EPJ Web of Conferences **138**, 02007 (2017) DOI: 10.1051/epjconf/201713802007
Baldin ISHEPP XXIII

Electroweak properties of ρ-meson in the instant form of relativistic quantum mechanics

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Abstract. Charge and magnetic radii, magnetic and quadrupole moments of the ρ-meson are calculated in framework of the developed by authors version of the instant form of relativistic quantum mechanics with fixed number of particles (IF RQM). The calculations are performed with different wave functions of quarks in the ρ-meson and using the so- called modified impulse approximation (MIA). The electromagnetic characteristics of ρ-meson are obtained without fitting parameters. The value of the magnetic moment coincides with available experimental data: $\mu_\rho = 2.1 \pm 0.5\, e/2M_\rho$.

1 Introduction

In recent years there has been significant progress in the experimental study of ρ-meson: measurement of the lepton decay constant from the process $\tau \to \rho\nu_\tau$ [1, 2], extraction of the ρ-meson magnetic moment from processes $\gamma^* \to 4\pi$ [3], measurement of the magnetic moment of decay $\rho \to \pi\gamma^*$ [4], the expected measurement of the electromagnetic form factors of the ρ-meson from the reaction $e^+ + e^- \to \rho^+ + \rho^-$ [5]. To the moment, there are a large number of calculations of the rho meson electroweak properties in different approaches (see, e.g., [6–11]). The theoretical calculations of the electroweak properties of the ρ-meson by different methods have important implications for understanding the processes in the transition region between nonperturbative and perturbative quark dynamics, for example. A fairly large amount of experimental information enables to estimate these approaches from the point of view of their ability to self-consistent description of these data.

In the present work the some electroweak characteristics of the ρ-meson (magnetic and quadrupole moments, the charge and magnetic mean-square radii, the lepton decay constant) are calculated in the framework of instant form of relativistic quantum mechanics (IF RQM)(see, e.g., review [12]). Developed by authors formulation of IF RQM is based on a direct realization of the Poincaré algebra on the set of the dynamic observables of the composite system and dates back to the work of P. Dirac [13]. Distinctive features of our variant of IF RQM are the original procedure of construction of the electroweak currents of a composite system and new formulation of the impulse approximation - the so-called modified impulse approximation (MIA). Unlike conventional impulse approximation MIA is formulated in the terms of the reduced current matrix elements on Poincaré group (form factors) and

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DOI: 10.1051/epjconf/201713802007

does not lead to violation of Lorentz-covariance and conservation law for the currents of composite systems.

In this work the model parameters are fixed from the calculations of electroweak properties of pion [14] and from description of the lepton decay constant of ρ-meson [15]. So, our calculations of the magnetic moment and the charge radius are performed without fitting parameters and give the good agreement with experimental data.

2 Electroweak characteristics of ρ-meson and numerical calculations

The electromagnetic current matrix element of the ρ-meson can be written in terms of conventional Sachs form factors for the system with the total angular momentum equal 1. To do this let us write the parameterization of the electromagnetic current matrix element in the Breit frame (see, e.g., [16]):

$$\langle \vec{p}_\rho\,, m_J | j_\mu(0) | \vec{p}_\rho\,'\,, m_J' \rangle = G^\mu(Q^2)\;,$$
$$G^0(Q^2) = 2p_{\rho 0}\left\{(\vec{\xi}\,'\vec{\xi}^{\,*})\,G_C(Q^2) + \left[(\vec{\xi}^{\,*}\vec{Q})(\vec{\xi}\,'\vec{Q}) - \frac{1}{3}Q^2(\vec{\xi}\,'\vec{\xi}^{\,*})\right]\frac{G_Q(Q^2)}{2M_\rho^2}\right\}\;,$$
$$\vec{G}(Q^2) = \frac{p_{\rho 0}}{M_\rho}\left[\vec{\xi}'\,(\vec{\xi}^{\,*}\vec{Q}) - \vec{\xi}^{\,*}(\vec{\xi}\,'\vec{Q})\right]G_M(Q^2)\;. \qquad (1)$$

Here G_C, G_Q, G_M are the charge, quadrupole and magnetic form factors, respectively, M_ρ is the ρ-meson mass.

The polarization vector in the Breit frame has the following form:

$$\xi^\mu(\pm 1) = \frac{1}{\sqrt{2}}(0\;,\;\mp 1\;,\;-i\;,\;0)\;,\quad \xi^\mu(0) = (0\;,\;0\;,\;0\;,\;1)\;. \qquad (2)$$

The variables in ξ are total angular momentum projections.

In the Breit frame:

$$q^\mu = (p_\rho - p_\rho')^\mu = (0\;,\;\vec{Q})\;,\quad p_\rho^\mu = (p_{\rho 0}\;,\;\frac{1}{2}\vec{Q})\;,\quad p_\rho'^{\;\mu} = (p_{\rho 0}\;,\;-\frac{1}{2}\vec{Q})\;, \qquad (3)$$
$$p_{\rho 0} = \sqrt{M_\rho^2 + \frac{1}{4}Q^2}\;,\quad \vec{Q} = (0\;,\;0\;,\;Q)\;.$$

Our approach gives following integral representation for the electromagnetic form factors of composite systems with total angular momentum equal 1 in MIA (see,e.g., [17]):

$$G_C(Q^2) = \int d\sqrt{s}\,d\sqrt{s'}\,\varphi(s)\,g_{0C}(s\,,Q^2\,,s')\,\varphi(s')\;,$$
$$G_Q(Q^2) = \frac{2\,M_\rho^2}{Q^2}\int d\sqrt{s}\,d\sqrt{s'}\,\varphi(s)\,g_{0Q}(s\,,Q^2\,,s')\,\varphi(s')\;, \qquad (4)$$
$$G_M(Q^2) = -\,M_\rho\int d\sqrt{s}\,d\sqrt{s'}\,\varphi(s)\,g_{0M}(s\,,Q^2\,,s')\,\varphi(s')\;.$$

Here $g_{0C}(Q^2)\,,\;g_{0Q}(Q^2)\,,\;g_{0M}(Q^2)$ are so-called free two-particle charge, quadrupole and magnetic form factors, respectively, $\varphi(s)$ is wave function of quarks in the ρ-meson in sense of RQM.

From physical point of view the free two-particle form factors describe the electromagnetic properties of system of the two non-interacting particle with discrete quantum numbers of the ρ-meson.

These form factors are reduced matrix elements of the electromagnetic current operator of the free two-particle system on the Poincaré group. Generally speaking, these form factors are regular generalized functions (distributions) (see [12] for details). These generalized functions can be calculated by the methods of relativistic kinematics and have the form given in [17]. Exactly the appearance of $g_{0C}(Q^2), g_{0Q}(Q^2), g_{0M}(Q^2)$ in the integral representation for the electromagnetic form factor of the ρ-meson is the essence of MIA.

The wave functions in sense of IF RQM in (4) at $J = S = 1$, $l = 0$ are defined by the following expressions (see, e.g., [12]):

$$\varphi(s) = \sqrt[4]{s}\,\psi(k)\,k\ , \quad k = \frac{1}{2}\sqrt{s - 4\,M^2}\ , \tag{5}$$

where M is mass of constituents quark.

Normalization is given by the following condition:

$$\int \psi^2(k)\,k^2\,dk = 1\ , \tag{6}$$

here $\psi(k)$ is a model wave function.

The magnetic moment μ_ρ and the quadrupole moment Q_ρ of ρ meson were calculated using the relations given in [16]:

$$G_M(0) = \frac{M_\rho}{M}\,\mu_\rho\ , \quad G_Q(0) = M_\rho^2\,Q_\rho\ . \tag{7}$$

The static limit in (4) gives the following relativistic expressions for moments:

$$\mu_\rho = \frac{1}{2}\int_{2M}^{\infty} d\sqrt{s}\,\frac{\varphi^2(s)}{\sqrt{s-4\,M^2}}\left\{1 - L(s) + (\kappa_u + \kappa_{\bar{d}})\left[1 - \frac{1}{2}\,L(s)\right]\right\}\ , \tag{8}$$

$$Q_\rho = -\int_{2M}^{\infty} d\sqrt{s}\,\frac{\varphi^2(s)}{\sqrt{s}}\left[\frac{M}{\sqrt{s}+2\,M} + \kappa_u + \kappa_{\bar{d}}\right]\frac{L(s)}{4M\sqrt{s-4M^2}}\ , \tag{9}$$

$$L(s) = \frac{2\,M^2}{\sqrt{s-4\,M^2}\,(\sqrt{s}+2\,M)}\left[\frac{1}{2\,M^2}\sqrt{s\,(s-4\,M^2)} +\ + \ln\frac{\sqrt{s} - \sqrt{s-4\,M^2}}{\sqrt{s} + \sqrt{s-4\,M^2}}\right]\ ,$$

where κ_q is the quark anomalous magnetic moment.

The ρ-meson charge $(\langle\, r_\rho^2\rangle_C)$ and magnetic $(\langle\, r_\rho^2\rangle_C)$ radii are calculated from relation:

$$\langle\, r_\rho^2\rangle_C = -6\,G_C'(0)\ , \quad \langle\, r_\rho^2\rangle_M = -6\,G_M'(0)\ . \tag{10}$$

The lepton decay constant of the ρ-meson, f_ρ is defined by the following matrix element of the electroweak current (see, e.g., [18]):

$$\langle 0|j_\mu^\rho(0)|\vec{P}_\rho, m_\rho\rangle = i\,\sqrt{2}\,f_\rho\,\xi_\mu(m_\rho)\frac{1}{(2\pi)^{3/2}}\ , \tag{11}$$

where $\vec{P}_\rho$ is the meson three-momentum, $m_\rho = -1, 0, 1$ is the spin projection, $\xi_\mu(m_\rho)$ is the polarization vector that in the Breit frame has the form (2).

For the calculation of the ρ-meson lepton decay constant we used the method of construction of current matrix element which is nondiagonal with respect to the total angular momentum [15]. Final expression for f_ρ in approximation of four-fermion interaction in our approach has form [15]:

$$f_\rho = \frac{\sqrt{3}}{\sqrt{2}\pi}\int_0^{\infty} dk\,k^2\,\psi(k)\frac{(\sqrt{k^2+M^2}+M)}{(k^2+M^2)^{3/4}}\left(1 + \frac{k^2}{3(\sqrt{k^2+M^2}+M)^2}\right)\ . \tag{12}$$

 DOI: 10.1051/epjconf/201713802007

For the description of the relative motion of quarks the following phenomenological wave functions are used:

1. A Gaussian or harmonic oscillator wave function (see, e.g., [19]):

$$\psi(k) = N_{HO}\exp\left(-k^2/2\,b^2\right). \tag{13}$$

2. A power-law wave function (see, e.g., [19]):

$$\psi(k) = N_{PL}\,(k^2/b^2 + 1)^{-n}\ , \quad n = 2\,,3\ . \tag{14}$$

For Sachs form factors of constituent quarks we used the following expressions:

$$G^q_E(Q^2) = e_q\, f_q(Q^2)\ , \quad G^q_M(Q^2) = (e_q + \kappa_q)\, f_q(Q^2)\ , \tag{15}$$

where e_q is the quark charge.

For $f_q(Q^2)$ the form proposed in [20] is chosen:

$$f_q(Q^2) = \frac{1}{1 + \ln(1 + \langle r^2_q\rangle Q^2/6)}\ , \tag{16}$$

here $\langle r^2_q\rangle$ is the MSR of constituent quark.

The choice of form factor of the constituent quark as (15), (16) due to the fact that the asymptotic of the electromagnetic pion form factor at $Q^2 \to \infty$ obtained in our nonperturbative approach with (16) does coincide with that predicted by QCD including the pre-asymptotic factor (see, e.g., [21] for details).

Parameters in our calculation are fixed from description of electroweak properties of pion [14]. Let us note that our RQM describes well the experimental data for the pion form factor including the recent points [22]. Let us emphasize that the parameters used in our calculations were obtained from the fitting to the experimental data up to $Q^2 \simeq 0.26$ GeV^2 [23]. At that time the data for higher Q^2 was not correlated in different experiments and had significant uncertainties. The later data for pion form factor in JLab experiments up to Q^2 =2.45 GeV^2 were obtained with rather good accuracy. All experimental points obtained in JLab up to now agree very well with our prediction of 1998.

So, by analogy with pion calculations from [14] we use the following set of parameters: M=0.22 GeV for the constituent quark mass; the quark anomalous magnetic moments enter our expressions through the sum $(\kappa_u + \kappa_{\bar d})$ and we take $\kappa_u + \kappa_{\bar d} = 0.0268$ in natural units; for the quark MSR we use the relation $\langle r^2_q\rangle \simeq 0.3/M^2$. The parameters of the wave functions b in (13), (14) were fixed from the requirements of the description of experimental values of ρ-meson lepton decay constants 152 ± 8 MeV [1]. The ρ-meson mass in our calculations is taken from [2] M_ρ = 775.5±0.4 MeV.

So, all parameters of our model are fixed. Results of numerical calculations are presented in table 1. Let us remark that all electroweak characteristics of the ρ-meson except the lepton decay constant are calculated without fitting parameters.

As it can be seen from table the calculated value of the magnetic moment is in agreement with experimental data. The values of $\langle r^2_\rho\rangle_C$, while not measured directly, are important for testing various conjectures about strongly interacting systems. One of the interesting related prediction was introduced as a consequence of the so-called Wu–Yang hypothesis [24] (see also [25]), though it is remarkable by itself. Namely, one may define the radius of a hadron either in terms of the electroweak interaction (the mean square charge radius, $\langle r^2_{\rm ch}\rangle$, calculated for the ρ meson in this paper) or in terms of the strong interaction (this radius, $\langle r^2_{\rm st}\rangle$, is defined by the slope of the cross section of

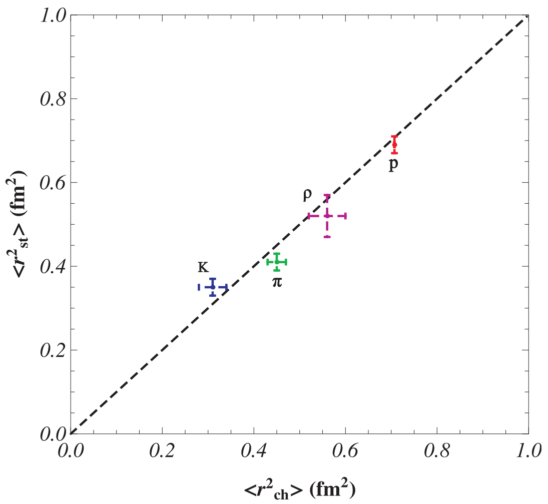

Figure 1. Relation between the strong-interaction hadronic radius $\langle r^2_{\rm st}\rangle$ and the charge radius $\langle r^2_{\rm ch}\rangle$ for light hadrons. Result for ρ-meson is obtained in our work with wave function (14) at $n = 3$.

Table 1. The ρ-meson electromagnetic moments and lepton decay constant obtained with the different model wave functions (13) – (14). $\langle r^2_\rho\rangle_C, \langle r^2_\rho\rangle_M$, are charge and magnetic MSR, respectively, in fm^2; μ_ρ is relativistic magnetic moment (8) in $e/2M_\rho$; Q_ρ is quadrupole moment (9) in fm^2. The parameters b in (13) and (14) are in GeV, f_ρ is ρ-meson lepton decay constant in MeV.

Wave functions	b	$\langle r^2_\rho\rangle_C$	$\langle r^2_\rho\rangle_M$	μ_ρ	Q_ρ	f_ρ
(13)	0.228	0.597	0.635	2.01	-0.0064	152.2
(14) n=2	0.217	0.579	0.532	2.12	-0.0064	153.6
(14) n=3	0.379	0.560	0.634	2.16	-0.0066	154.9
Experiment				2.1 ± 0.5 [3]		152 ± 8 [1]

hadron–proton scattering). The conjecture [25], which may be derived from, though not necessary implies, the hypothesis of [24], is the equality of the two radii,

$$\langle r^2_{\rm st}\rangle = \langle r^2_{\rm ch}\rangle \ . \tag{17}$$

This remarkable equality has been verified experimentally with a great degree of accuracy for the proton, π- and K- mesons. Even more demonstrative is figure 1, analogous to a figure from the paper [25], but presenting more recent data. We can see that the value of the ρ-meson charge radius obtained in this paper fits perfectly the conjecture (17).

3 Conclusions

In the present work the calculations of electroweak characteristics of the ρ-meson in the framework IF RQM are performed with different wave functions in modified impulse approximation (MIA). MIA does not violate the Lorentz covariance and conservation law for the electroweak currents in contrast with conventional impulse approximation. Other distinctive feature of our work is that all calculations were made without fitting parameters. The results for magnetic moment of the ρ-meson are consistent with recent experimental data [3]. The values of the charge radius satisfy the hypothesis of Wu-Yang that has been experimentally verified for a number of hadrons. One of the authors (VT) thanks S.Troitsky for interesting discussions. Authors (AK and RP) thank the Organizing committee for their kind invitation to XXIII International Baldin Seminar on High Energy Physics Problems and hospitality. This work was supported in part (AK and RP) by the Ministry of Education and Science of the Russian Federation (grant No. 1394, state task).

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