Branching ratio estimates of $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ decays

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Abstract. We study the rare $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ decays. These processes occur via isospin violation and belong to the so-called second-class currents unseen in Nature so far. Our analysis is based on the framework of resonance chiral theory supplemented by dispersion relations. In this contribution we discuss the prospects for their discovery at forthcoming B-factories such Belle-II. While we find a total branching fraction for the $\pi^- \eta$ decay mode of $\sim 1.7 \times 10^{-5}$, well within the reach of Belle-II, the $\pi^- \eta'$ channel might be one or two orders of magnitude more suppressed.

1 Introduction

The $\tau$ is the only lepton heavy enough to decay into hadrons. Actually, its partial decay width involving hadrons in the final state is of $\sim 65\%$. At the exclusive level, hadronic $\tau$ decays represent a clean laboratory to access the non-perturbative regime of QCD, i.e. they are useful to understand the hadronization of QCD currents, to study form factors and to extract resonance parameters. Rare decay channels of the $\tau$ are a very promising area to pursue at forthcoming B-factories since the interaction is suppressed and the sensitivity to new physics might eventually be enhanced. In the Standard Model (SM), the suppression of rare $\tau$ decays might be due to several reasons e.g.:

- Cabbibo suppression: strange hadronic final states are suppressed with respect to non-strange ones since the $V_{us}$ element of the CKM matrix enters the description instead of $V_{ud}$. Examples of Cabbibo suppressed decays are $\tau^- \to K^- \eta \nu_\tau$ and $\tau^- \to K^- \eta' \nu_\tau$ that we have analyzed in Refs. [1, 2].

- Phase-space: decays involving kaons, $\eta$ or $\eta'$ mesons in the final state lead, usually, to phase-space suppression because of the large mass of these states.

- Second-class currents: decays occurring via $G$-parity violation i.e. the $G$-parity of the vector current is opposite to the $G$-parity of the hadronic system. In the isospin limit, $G$-parity is exact and these processes are forbidden in the SM. However, isospin is an $SU(2)$ approximate symmetry broken both by the up and down quark mass and electric charge differences leading the suppression.

In this contribution we address the study of the rare second-class currents $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ decays [3]. Our goal is twofold: to describe the participating hadronic form factors, predict the decay spectra and estimate the corresponding branching ratios in order to stimulate the experimental collaborations to measure these decays for the first time.

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2 Decay distribution and form factors

The expression for the invariant mass distribution has been derived in detail in Ref. [3] and reads

\[
\frac{d\Gamma(\tau^- \to \pi^- \eta^{(\prime)} \nu_{\tau})}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{24\pi^3 s} \sum_{V} |V_{ud}|^2 |F_{\pi^- \eta^{(\prime)}}(0)|^2 \left( 1 - \frac{s}{M_\tau^2} \right)^2 \times \left[ \left( 1 + \frac{2s}{M_\tau^2} \right) q_{\pi^- \eta^{(\prime)}}(s) |\bar{F}_{\pi^- \eta^{(\prime)}}(s)|^2 + \frac{3\Delta_{\pi^- \eta^{(\prime)}}}{4s} q_{\pi^- \eta^{(\prime)}}(s) |\bar{F}_{0}(s)|^2 \right],
\]

(1)

where \( q_{\pi\eta}(s) \) is a kinematical factor and \( \bar{F}_{\pi^- \eta^{(\prime)}}(s) \) are the vector (+) and scalar (0) form factors normalized to unity at the origin. Notice that \( F_{\pi^- \eta^{(\prime)}}(0) \), an isospin-violating term (cf. Eq. (2)), factorizes in Eq. (1) explaining the smallness of these decays.

2.1 Form factors

- **Vector form factor:** Our initial approach to build the \( \pi^- \eta^{(\prime)} \) vector form factor is through resonance chiral perturbation theory (RChT). It reads [3]

\[
F_{\pi^- \eta^{(\prime)}}(s) = \varepsilon_{\pi\eta} \left( 1 + \sum_{V} \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s} \right),
\]

(2)

where \( \varepsilon_{\pi\eta} = 9.8(3) \cdot 10^{-3} \) and \( \varepsilon_{\pi\eta'} = 2.5(1.5) \cdot 10^{-4} \) are isospin-violating prefactors we derived in Ref. [3], accounting for the \( \pi \eta \) and \( \pi \eta' \) mixing, respectively. In this framework, the term inside the parenthesis is the \( \pi^- \pi^0 \) vector form factor. Therefore, for our analysis we implement the pion form factor extracted from the experimental data on the well-known \( \tau^- \to \pi^- \pi^0 \nu_{\tau} \) decay mode [4].

- **Scalar form factor:** we have considered several parameterizations ordered according to their increasing fulfillment of analyticity and unitarity (See Ref. [3] for details and expressions).

  - Breit-Wigner: we start again with the RChT framework by considering first the exchange of the \( a_0(980) \) resonance and then including the \( a_0(1450) \) into the description. We have taken into account some (elastic and inelastic) final state hadronic interactions effects by resuming the imaginary part of the self-energy loop function into the propagator. However, this description does not incorporate the real part of the loop function and, therefore, it violates analyticity.
  
  - Elastic dispersion relation: to fulfill analyticity and elastic unitarity we rely on the Omnès integral. Unfortunately, we lack of any kind of experimental data either for the phase shift or for the decays spectra. Therefore, for our analysis we get a model for the phase from the elastic scattering amplitudes computed in Ref. [5] in RChT at one-loop unitarized through the \( N/D \) method.
  
  - Coupled channels: in Ref. [3] we have provided an alternative expression equivalent to the Omnès solution with one subtraction, given as a closed-form expression, for the single elastic case. We have also shown that one might accommodate inelasticities and obtain form factors in coupled-channels reasoning in the same way. In particular, for our analysis we have considered two- and three-coupled channels problem for the \( \pi^- \eta, K^- K^0 \) and \( \pi^- \eta' \) scalar form factors.

3 Branching ratio predictions

In Fig. 1 we show the distribution for the \( \tau^- \to \pi^- \eta \nu_{\tau} \) decay depending on the scalar form factor we have employed. The vector contribution dominates the distribution at low-energies with a clear
Table 1. Branching ratio predictions for $\tau \rightarrow \pi \eta^{'}/\nu_\tau$ [3].

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>Vector contribution</th>
<th>Scalar contribution</th>
<th>Branching ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^- \rightarrow \pi^- \eta \nu_\tau$</td>
<td>$0.26(2) \cdot 10^{-5}$</td>
<td>$1.41(9) \cdot 10^{-5}$</td>
<td>$1.67(9) \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\tau^- \rightarrow \pi^- \eta' \nu_\tau$</td>
<td>$[0.3, 5.7] \cdot 10^{-10}$</td>
<td>$[1 \cdot 10^{-7}, 1 \cdot 10^{-6}]$</td>
<td>$[1 \cdot 10^{-7}, 1 \cdot 10^{-6}]$</td>
</tr>
</tbody>
</table>

signal of the $\rho$ while the scalar contribution might dominate at intermediate-and high-energies with a significant signal of the $a_0(980)$ or of the $a_0(1450)$ resonances. Our predictions are collected in Table 1 from where we observe that both the vector and scalar contributions are of the same order, though the scalar ones tend to dominate the decay, and that we respect the current experimental upper bounds of $9.9 \cdot 10^{-5}$ at 95% CL, $7.3 \cdot 10^{-5}$ at 90% CL and of $1.4 \cdot 10^{-4}$ at 95% CL reported by BaBar, Belle and CLEO collaborations, respectively [6–8].

Regarding the $\pi^- \eta'$ mode, the scalar form factor tends to dominate the decay instead. Apart from phase-space suppression, this channel is one or two orders of magnitude smaller than the $\pi^- \eta$ channel because of the smallness of the value of the $\pi^- \eta'$ mixing, $\varepsilon_{\pi \eta'}$.

Figure 1. $\tau^- \rightarrow \pi^- \eta \nu_\tau$ decay distribution. The vector contribution (red-dashed curve) and the full distribution employing the scalar form factor in its elastic version (black solid curve), in the three coupled-channels analysis (green dot-dashed curve) and using a Breit-Wigner with two resonances (blue dotted curve) are displayed.

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