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XIIth Quark Confinement & the Hadron Spectrum

Confinement in F_4 Exceptional Gauge Group Using Domain Structures

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Abstract. We calculate the potential between static quarks in the fundamental representation of the F_4 exceptional gauge group using domain structures of the thick center vortex model. As non-trivial center elements are absent, the asymptotic string tension is lost while an intermediate linear potential is observed. *SU(2)* is a subgroup of F_4. Investigating the decomposition of the 26 dimensional representation of F_4 to the *SU(2)* representations, might explain what accounts for the intermediate linear potential, in the exceptional groups with no center element.

1 Introduction

Quarks -the fundamental particles of nature- interact via non-Abelian gauge fields namely gluons. However, no isolated quark and gluon have been detected in labs yet, which is traditionally called quark confinement. In fact, confinement could be related to the formation of an electric flux tube and a linear potential between static quarks. To study confinement, there are vast variety of methods, from Lattice Gauge Theory to different Phenomenological Models. In the latter, QCD vacuum is assumed to be filled with topological field configurations such as magnetic monopoles, center vortices, merons and caloron gas [1]. These objects cause the expectation value of a large Wilson loop to obey the area-law falloff which implies a linear potential between static quarks.

In the center vortex model that has been put forward by G.'t Hooft [2], the potential between static color sources is a result of the interaction between center vortices and Wilson loop. A center vortex is a topological field configuration which is Line-like (in $D = 3$ dimensions) or Surface-like (in $D = 4$ dimensions) having some finite thickness. A center vortex carries quantized magnetic flux with respect to the non-trivial center elements of the gauge group [3]. The original center vortex theory was capable of explaining quark confinement at large distances, yet unqualified to illustrate the intermediate linear potential, in particular, for higher representations. The thick center vortex model [3] is the generalization of this theory that produces the intermediate linear potential consistent with Casimir scaling for higher representations, qualitatively.

In an attempt to understand what feature of vacuum fluctuations accounts for Casimir scaling at intermediate distances, Greensite *et al.* have improved the previous model using both trivial and non-trivial center elements resulting in the vacuum domain structure model [4]. Using the thick center vortex model with the idea of vacuum domain structures, Deldar *et al.* have found the potential

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between static color sources in the G_2 gauge group [5],[6],[7]. Their results are compatible with numerical Lattice calculations for the G_2 gauge group [8],[9],[10],[11]. As a trivial result, the potential of every representation is screened at large distances, since G_2 gauge group contains only one trivial center element. The intermediate linear potential is claimed to be related to the *SU(2)* and *SU(3)* subgroups of the G_2 gauge group.

In this paper, we calculate the potential between static quarks in the fundamental representation of the F_4 group. We try to interpret the confinement of quarks at intermediate distances by means of the subgroups of F_4, using the same procedure done by Deldar *et. al* for the G_2. In Sec. 2 we introduce the thick center vortex and vacuum domain structure models, briefly. Some properties of F_4 exceptional group and its subgroups are presented in Sec. 3. In Sec. 4, we apply the thick center vortex model to F_4 and by plotting the group factor of this group, we claim that confinement is the outcome of the center elements of the F_4 subgroups.

2 Thick Center Vortex Model with Trivial and Non-Trivial Center Elements

In the thick center vortex model, the potential between static color sources is the result of the interaction between thick center vortices and Wilson loop, which could be partially or completely. In an attempt to increase the intermediate linear regime, it was assumed that both center vortices and vacuum domains exist in the QCD vacuum [4]. Center vortices are quantized to non-trivial center elements, while vacuum domains are quantized to the trivial one and carry a zero magnetic flux. Therefore, the potential energy between static sources induced by center vortices and vacuum domains is

$$V(R) = -\sum_{m=-\infty}^{+\infty} \ln\left\{1 - \sum_{n=0}^{N-1} f_n (1 - \mathrm{Re}\, \mathcal{G}_r[\vec{\alpha}_C^{(n)}(x_m)])\right\}, \tag{1}$$

in which $x_m = m + \frac{1}{2}$ is the location of the center of the vortex. f_n is the probability that any plaquette on the lattice is pierced by a vortex type n . $n = 0$ represents the vacuum domain and $n = 1, \cdots, N-1$ indicates the center vortices. C represents the Wilson loop and the group factor, $\mathcal{G}_r[\vec{\alpha}^{(n)}]$, is

$$\mathcal{G}_r\left[\vec{\alpha}^{(n)}\right] = \frac{1}{d_r} \mathrm{Tr}\ \exp\left[i\, \vec{\alpha}^{(n)}.\vec{H}\right], \tag{2}$$

where the H_i's, $\{i = 1, \cdots, N-1\}$ are the generators spanning the Cartan sub-algebra and d_r is the dimension of the representation r. $\alpha_C^{(n)}(x)$ indicates the vortex profile function and depends on what fraction of the vortex flux enters the loop C. Thus, it depends on both the shape of the loop C, and the position x of the center of the vortex core, relative to the perimeter of the loop. Vortices which pierce the plaquettes far from Wilson loop do not have any effect on the loop. In addition, when the distance R between the quark and anti-quark goes to zero, the percentage of the vortex core which is inside the loop must go to zero. Moreover, when the vortex core is completely inside the Wilson loop area, then the influence of the vortex on the Wilson loop is given by a center element:

$$\exp\left[i\vec{\alpha}^{(n)}.\vec{H}\right] = z_n \mathbb{I} \tag{3}$$

There are different ansatzes which satisfy these conditions. One of them, introduced in Ref. [3], is as the following:

$$\vec{\alpha}_i^{(n)} = \frac{\alpha_i^{n(max)}}{2}[1 - \tanh(a\, y(x) + \frac{b}{R}], \tag{4}$$

where R represents the space-extent of the Wilson loop with finite but large time-extent and x determines the position of the vortex core. a is proportional to the inverse of the vortex thickness. The parameter b introduces a dependency on the space-extent R of the Wilson loop into the vortex profile. The value of $\alpha_i^{n(max)}$ is calculated from the condition of Eq. (3) and $y(x)$ is

$$y(x) = \begin{cases} x - R & \text{for} \quad |R - x| \leq |x| \\ -x & \text{for} \quad |R - x| > |x|. \end{cases} \tag{5}$$

3 Some Properties of F_4 Exceptional Group

In general, there are five distinguishable exceptional groups named as G_2, F_4, E_6, E_7 and E_8. The subscripts point out the rank of the groups. Therefore, F_4, similar to *SU(5)* , has rank 4 and possesses four Cartan generators and four simple roots. It is, in terms of the size, the second exceptional lie group. The fundamental and adjoint representations of F_4 are 26 and 52 dimensional representations. The universal covering group of F_4 is the group itself and all the irreducible representations are real. So, there will be no complex conjugate in this group. F_4 has three maximal regular subgroups: *SO(9)*; *SU(3)*×*SU(3)* and *SP(6)*×*SU(2)* which could be obtained from extended Dynkin diagram. In addition, it has two maximal singular subgroups including *SU(2)* and $G_2 \times$ *SU(2)*.

Four Cartan generators of F_4 are [12]:

$$H_1 = D_5^5 + D_6^6 - D_7^7 + D_8^8 - D_9^9 - D_{10}^{10}, \tag{6}$$

$$H_2 = D_3^3 + D_4^4 - D_5^5 - D_6^6 + D_{10}^{10} - D_{11}^{11}, \tag{7}$$

$$H_3 = \frac{1}{2}(D_2^2 - 2D_3^3 - D_4^4 + D_6^6 - D_8^8 + D_9^9 - D_{10}^{10} + D_{11}^{11} - D_{12}^{12}), \tag{8}$$

$$H_4 = \frac{1}{2}(-2D_2^2 + D_3^3 - D_4^4 + D_5^5 - D_6^6 + D_7^7 - D_9^9 + D_{12}^{12} - D_{13}^{13}). \tag{9}$$

where $D_{ab} = I_{ab} - I_{\overline{ba}}$ and I_{ab} is the 26×26 matrix with elements as follows :

$$(I_{ab})_{jk} = \delta_{aj}\,\delta_{bk}, \tag{10}$$

where labels j and k have the same value as a and b such that $a, b\,:\, -13 \leqslant j,k \leqslant 13$. It should be noted that by using the standard normalization condition

$$\mathrm{Tr}\,[H_a H_b] = \frac{1}{2}\delta_{ab}, \tag{11}$$

the normalization factors $N_1 = N_2 = \frac{1}{2\sqrt{6}}$ and $N_3 = N_4 = \frac{1}{2\sqrt{3}}$ are obtained for H_1 to H_4, respectively.

In this paper, we report the results of the decomposition of F_4 to *SU(2)* $\times$ G_2. According to the branching rule of F_4, the fundamental representation might be decomposed into the mentioned subgroups as the following [14],[15] :

$$26 = (5, 1) \oplus (3, 7) = 5\{1\} \oplus 3\{7\}. \tag{12}$$

The first number in the parentheses represents the dimension of the irreducible representations of the *SU(2)* and the second one belongs to the G_2 gauge group. By adopting the same approach used in Refs.[6],[7] and utilizing Eq. (12), we can reconstruct the Cartan generator H of F_4 for the fundamental 26-dimensional representation:

$$H^{26}_{F_4 \supset SU(2) \oplus G_2} = \frac{1}{\sqrt{3}}\,\mathrm{diag}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & H_8^7 & H_8^7 & H_8^7 \end{bmatrix} \tag{13}$$

where H_8^7 is the Cartan generator of G_2 [6]:

$$H_8^7 = \frac{1}{\sqrt{6}} \begin{pmatrix} \sigma_3^2 & 0 & 0 \\ 0 & \sigma_3^2 & 0 \\ 0 & 0 & \sigma_3^3 \end{pmatrix} \tag{14}$$

where σ_3^2 and σ_3^3 are the Cartan generators of *SU(2)* in the fundamental and adjoint representations, respectively. It should be mentioned that the Cartan generator of Eq. (14) has been constructed from the decomposition of the fundamental 7-dimensional representation of the G_2 to its *SU(2)* subgroups:

$$7 = (2, 2) \oplus (1, 3) = 2\{2\} \oplus \{3\}. \tag{15}$$

Using Eqs. (12) and (15), $\mathbb{Z}^{26}$, the center element of the *SU(2)* $\times G_2$ subgroup, could be written as follows:

$$\mathbb{Z}^{26} = \mathrm{diag}\,(1, 1, 1, 1, 1, z\mathbb{I}_{2\times2}, z\mathbb{I}_{2\times2}, \mathbb{I}_{3\times3}, z\mathbb{I}_{2\times2}, z\mathbb{I}_{2\times2}, \mathbb{I}_{3\times3}, z\mathbb{I}_{2\times2}, z\mathbb{I}_{2\times2}, \mathbb{I}_{3\times3})\,, \tag{16}$$

in which $z = e^{i\pi}$ is the non-trivial center element of the *SU(2)* and $\mathbb{I}$ is the unit matrix.

4 Potentials of the F_4 Exceptional Group

To acquire $\alpha_i^{\max}$, we use the condition of Eq. (3) and obtain 12 independent equations. Solving these equations, we obtain different sets of answers that differ by a coefficient. One set is

$$\alpha_1^{max} = 2\pi\sqrt{24}\,; \qquad \alpha_2^{max} = 6\pi\sqrt{24}\,; \qquad \alpha_3^{max} = 4\pi\sqrt{48}\,; \qquad \alpha_4^{max} = 2\pi\sqrt{48} \tag{17}$$

Fig. 1 shows $V(R)$ versus $R \in [0, 100]$ for the 26-dimensional fundamental representation. The free parameters of the model has been chosen as $f_0 = 0.1$, $a = 0.05$ and $b = 4$. Screening is clearly observed

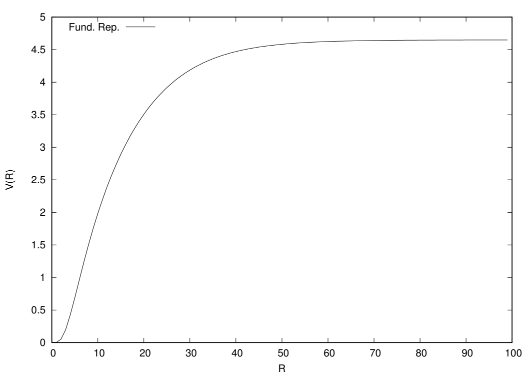

Figure 1. The Potential between two static sources in the fundamental representation of the F_4. Screening is clearly observed at large quark separations while the potential is linear at intermediate distances.

at large quark separations but what accounts for the linearity of the potential at intermediate distances? We try to relate the intermediate linearity to the subgroups of F_4. So, we apply the model to *SU(2)*$\times G_2$ subgroups of F_4. Utilizing Eqs. (16) and (13) in the condition $\exp(i\alpha^{max}_{SU(2)\oplus G_2} H^{26}_{F_4 \supset SU(2)\oplus G_2}) = \mathbb{Z}^{26}$, we attain $\alpha^{max}_{SU(2)\oplus G_2} = 2\pi\sqrt{18}$. Using this value, we are able to calculate the potential of Eq. (1) for the *SU(2)* subgroup of F_4.

Fig. 2 represents the potential of the fundamental representation of F_4 itself and when it is decomposed to its subgroup in the range $R \in [0, 100]$. The free parameters of the model have been chosen as $f = 0.1$, $a = 0.05$ and $b = 4$. To be more accurate, at the interval $R \in [9, 14]$, the discrepancy

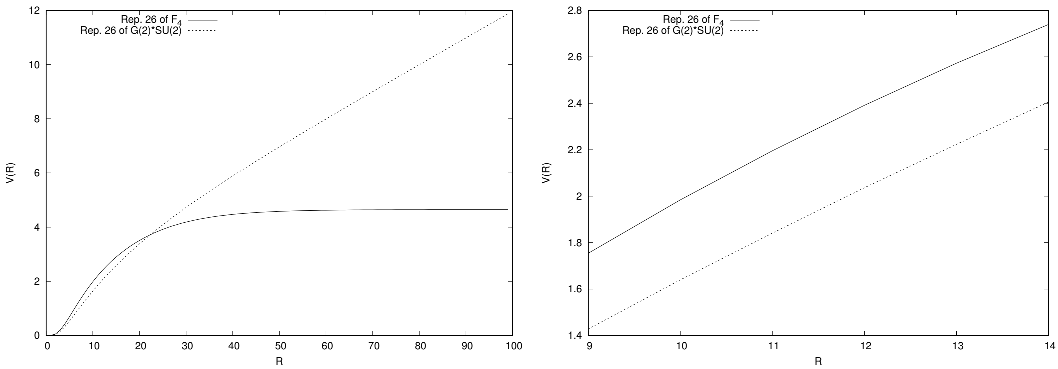

Figure 2. Left digram: The potentials for the F_4 and its subgroup *SU(2)* $\times G_2$ in the fundamental representation in the range $R \in [0, 100]$. Right diagram: The same the left diagram but for $R \in [9, 14]$. The two curves are nearly parallel to each other.

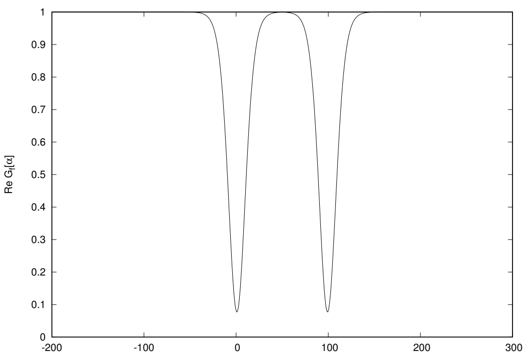

Figure 3. The group factor $\mathrm{Re}\,\mathcal{G}_f[\alpha]$ versus the location x of the center of the vortex. This factor changes between 1 and 0.08.

between the slopes of the two curves is about 0.8%. So, it seems that subgroups of F_4 can interpret the linearity of the potentials at intermediate distances.

Investigating the behavior of the group factor $\mathcal{G}_f[\vec{\alpha}]$, for the fundamental representation of F_4, might be interesting. Fig. 3 pictures the real part of the group factor of the fundamental representation of the F_4 group in the range $[-200, 300]$. It is clear that the maximum value of $\mathrm{Re}\mathcal{G}_f[\alpha]$ is equal to 1 when the vortex core is far from the Wilson loop or when it is completely inside it. The lower limit for the real part of the group factor equals to 0.08. Using the same method in Refs.[5],[6],[7] one may write:

$$\begin{aligned}\min\left\{\mathrm{Re}\mathcal{G}_f[\vec{\alpha}(x)]\right\}_{SU(2)\oplus G_2} = \frac{1}{26}\mathrm{Re}\left[\mathrm{Tr}\left(\exp(i\alpha H)\right)_{min}\right] &= \frac{1}{26}\mathrm{Re}\mathrm{Tr}\left(\mathbb{Z}^{26}\right)\\ &= \frac{1}{26}(5-2-2+3-2-2+3-2-2+3)\\ &= 0.0769 \simeq 0.08\end{aligned} \quad (18)$$

5 Conclusion

The simple phenomenological thick center vortex model has been applied to the F_4 exceptional group. The potential between quarks in the fundamental representation is screened at large distances due to the absence of non-trivial center elements. While, confinement is observed in the form of linearity of the potential at intermediate regime. Hence, the exceptional group F_4 might represent the possibility of confinement without the center. Despite unavailable Lattice Monte Carlo calculations concerning F_4, one might claim that the results it similar to G_2 and could be another laboratory to study the existence of confinement in groups with the trivial center. On the other hand, arguments about the reason of the linearity of the potential has led to the agreement on the imposing the role of *SU(2)* as a subgroup of F_4 in the structure of $SU(2) \times G_2$. It appears as if the F_4 QCD is dominated by *SU(2)* center elements. Ultimately, for future research, the role of other subgroups of F_4 and also the potential of the adjoint representation are under investigations.

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