{y=0} & p_T \frac{m_T}{T} \left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)} \\ & \times (\text{factors independent of } p_T)\end{aligned} \tag{7}$$

At large transverse momenta the asymptotic behavior is

$$\lim_{p_T\to\infty} \frac{d^2N}{dp_T\ dy}\bigg|_{y=0} \propto p_T \left[\frac{p_T}{T}\right]^{-1/(q-1)}$$

which shows that the scale is being set by the temperature T and the asymptotic behavior is set by q. It is highly sensitive to small deviations from 1 in this last variable. The upper limit for q is given by

$$q < \frac{4}{3}. \tag{8}$$

For larger values of q the integrals become divergent [19].

The above power law has been used to fit the p_T spectra of charged particles measured by the ATLAS [20] and CMS [21] collaborations in [14]. The ATLAS collaboration has reported the transverse momentum in an inclusive phase space region taking into account at least two charged particles in the kinematic range $|\eta| < 2.5$ and $p_T > 100$ MeV [20]. The CMS collaboration has presented the differential transverse momentum distribution covering a p_T range up to 200 GeV/c, the largest range ever measured in a colliding beam experiment [21]. The fits are presented in Figs. (1) and (2).

The results can be compared to those obtained in [7, 11, 12] where very good fits to transverse momentum distributions were presented. The quality of the fits s confirmed albeit with different values of the parameters since a different version is being used.

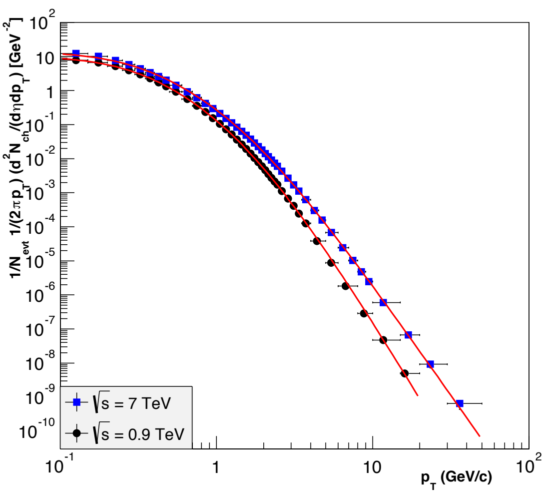

Figure 1. Charged particle multiplicities as a function of the transverse momentum measured by the ATLAS collaboration for events with $n_{ch} \geq 2$, p_T >100 MeV and $|\eta| < 2.5$ at $\sqrt{s} = 0.9$ and 7 TeV in proton - proton collisions [20] fitted with Tsallis distribution [14].

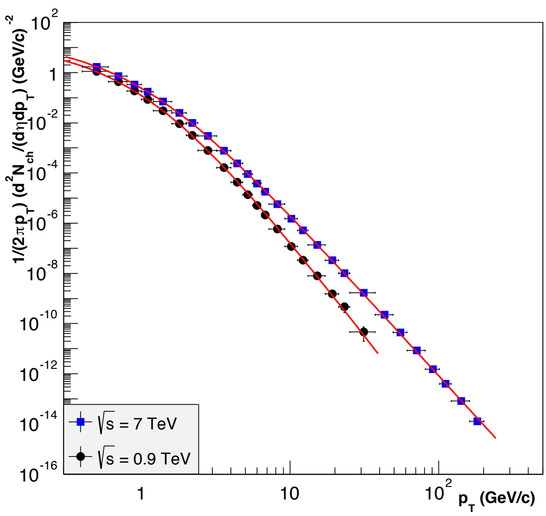

Figure 2. Charged particle differential transverse momentum yields measured within $|\eta| < 2.4$ by the CMS collaboration in proton - proton collisions at $\sqrt{s} = 0.9$ and 7 TeV [21] fitted with the Tsallis distribution [14].

The resulting parameters are listed in table 1.

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Table 1. Values of the q, T and R parameters and χ $/NDF$ obtained from fits to the p_T spectra measured by the ATLAS [20] and CMS [21] collaborations.

Experiment	$\sqrt{s}$ **(TeV)**	q	T **(MeV)**	R **(fm)**	χ^2/NDF
ATLAS	0.9	1.129 ± 0.005	74.21 ± 3.55	4.62 ± 0.29	0.66/36
ATLAS	7	1.150 ± 0.002	75.00 ± 3.21	5.05 ± 0.07	4.35/41
CMS	0.9	1.129 ± 0.003	76.00 ± 0.17	4.32 ± 0.29	0.65/17
CMS	7	1.153 ± 0.002	73.00 ± 1.42	5.04 ± 0.27	0.52/24

3 Energy Dependence of Tsallis Parameters

The dependence of the parameters q and T on beam energy have been analyzed in detail in [10]. This was recently extended to lower energies in [24] where the pion spectra were analyzed consistently so as to include also the results of the NA61 [25], PHENIX [26], STAR [27], CMS [28] and ALICE [29] collaborations. As can be seen, the value of q increases smoothly with beam energy which indicates

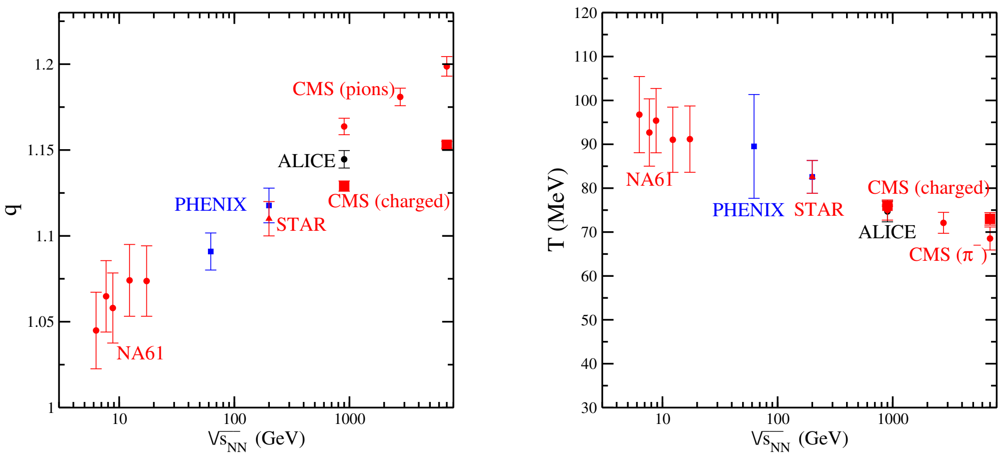

Figure 3. Energy dependence of the Tsallis parameters q (left-hand pane) and T (rigt-hand pane) as deduced from the transverse momentum spectra for π^-.

that the distribution is steadily moving away from a Boltzmann distribution, this is in agreement with a previous analysis [10]. It is a bit of a surprise that the T is smoothly decreasing as a function of beam energy as this behaviour was not seen as clearly in a previous analysis [10].

4 Conclusions

The Tsallis distribution gives a very good description of the transverse momenta distributions observed in $p-p$ collisions at high energies. The parameters obtained show a smooth increase for the values of q and a decrease in the parameter T. It is remarkable that the transverse momentum distributions measured up to 200 GeV/c in p_T can be described consistently over 14 orders of magnitude by a straightforward Tsallis distribution.

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