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Zero-momentum correlator in SU(2) Landau gauge gluodynamics under various boundary conditions

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Abstract. We make new simulations of $SU(2)$ zero spatial momentum (ZM) gluon correlator in Landau gauge gluodynamics paying special attention to possible lattice artefacts and Gribov copy effect. In particular we started investigation of the correlator dependence on the choice of boundary conditions (BCs) by comparing results for periodic and zero-field (ZF) boundary conditions at various β values. Time behaviour of the ZM correlator for ZF BCs corresponds to approximately constant in time effective gluon mass thus providing additional evidence in favour of decoupling behaviour of momentum-dependent gluon propagator in the IR region. We have found that at fixed lattice sizes and β values the ZM gluon correlator and effective gluon mass for periodic and ZF boundary conditions can differ considerably.

1 Introduction

The only solution which has been found in lattice studies of momentum-dependent gluon and ghost propagators within Landau gauge gluodynamics on sufficiently large lattices

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z(q^2)}{q^2}\,, \quad G^{ab} = \delta^{ab}\frac{J(q^2)}{q^2}$$

is *decoupling*, or *regular* one, which is characterized by the finite nonzero gluon propagator $D(p^2)$ and the finite ghost dressing function $J(q^2)$ for $q^2 \to 0$ (for details see [1–3]). This solution describes the "effectively massive" gluon and breaks the BRST symmetry contrary to the *scaling*, or *conformal* solution, corresponding to massless gluon, zero $D(0)$ and infinite $J(q^2 \to 0)$. After thorough checking of possible lattice artefacts due to finite-volume, finite-size and Gribov copy effects the *decoupling* solution has been widely accepted by the "community of nonperturbative QCD studies" as the only physical solution. However up to now dependence of the lattice propagators on the choice of boundary conditions has not been reported, to our knowledge (note, that in most lattice studies of gluodynamics periodic boundary conditions (PBCs) are traditionally used). Here we present results of the comparative study of the ZM gluon correlator in $SU(2)$ gluodynamics under periodic and zero-field (sometimes called "open") boundary conditions.

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2 Zero-momentum gluon correlator

2.1 Definition of correlators

Lattice studies of the gauge-dependent "space",

$$S(t) = \sum_{\vec{x}} \sum_{i=1,2,3} \mathrm{Tr}\, \langle\, A_i(t,\vec{x})\, A_i(0,\vec{x})\, \rangle\,,$$

and "time"

$$T(t) = \sum_{\vec{x}} \mathrm{Tr}\, \langle\, A_0(t,\vec{x})\, A_0(0,\vec{x})\, \rangle\,,$$

ZM gluon correlators in the Landau gauge was started in [4] and continued in [5–7].

2.2 Landau gauge fixing

To fix the Landau gauge, we apply to all links a gauge transformation $g \in G$ ($G = SU(2)$) mapping $U_{x,\mu} \to {}^gU_{x,\mu} = g_x U_{x,\mu}\, g^\dagger_{x+\hat{\mu}}$, with the aim to maximize a gauge functional

$$F_U[g] = \frac{1}{8V} \sum_{x,\mu} \Re\mathrm{e}\; \mathrm{Tr}\; {}^gU_{x,\mu}\;, \tag{1}$$

or, more exactly, to find the *global* maximum of $F_U[g]$ [8]. Search for the global maximum in practice is a complicated problem which becomes exceedingly time consuming with increasing lattice volume. Starting from an initial random gauge transformation g_x one generally arrives at one of many *local* maxima of $F_U[g]$. The corresponding gauge-fixed configurations are called Gribov copies; they all satisfy the differential gauge condition $\partial_\mu A_\mu = 0$. To avoid appearance of lattice Gribov copies we successfully applied the Simulated Annealing (SA) method (see, e.g., [3] and references therein), finalized by overrelaxation (OR) iterations. The SA method – often called a "stochastic optimization method"– allows quasi-equilibrium tunneling through functional barriers, in the course of a "temperature" T decrease. It is proven that *infinitely slow* "cooling" of a system under consideration (decreasing of T) allows to reach the *global* maximum of its underlying functional. In practice, the greater is the number of SA "cooling" steps, the closer is the value of $F_U[g]$ local maxima reached in simulations to its global maximum. In this work we for the first time applied the SA technique to the study of the ZM gluon correlator which allowed to diminish dispersion of the correlator $S(t)$ values and thus raise accuracy of "effective" gluon mass calculation.

2.3 Periodic vs Zero-field BCs; ZM correlators

We accomplished high-statistic Monte-Carlo studies of ZM correlators on lattices $L_s^3 \times L_t$, $L_t = 2L_s$ (in most cases), with $L_s = 10, 12, 14, 16, 18$; our maximal lattice extension was $22^3 \times 30$.

The typical number of MC configurations was of order 10^4 with one gauge fixed copy obtained for each MC configurations. For gauge fixing we use $SA + OR$ method with rather slow cooling (large number of SA steps) thus reaching values of the gauge functional G_F very close to its global maximum. We have checked that $T(t)$ is constant in t with very high accuracy (in Landau gauge it should be t-independent [4]), which shows reliability of our simulations.

In the first series of simulations we use zero-field boundary conditions (ZF BCs). It means that link variables $U_{x,\mu}$ are given by unit $SU(2)$ matrix for all links out of the minimal hypercubic volume containing all $L_s^3 \times L_t$ lattice sites.

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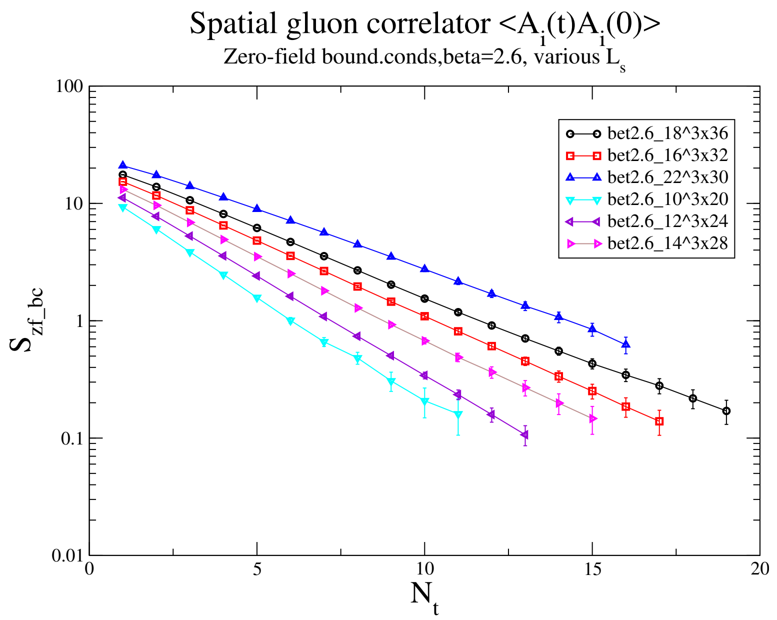

Figure 1. ZM gluon correlator $S(t)$ at $\beta = 2.6$ and various lattice extensions for ZF BCs.

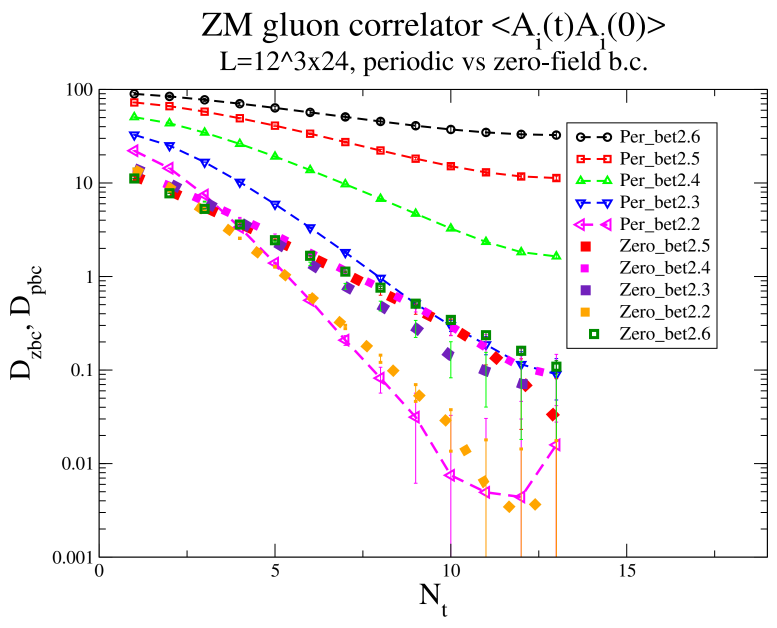

Figure 2. ZM gluon correlator $S(t)$, $L = 12^3 \times 24$, periodic vs zero-field BCs.

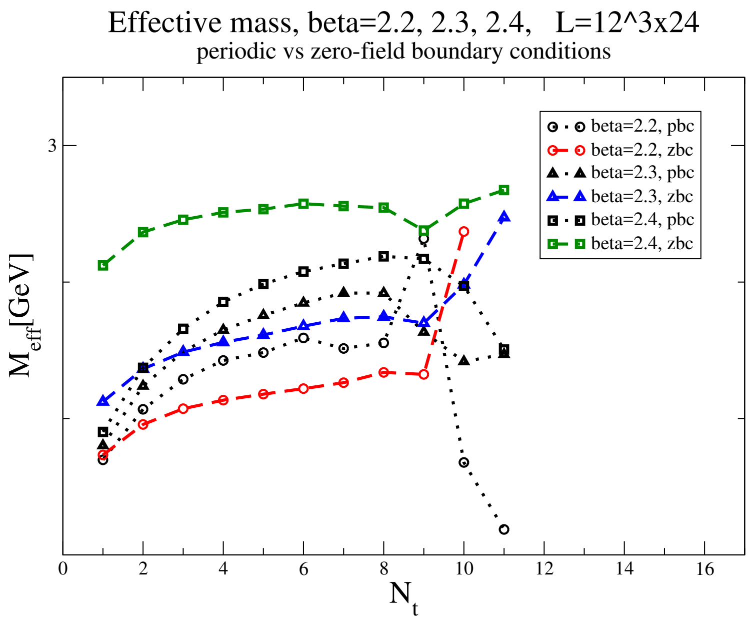

Figure 3. Comparison of dependencies of gluon effective masses on N_t under PBSc and ZF BCs at $L = 12^3 \times 24$ for $\beta = 2.2, 2.3, 2.4$.

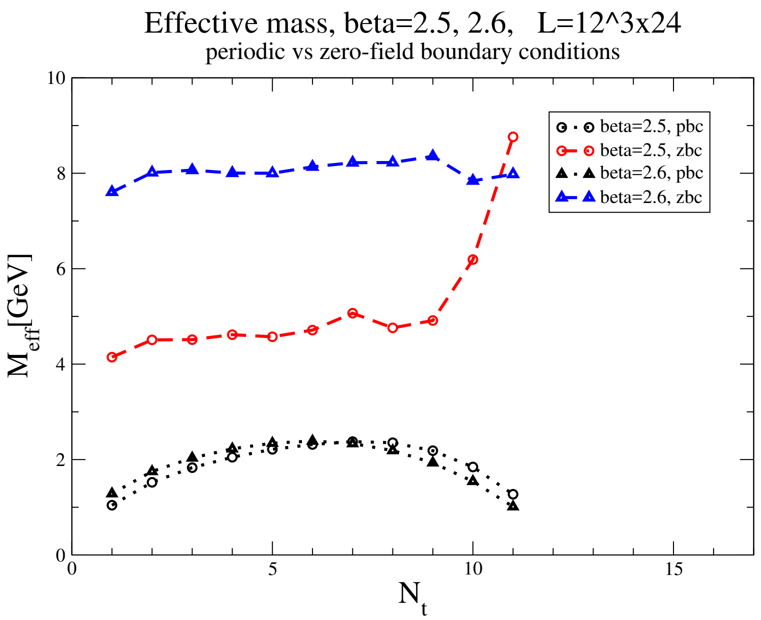

Figure 4. Comparison of dependencies of gluon effective masses on N_t under PBSc and ZF BCs at $L = 12^3 \times 24$ for $\beta = 2.5, 2.6$.

The varying correlator $S(t)$ obtained for $\beta = 2.6$ is plotted in figure 1. It worth noting that correlators $S(t)$ found for ZF BCs at various lattice volumes are presented in the logarithmic scale by nearly straight lines which correspond to gluon "effective" masses independent of N_t contrary to what was found for periodic boundary conditions (PBCs) in [4–7] (see also below).

In the second series of simulations we compare results obtained on the lattice $L = 12^3 \times 24$ for 2 types of boundary conditions: periodic (which are traditionally used in lattice experiments, PBCs) and zero-field (ZF BCs) ones; the results are presented in figure 2. One can see that the $S(t)$ correlators obtained for PBCs and ZF BCs can differ considerably.

Finite-volume effects (mainly) and insufficient statistics (number of MC configurations) can lead to distortion of awaited exponential decrease of $S(T)$ correlator; in particular, the decrease is in most cases smaller at small N_t, and relative statistical errors become progressively larger when $N_t \to L_t/2 + 1$. That is why the notion of N_t-dependent "effective" mass

$$M_{eff}(N_t) = -\,ln\,\frac{S(N_{t+1})}{S(N_t)}$$

is introduced. In figure 3 and figure 4 we compare dependencies of "effective" masses for PBCs and ZF BCs on the lattice $L = 12^3 \times 24$ and various $\beta = \dfrac{2}{g^2}$ (figure 3 for $\beta = 2.2, 2.3, 2.4$ and figure 4 for $\beta = 2.5, 2.6$). One can see the quick increase of $M_{eff}(N_t)$ found under ZF BCs at larger $\beta = 2.5, 2.6$ for which correlation length becomes larger than the physical lattice extension $L_s \times a(\beta)$ due to small values of $a(\beta)$. One might assume that for larger β ZF BCs introduce stronger finite-volume effects, which however should become weaker with growing lattice extension L_s.

Our simulations of ZM gluon correlators at $\beta = 2.6$ and various $L = L_s^3 \times L_t$ with $L_s = 2\,L_t$ allow to find out whether it is really the case. In figure 5 it is clearly seen that the effective mass curves for

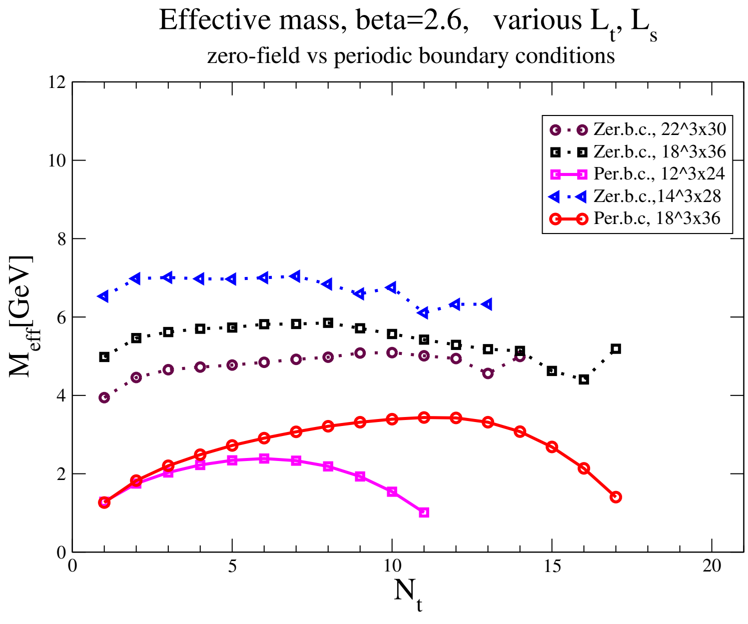

Figure 5. Convergence of dependencies of effective masses at $\beta = 2.6$ for (i) PBCs and (ii) ZF BCs when increasing lattice extensions

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PBCs and ZF BCs mutually converge at growing lattice extensions, tending to some limiting curve which could be used for estimation of effective gluon mass being obtained from the ZM correlator $S(t)$.

Note here, that mass estimates for larger β's are physically preferable because for these β's finite-size effects are considerably lower. Relying on the simulations of the ZM correlator with $\beta = 2.6$ one might propose the following bounds for gluon mass M_g : $2MeV \leq M_g \leq 3.5MeV$.

3 Conclusions

We studied behaviour of zero-momentum gluon correlators having in mind existence of Gribov ambiguity which appears when Landau gauge fixing procedure is applied within Monte-Carlo lattice approach. We used one-copy $SA + OR$ technique with slow cooling which provides good practical solution of the Gribov copy problem. Numerical studies of ZM correlators were accomplished on lattices of various space and time extensions L_s and L_t and various β values.

We have studied dependence of ZM correlator $S(t)$ on the choice of boundary conditions. To this aim we have made simulations of the $S(t)$ ZM correlator both at periodic and zero-field boundary conditions and have studied behaviour of effective masses found from this correlator. In all cases we have found that effective $SU(2)$ gluon mass turns out to be nonzero (thus confirming qualitative results of [4–7] on this issue) and have got the following estimate of M_g in case of $SU(2)$ group: $2MeV \leq M_g \leq 3.5MeV$.

Appearance of nonzero effective gluon mass found when studying zero-momentum gluon correlator can be considered as additional argument in favour of physical importance of decoupling, or regular solution for gluon and ghost momentum-dependent propagators which has been found in Landau gauge gluodynamics in [3] and is characterized by nonzero effective gluon mass.

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