Strong Couplings of Three Mesons with Charm(ing) Involvement

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Abstract. We determine the strong couplings of three mesons that involve, at least, one η_c or J/ψ meson, within the framework of a constituent-quark model by means of relativistic dispersion formulations. For strong couplings of J/ψ mesons to two charmed mesons, our approach leads to predictions roughly twice as large as those arising from QCD sum rules.

1 Three-meson strong coupling from meson–meson transition amplitudes

We determine the strong couplings of three mesons at least one of which is one of the charmonia η_c and J/ψ , generically called $g_{PP'V}$ and $g_{PV'V}$ for pseudoscalar mesons P of mass M_P and vector mesons V of mass M_V and polarization vector ε_μ and defined, for momentum transfer $q \equiv p_1 - p_2$, by the amplitudes

$$\begin{aligned}\langle P'(p_2) V(q) | P(p_1) \rangle &= -\frac{g_{PP'V}}{2} (p_1 + p_2)^\mu \varepsilon_\mu^*(q), \\ \langle V'(p_2) V(q) | P(p_1) \rangle &= -g_{PV'V} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\mu}(q) \varepsilon^{*\nu}(p_2) p_1^\rho p_2^\sigma,\end{aligned}$$

from the residues of *poles* situated at the masses M_{P_R} and M_{V_R} of (appropriate) pseudoscalar and vector resonances P_R and V_R and contributing to *transition form factors* $F_+^{P>P'}(q^2)$, $V^{P>V}(q^2)$ and $A_0^{P>V}(q^2)$, in terms of vector quark currents $j_\mu \equiv \bar{q}_1 \gamma_\mu q_2$ and axial-vector quark currents $\hat{j}_\mu \equiv \bar{q}_1 \gamma_\mu \gamma_5 q_2$ defined by

$$\begin{aligned}\langle P'(p_2) | j_\mu | P(p_1) \rangle &= F_+^{P>P'}(q^2) (p_1 + p_2)_\mu + \dots, & F_+^{P>P'}(q^2) \Big|_{\text{pole}} &= \frac{g_{PP'V_R} f_{V_R}}{2 M_{V_R} (1 - q^2/M_{V_R}^2)}, \\ \langle V(p_2) | j_\mu | P(p_1) \rangle &= \frac{2 V^{P>V}(q^2)}{M_P + M_V} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu}(p_2) p_1^\rho p_2^\sigma, & V^{P>V}(q^2) \Big|_{\text{pole}} &= \frac{(M_V + M_P) g_{PVV_R} f_{V_R}}{2 M_{V_R} (1 - q^2/M_{V_R}^2)}, \\ \langle V(p_2) | \hat{j}_\mu | P(p_1) \rangle &= i q_\mu (\varepsilon^*(p_2) p_1) \frac{2 M_V}{q^2} A_0^{P>V}(q^2) + \dots, & A_0^{P>V}(q^2) \Big|_{\text{pole}} &= \frac{g_{PP_RV} f_{P_R}}{2 M_V (1 - q^2/M_{P_R}^2)},\end{aligned}$$

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where the P or V decay constants $f_{P,V}$ parametrize the matrix elements of the interpolating currents $j_\mu^{(5)}$

$$\langle 0 | j_\mu^{(5)} | P(q) \rangle = i f_P q_\mu, \quad \langle 0 | j_\mu | V(q) \rangle = f_V M_V \varepsilon_\mu(q).$$

Such strong-coupling results may prove to be useful for studies of long-distance QCD effects in hadron decays involving charmed mesons or charmonia in the final state of a kind similar to the one in Ref. [1].

2 Quark-model-underpinned dispersion analysis of transition form factors

We describe the relevant properties of the involved strongly coupling mesons by means of a relativistic constituent-quark model [2–4]. Of course, this requires us to match the QCD currents $j_\mu^{(5)}$ to associated constituent-quark currents, which is, for heavy quarks, easily effected by introducing form factors $g_{V,A}$,

$$j_\mu = g_V \bar{Q}_1 \gamma_\mu Q_2 + \text{other Lorentz structures}, \quad j_\mu^{(5)} = g_A \bar{Q}_1 \gamma_\mu \gamma_5 Q_2 + \text{other Lorentz structures},$$

for which we choose $g_V = g_A = 1$ [5] but, for light quarks, rendered rather involved [6, 7], for instance, if embedding partial axial-current conservation. For the radial meson wave functions, Gaussian shapes

$$w_{P,V}(k^2) \propto \exp\left(-\frac{k^2}{2\beta_{P,V}^2}\right), \quad \int dk^2 w_{P,V}^2(k^2) = 1,$$

with slopes $\beta_{P,V}$ given, together with all relevant mesonic features, in Table 1 [8–13], turn out to suffice for our purposes. Table 2 lists the numerical values adopted for the masses of the constituent quarks Q . Within the framework of a relativistic dispersion formalism (reviewed, e.g., in Ref. [14]), we represent each transition form factor $\mathcal{F}(q^2) = F_+^{P>P'}(q^2)$, $V^{P>V}(q^2)$, $A_0^{P>V}(q^2)$ by a double dispersion integral of a double spectral density $\Delta_{\mathcal{F}}(s_1, s_2, q^2)$ the one-loop contributions to which derive from Feynman graphs like the ones in Fig. 1 and each decay constant $f_{P,V}$ by a dispersion integral of a spectral density $\rho_{P,V}(s)$,

$$\mathcal{F}(q^2) = \int ds_1 ds_2 \phi_1(s_1) \phi_2(s_2) \Delta_{\mathcal{F}}(s_1, s_2, q^2), \quad f_{P,V} = \int_{(m_1+m)^2}^{\infty} ds \phi_{P,V}(s) \rho_{P,V}(s),$$

Table 1. Relevant parameters of the mesons: numerical values of mass M , leptonic decay constant f and slope β .

Meson	M (GeV)	f (MeV)	β (GeV)
D	1.87	206 ± 8	0.475
D^*	2.010	260 ± 10	0.48
D_s	1.97	248 ± 2.5	0.545
D_s^*	2.11	311 ± 9	0.54
η_c	2.980	394.7 ± 2.4	0.77
J/ψ	3.097	405 ± 7	0.68

Table 2. Constituent mass of each quark flavour $Q = u, d, s, c$ [5] involved in charm(ing) three-meson couplings.

Quark flavour	Quark mass m (GeV)
u	0.23
d	0.23
s	0.35
c	1.45

Figure 1. Feynman graphs of transitions among the mesons η_c , J/ψ , D , D^* , induced by the currents $\bar{c} \gamma_\mu c$ or $\bar{c} \gamma_\mu d$.

and of the wave functions of all mesons entering the corresponding one- or two-meson matrix elements

$$\phi_{P,V}(s) = \frac{\pi}{s^{3/4}} \sqrt{\frac{s^2 - (m_1^2 - m^2)^2}{2[s - (m_1 - m)^2]}} w_{P,V} \left(\frac{(s - m_1^2 - m^2)^2 - 4m_1^2 m^2}{4s} \right).$$

3 Three-meson strong coupling: determination from transition amplitudes

We fix the slopes $\beta_{P,V}$ such that the decay constants $f_{P,V}$ are reproduced by their spectral representation. Equipped with these $\beta_{P,V}$ values, we deduce all strong couplings from the spectral representation of the relevant form factors $\mathcal{F}(q^2)$ derived sufficiently off the resonances at M_R ($R = P_R, V_R$), by interpolating pointwise given momentum dependences of $\mathcal{F}(q^2)$ by three-parameter ($\sigma_{1,2}, \mathcal{F}(0)$) ansätze of the form

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{1 - \sigma_1 q^2/M_R^2 + \sigma_2 q^4/M_R^4} \frac{1}{1 - q^2/M_R^2}, \quad \text{Res } \mathcal{F}(M_R^2) = \frac{\mathcal{F}(0)}{1 - \sigma_1 + \sigma_2}, \quad R = P_R, V_R,$$

and extrapolating $\mathcal{F}(q^2)$ to the poles at $q^2 = M_R^2$, where the strong couplings emerge from the residues. Using $\sigma_{1,2}, \mathcal{F}(0)$, and M_R as fit parameters, all arising masses M_R come close to the known resonances. Quite generally, a given strong coupling may show up in and therefore can be extracted from more than one meson–meson transition form factor, for example, $g_{\eta_c \eta_c \psi}$ from $F_+^{\eta_c \rightarrow \eta_c}$ or $A_0^{\eta_c \rightarrow \psi}$ (see Fig. 2) [15–17], $g_{DD\psi}$ from $F_+^{D \rightarrow D}$ or $A_0^{D \rightarrow \psi}$ (see Fig. 3(a)) [15–17] and $g_{DD^* \eta_c}$ from $F_+^{\eta_c \rightarrow D}$, $A_0^{\eta_c \rightarrow D^*}$ or $A_0^{D \rightarrow D^*}$ (see Fig. 3(b)) [15–17]; for further examples of such multiple involvements, consult Tables III, V, and VI of Ref. [15].

Figure 2. Behaviour of the off-shell η_c - η_c - J/ψ strong coupling $g_{\eta_c \eta_c \psi}$ with increasing resonance-mass-normalized momentum transfer $x \equiv q^2/M_R^2$ for the transition of η_c to the η_c (solid red line) or the J/ψ meson (dotted blue line).

Figure 3. Behaviour of the “off-shell” D - D - J/ψ and D - D^* - η_c strong couplings $g_{DD\psi}$ and $g_{DD^*\eta_c}$, respectively, with increasing “resonance-mass-normalized” momentum transfer $x \equiv q^2/M_R^2$: (a) $g_{DD\psi}(x) = 2 M_\psi (1-x) A_0^{D \rightarrow \psi}(q^2)/f_D$ (blue dotted line and squares ■) and $g_{DD\psi}(x) = 2 M_\psi (1-x) F_+^{D \rightarrow D}(q^2)/f_\psi$ (red solid line and triangles ▲), relying on interpolation (blue or red lines) or not (squares ■, triangles ▲); (b) $g_{DD^*\eta_c}(x)$ (solid red line), $g_{DD^*\eta_c}(x)$ (dotted blue line) and $g_{DD^*\eta_c}(x)$ (dashed green line). For each transition, the relevant resonance, R , is identified by a circumflex.

4 Strong coupling predictions from relativistic constituent-quark approach

We collect our emerging strong-coupling findings — extracted, in the case of multipresence of one and the same three-meson coupling in more than one meson–meson transition amplitude, by a combined fit — in Table 3: Strange quark content instead of a down quark implies a reduction of the involved strong couplings, by roughly 10%. Confronting, in Table 4, our $D_{(s)}\text{-}D_{(s)}^{(*)}\text{-}J/\psi$ predictions with QCD sum-rule outcomes [18–20], the QCD sum-rule estimates prove to be lower than ours [15–17] by a factor of two.

Table 3. Charm(ing) three-meson strong couplings: quark-model-based dispersion-approach outcomes [15–17].

$PP'V$ Coupling	Strong coupling $g_{PP'V}$	$PV'V$ coupling	Strong coupling $g_{PV'V}$ (GeV ⁻¹)
$\eta_c\text{-}\eta_c\text{-}J/\psi$	25.8 ± 1.7	$\eta_c\text{-}J/\psi\text{-}J/\psi$	10.6 ± 1.5
$D\text{-}D\text{-}J/\psi$	26.04 ± 1.43	$D\text{-}D^*\text{-}J/\psi$	10.7 ± 0.4
$D\text{-}D^*\text{-}\eta_c$	15.51 ± 0.45	$D^*\text{-}D^*\text{-}\eta_c$	9.76 ± 0.32
$D_s\text{-}D_s\text{-}J/\psi$	23.83 ± 0.78	$D_s\text{-}D_s^*\text{-}J/\psi$	9.6 ± 0.8
$D_s\text{-}D_s^*\text{-}\eta_c$	14.15 ± 0.52	$D_s^*\text{-}D_s^*\text{-}\eta_c$	8.27 ± 0.37

Table 4. Strong couplings of the J/ψ meson to two charmed mesons: relativistic quark model vs. QCD sum rule.

Coupling \ Approach	Quark model [15–17]	QCD sum rules	References
$D\text{-}D\text{-}J/\psi$	26.04 ± 1.43	11.6 ± 1.8	[18]
$D\text{-}D^*\text{-}J/\psi$	$(10.7 \pm 0.4) \text{ GeV}^{-1}$	$(4.0 \pm 0.6) \text{ GeV}^{-1}$	[18]
$D_s\text{-}D_s\text{-}J/\psi$	23.83 ± 0.78	$11.96^{+1.34}_{-1.16}$	[19]
$D_s\text{-}D_s^*\text{-}J/\psi$	$(9.6 \pm 0.8) \text{ GeV}^{-1}$	$(4.30^{+1.53}_{-1.22}) \text{ GeV}^{-1}$	[20]

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