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Photo-absorption sum rules σ_{-1} and further questions about correlations of constituents inside bound hadrons

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Abstract. Within the non-relativistic approach to photo-absorption sum rules for the 3N(4N)-nuclei the new σ_{-1} sum rules are proposed which are based on general charge-symmetry (CS) consequences for the "CS-conjugated" triton and ^{3}He and the CS-self-conjugated ^{4}He. Combining the relativistic constituent quark model of the nucleon and the known spin-,isospin- and the dipole-moment fluctuation sum rules, the relevant moments of the quark coordinate distribution and correlation functions in the ground state of the nucleon are derived and discussed.

1 Introduction

The observed effects of the structure-dependent asymmetries in the static electromagnetic characteristics of hadrons, such as parameters of form-factors or coefficients of the electromagnetic polarizability, or in characteristics of processes induced by lepton- or photon-hadron interactions, e.g., total or partial cross sections, are known as very useful means to study and to understand the underlying dynamics.

The sum rules, in particular, have long served as a reliable constraints on, or relations between measured quantities, depending only on most general principles and statements of the theory.

In this work we shall concentrate mainly on two topics: the sum rules for static electromagnetic characteristics of hadrons and the role of the dynamical short-range correlations of nuclear constituents in description of the behavior of the total and polarized photonuclear or photon-nucleon cross section.

As it is known, the non-relativistic dipole sum rules continue to be a useful tool in the theory of the atomic and nuclear photo-effect.

$$\sigma_n(E1) = \int_{thr}^{\infty} d\omega\, \omega^n\, \sigma_{E1}(\omega).$$

Examples: $n = -2$ $\rightarrow$ Kramers-Heisenberg sum rule (SR) for static electric-dipole polarizability of a given quantum system;
$n = -1$ $\rightarrow$ the bremsstrahlung-weighted SR, dependent of charged-"parton" correlation in a given system;
$n = 0$ $\rightarrow$ the famous Thomas-Reiche-Kuhn SR, known as a precursor of the Quantum Mechanics. As far as the nucleon is the relativistic 3q system, the relativistic generalization of sum rules is needed, including the spin degrees of freedom and spin-dependent interactions and correlations of partons.

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2 The non-relativistic σ_{-1} sum rules for few-body nuclei

Besides sum rules including the measurable physical quantities and obtained on the base of the current algebras and the $p_z \to \infty$ techniques, the similar sum rules (but without the explicitly spin-dependent terms) were obtained in the non-relativistic (NR) nuclear physics context. In particular, the NR sum rules σ_{-1} for the s-shell nuclei ($A = Z + N \leq 4$) were earlier obtained from the assumption that the ground-state wave function is symmetric in the space coordinates of nucleons [1],[2] for each nucleus of the isotopic doublet ^3H and ^3He and for ^4He

$$4\pi^2\alpha\frac{NZ}{A-1}(\frac{1}{3})\cdot\langle r_{ch}^2\rangle_{NR} = \int\frac{d\nu}{\nu}\sigma_{E1}(\nu). \tag{1}$$

The present-day situation is that the accuracy of the theoretical results [3] is higher than the experimental uncertainties in the important energy range up to the pion production threshold [4] suggesting that new experimental data would be timely. The main objective of this section is a comparison of the Charge Independence (CI) and Charge Symmetry (CS) initial assumptions taken for the derivation of the σ_{-1} sum rules for the "mirror" ^{3}He -^{3}H and the "charge-self-conjugate" ^{4}He nuclei. The validity of the Foldy [1] and Khokhlov [2] sum rules assumes validity of the (nucleon) charge independence (CI) symmetry. Our objective in this section is to apply stronger charge symmetry (CS) requirements in derivation of the F-Kh-type sum rules. In accord with the CS-provisions for the mean values of the nucleon radii correlations in the mirror pair ^{3}H- and ^{3}He nuclei and the "self-conjugate" ^{4}He nucleus, we have get that the values of $< D^2 >_{^3He}$, $< D^2 >_{^3H}$, and $\langle r_{ch}^2\rangle_{^3H}$ can be expressed via one and the same linear combination of the introduced and undetermined parameters which is different from the analogues expression for $\langle r_{ch}^2\rangle_{^3He}$. Taken at face value this means that with our stricter assumption on the underlying symmetry i.e. the CS instead of CI-option, we get only one "mixed" sum rule instead of three (for ^{3}H,^{3}He and ^{4}He) earlier mentioned Foldy-Khokhlov sum rules:

$$\sigma_{-1}(^3H)[2.45mb] = \frac{4\pi^2\alpha}{3}\cdot\langle r_{pp}^2\rangle_{ch}^{^3H}[2.42mb] = \sigma_{-1}(^3He)[2.54 \pm .09mb]. \tag{2}$$

The nuclei charge radii in the above relations and in what follows are understood as corresponding to the case of "point-charge-protons" ones which reached by subtracting from "physical" values standard corrections, e.g.[5].

The problem of the radii difference in the charge-symmetry-conjugated 3N-nuclei is of similar degree of complexity as the long-discussed binding energy difference of the ^{3}He and ^{3}H: $E_b(^3H) = 8.52MeV$ and $E_b(^3He) = 7.76MeV$, e.g. [6, 7]. For a qualitative understanding of the intended estimation, we resort to dimensional arguments and factors as follows:

$$\frac{E_b(^3He)}{E_b(^3H)}[.911] \cong \frac{\langle r_{ch}^2\rangle_{^3He}^{-1/2}}{\langle r_{ch}^2\rangle_{^3H}^{-1/2}}[0.899]. \tag{3}$$

The unexpected numerical closeness of two ratios written down on exclusively dimensional ground signals nevertheless that for its full dynamical explanation one should include into consideration not only the Coulomb interaction effects but all collection of the CS-breaking-effects as in [6, 7], that represent on the nuclei and hadronic scales the fundamental current u- and d-quark mass difference.

The modified $\sigma_{-1}(^4He)$ sum rule keeping the basic CS-symmetry limitations can be presented in the form

$$\sigma_{-1}(E1)^{^4He} = \frac{16}{9}\pi^2\alpha(\langle {r_c}^2\rangle_{^4He} + \beta_{pp} - \beta_{pn}). \tag{4}$$

The notations for the $\beta_{NN'}$ through the scalar products of the nucleon radial coordinates were introduced in the paper [1]. For the CS-self-conjugate $^4He_2$-nucleus all $\beta_{pp} = \beta_{nn} \neq \beta_{pn}$ can be computed if the explicit form of the ground state radial wave function is available.

3 Relativistic σ_{-1} sum rule and its application for one-electron atom

In what follows we consider relativistic dipole moment fluctuation sum rules in the "valence-parton" approximation, that is neglecting virtual particle-antiparticle configurations in the ground state of the considered systems or diffractively produced in the final states of photo-absorption reactions. The anomalous magnetic moment sum rules express a model-independent correspondence between static properties of a particle (or bound system of particles) and integrals over the photo-absorption spectrum. For particles with the spin $S = 1/2$ the sum rule for the anomalous magnetic moment κ (or GDH-sum rule) reads [8–10]

$$\frac{2\pi^2\alpha\kappa^2}{m^2} = \int_{thr}^{\infty} \frac{d\nu}{\nu}(\sigma_p(\nu) - \sigma_a(\nu)) \tag{5}$$

where $\sigma_{p(a)}$ refers to the cross-section with the parallel(anti-parallel) spins of the target and photon.

In QED,the validity of the sum rule for free electron was checked in two lowest orders of the perturbation theory [11, 12]. Later on, for the physical reasons, we shall replace κ^2 entering different sum rules just by its integral expression in the GDH sum rule. In particular, for the relativistic dipole moment fluctuation sum rule

$$4\pi^2\alpha[\frac{1}{3} < D^2 > -\frac{\kappa^2}{4m^2}] = \int_{thr}^{\infty} \frac{d\nu}{\nu}\sigma_{tot}(\nu), \tag{6}$$

we obtain another form to be used later

$$4\pi^2\alpha[\frac{1}{3} < D^2 >] = \int_{thr}^{\infty} \frac{d\nu}{\nu}(\sigma_p(\nu). \tag{7}$$

We apply first derived sum rule to the system of the highly ionized atom Pb^{81+}, initiated about half-century ago by J.S. Levinger and co-workers [13]. Using the form of the sum rule with our included term κ_{atom} we reduced deviation between left- and right-hand sides of the sum rule, discovered in earlier works, to one-half percent. Numerically:

$$4\pi^2\alpha\frac{1}{3} < D^2 > [937.2b] - 4\pi^2\alpha(\frac{\kappa}{2M})^2[67.9b] = \int_{thr}^{\infty} \frac{d\nu}{\nu}\sigma_{tot}(\nu)[874b], \tag{8}$$

where for the anomalous magnetic moment of the atom we used the total magnetic moment of electron bound in the lowest S-wave orbit [14].

4 Relativistic constituent quark model and nucleons

Following formally to the $p_z \to \infty$ techniques derivation of the Cabibbo-Radicati [15] or GDH sum rule [16], we can obtain the relation

$$4\pi^2\alpha(\frac{1}{3} < \vec{D}^2 >) = \int \frac{d\nu}{\nu}\sigma_p^{res}(\nu). \tag{9}$$

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We use the definitions

$$\hat{D} = \int \vec{x}\hat{\rho}(\vec{x})d^3x = \sum_{j=1}^{3} Q_q(j)\vec{d}_j, \hat{r_1^2} = \int \vec{x}^2\hat{\rho}(\vec{x})d^3x = \sum_{j=1}^{3} Q_q(j)\vec{d}_j^{\,2}. \quad (10)$$

The defined operators $Q_q(j)$ and $\vec{d}_j$ are the electric charges and configuration and spin variables of point-like interacting quarks in the infinite-momentum frame of the bound system.

Finally, we relate the electric dipole moment correlators consecutively for the proton, the neutron, the pure "isovector-nucleon" and the iso-vector part of the mean-squared radii operators, which all are sandwiched by the nucleon state vectors in the "infinite - momentum frame", with experimentally measurable data on the resonance parts of the photo-absorption cross sections on the proton and neutron presently known below $\sim$ 2 GeV [17]. The listed operator mean values are presented as follows

$$R_V = \frac{1}{2}(< r_1^2 >_P - < r_1^2 >_N) = \alpha - \frac{1}{2}\beta. \quad (11)$$

$$J_P = \frac{1}{3} < \hat{D}^2 >_P = \frac{8}{27}\alpha + \frac{1}{27}\beta + \frac{8}{27}\gamma - \frac{8}{27}\delta. \quad (12)$$

$$J_N = \frac{1}{3} < \hat{D}^2 >_N = \frac{2}{27}\alpha + \frac{4}{27}\beta + \frac{2}{27}\gamma - \frac{8}{27}\delta. \quad (13)$$

$$J_V = \frac{1}{3} < \hat{D}^2 >_V = \frac{2}{3}\alpha + \frac{1}{3}\beta + \frac{2}{3}\gamma - \frac{4}{3}\delta, \quad (14)$$

where $< \vec{d}_1^{\,2} >=< \vec{d}_2^{\,2} >= \alpha, < \vec{d}_3^{\,2} >= \beta, < \vec{d}_1 \cdot \vec{d}_2 >= \gamma, < \vec{d}_1 \cdot \vec{d}_3 >=< \vec{d}_2 \cdot \vec{d}_3 >= \delta$

indices "1" and "2" refer to the like quarks (i.e. to the $u(d)$- quarks inside the proton (neutron)), and "3" to the odd quark. Evaluation of the relativistic electric dipole moment fluctuation and the isovector charge radius sum rules for the nucleon was carried out with the available compilation [17] of the resonance pion-photoproduction data on the proton and neutron with the helicity $\lambda = 1/2(3/2)$ $A^{P(N)}_{1/2}$ and $A^{P(N)}_{3/2}$ and all integrals over photo-excited nucleon resonances were taken in the narrow resonance approximation:

$$J^{res}_{p(a)} \simeq \frac{4\pi m_n |A^{res}_{3/2(1/2)}|^2}{m^2_{res} - m^2_n}, \quad (15)$$

where $m_{n(res)}$ is the nucleon (or resonance) mass. Solving the system of the linear equations and evaluating the R_V,$J_{P,N,V}$ with the help of experimentally known partial amplitudes of main photo-excited resonances, we find our final results for the numerical values α, β and the opening angle θ_{12} and θ_{13} between vectors $\vec{d}_1$ and $\vec{d}_2$ and vectors $\vec{d}_{1(2)}$ and $\vec{d}_3$:

$$\alpha^{1/2} = 0.75 \pm 0.06 f, \quad (16)$$

$$\beta^{1/2} = 0.77 \pm 0.12 f, \quad (17)$$

$$\theta_{12} \simeq 120^0, \quad (18)$$

$$\theta_{13} \simeq \theta_{23} \sim 120^0, \tag{19}$$

$$< {r_1}^2 >_V = 0.25 \pm 0.02 f^2 [exp : .29 f^2]. \tag{20}$$

Having got the values of the "global" parameters α,β, etc. defined in the infinite momentum frame, we put all three vectors with the pertinent numerical parameters into the plane transverse to $p_z \to \infty$ momentum and define parameters of the closed contour circumscribing their ends. The resulting curve is either ellipse with very small eccentricity or the circle. This fact is, in our opinion, at variance with rather popular "quark-diquark" models of the nucleon with, presumably, more noticeable asymmetry of the spatial correlations of the *uu*- and *ud* - quark pairs. It should be stressed that the accepted parametrization of the integrals $R_V \div J_V$ is corresponding to the CS-breaking option, that is, the cited pairwise closeness of the α and β, or γ and δ may be due to the approximate nature of our numerical estimates. Their actual difference may be connected, e.g., with the mass and coupling constant difference of the charged and neutral pion within the valence quarks of nucleons, that is with the mass-difference of the u- and d-quarks, as the fundamental reason of the charge-symmetry breaking.

5 Concluding remarks

1. The generalization of the non-relativistic Foldy-Khokhlov sum rules is proposed for the "CS-mirror" ^{3}He -^{3}H and the "CS-self-conjugate" ^{4}He nuclei. The introduced corrections replace the charge-independence, (CI)- approximations used earlier.

2. The relativistic current-algebra and dispersion relation based sum rule establishing new relation between characteristics of the electron bound in the hydrogen-like, highly-charged ions is checked numerically.

3. With the accuracy about 15%, the isovector charge radius of the nucleon was calculated via the Cabibbo-Radicati sum rule through the integral of total resonance photo-production of hadrons on the nucleons. The mentioned deficit should be referred to the "pion-sea" presence in the nucleon state vector.

4. Approximately derived values of the parameters defining quark correlations in the nucleon turn out qualitatively close to each other contrary to expectations based on the implementation of the CS-vs-CI-symmetry, as it is seen in pairwise comparison of α-vs-β or γ-vs-δ. Thus, these data don't seemingly be consistent with the asymmetric quark-diquark model of the nucleon.

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