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Hadron rapidity spectra within a hybrid model

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Abstract. A multistage hybrid model is constructed what joins the initial non-equilibrium stage of interaction, described by the hadron string dynamics (HSD) model, to subsequent evolution of the expanding system treated within ideal hydrodynamics (the second stage). Particles can still rescatter after hydrodynamical expansion that is the third interaction stage. The developed hybrid model is assigned to describe heavy-ion collisions in the energy range of the NICA collider. Generally, the model is in reasonable agreement with the available data on proton rapidity spectra.

1 Introduction

Application of hydrodynamics to high-energy nuclear collisions has a long and vivid history which began almost 65 years ago with Landau's original work [1]. In this history a lot of papers have been written covering a wide spectrum of various and important problems of hydrodynamics. Modern phenomenological status of hydrodynamics is well reflected in recent review-articles (for example, see [2–4] and reference therein).

Hydrodynamics is a collective model of nuclear motion characterized by such physics parameters as temperature, pressure, equation of state (EoS), transport coefficients and so on. Unfortunately, the direct access to this information is impossible since the only available experimental information is contained in observable spectra of particles. Hydrodynamics gives the connection between observables and thermodynamic properties of excited nuclear matter. One has to keep in mind that hydrodynamics is applicable when the mean free path of a quasiparticle is smaller than the size of the system. In nuclear collisions this condition may be violated in rarefied matter at the initial stage of interaction, in peripheral collisions of nuclei and at the end of a hydrodynamical expansion. In this respect, the initial state in the hydrodynamic approach, i.e. space distributions of the energy density, charge density and velocity field, is postulated or calculated within other dynamical models. The subsequent second interaction stage is actually hydrodynamical expansion of dense matter with its EoS which can contain a phase transition (PT) of hadrons into a quark phase. Our model can also include post-hydrodynamic rescattering which is the final collision stage.

The ultimate aim of this work is the development of a multistage hybrid hydrokinetic approach to heavy-ion collisions in the range of moderate collision energies $\sqrt{s} \lesssim 10$ GeV, which are planned to be reached at the heavy-ion collider NICA (Dubna) [5] and heavy-ion accelerator FAIR (Darmstadt) [6].

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The ideology of our model completely coincides with the hybrid hydro+UrQMD approach which has been successfully developed in recent years. The theoretical difficulty lies in the fact that one should construct a model describing available data in a unified way in the whole energy range considered and having a predictive power. So far the only hydrodynamic model, which well describes a variety of experimental data in this energy range is the three-fluid hydro model [7].

2 Hybrid model

2.1 Kinetic stage – hadron string dynamics model (HSD) [8]

Since the hydrodynamic equations are equations in partial derivatives, one needs to define initial conditions. In hybrid models these conditions are obtained from calculations in a kinetic model. In such way one takes into account the nonequilibrium evolution of the system at the beginning of the collision. As a model we choose the hadron string dynamics model [8], which describes a large variety of experimental data in the energy range of interest $E_{\rm lab} = 2 - 50$ AGeV.

In this paper, as a basis we consider both the AGS ($E_{\rm lab} = 6$ and 10.7 AGeV), and SPS energy ($E_{\rm lab} = 40$ AGeV, where $E_{\rm lab}$ is the kinetic energy of the bombarding nucleus in the laboratory system). To get smooth distributions in the density of energy and a baryon number in the initial state, the averaging over $5 \cdot 10^4$ events is used.

The transition from the kinetic description to the hydrodynamic one occurs at some time moment $t_{\rm start}$. It is assumed that at this time the excited system is close to an equilibrium state that may be characterized by conserving quantities, entropy or the ratio of entropy to the baryon charge [9]. As a criterium of the hydrodynamic stage start, we take the constancy of the ratio $S(t)/N_B(t)$, see [10] for details. Such approach to determination of the transition time to hydrodynamics allows one to easily take into account the effect of nuclear opacity. We also have to mention that only particles that have suffered interactions are taken into account to construct the initial state.

2.2 Hydrodynamic stage

The hydrodynamic equations express simply the conservation laws for the energy, the momentum and the baryon charge:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J^\mu = 0, \tag{1}$$

The ideal hydrodynamics assumes that matter is in local equilibrium. Then the energy-momentum tensor $T^{\mu\nu}$ and the vector of the baryon current J^μ are

$$T^{\mu\nu} = (\varepsilon + P)\, u^\mu u^\nu - P g^{\mu\nu}, \quad J^\mu = n\, u^\mu. \tag{2}$$

Here $u^\mu = \gamma(1, \mathbf{v})$ is the 4-velocity of liquid, $\mathbf{v}$ is the 3-velocity, the Lorenz-factor is $\gamma = (1 - v^2)^{-1/2}$, quantities ε, n, P are the energy density, baryon density and pressure, and $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is the metric tensor. Equations (1) should be completed by EoS $P = P(\varepsilon, n)$, then the system becomes closed.

Hydro equations (1) are solved by means of the SHASTA (the SHarp and Smooth Transport Algorithm) algorithm [11]. The details of our code can be found in [10]. We use the EoS for a hadron gas in the mean field proposed in [12] with included σ-meson.

2.3 Evaluation of observable: particlization procedure

A special task is the calculation of observables: rapidity distributions, transverse momentum spectra and azimuthal flows of particles. We use the approximation of "instantaneous freeze-out" which is frequently used in hydrodynamic models and assumes that on some space-time hypersurface there occurs an instantaneous transition from local equilibrium to collisionless particle expansion. In most cases it is postulated that the existence of some freezed-out cells do not essentially influence the dynamics of other parts of the system (as an example, see further isothermal and isoenergetic freeze-out). However, there are models where such influence is accounted, e.g., see [7].

The calculation can be finished when all cells are frozen. This is the short two-stage version of our model. However models of such kind neglect the third (again nonequilibrium) stage of nuclear collision which consists in possible particle rescatterings after fireball expansion. We construct also the full hybrid model including all three stages. But the main attention is paid to the short model since it gives better results.

To define an integration hypersurface on which the "instantaneous" transition from a liquid fluid to freely expanding particles occurs, we use the CORNELIUS algorithm [13], whose realization is available freely. We consider different freeze-out scenarios: 1) isochronous, when the calculation is completed at the given time moment; 2) isothermal (iso-T), when the cell is frozen if its temperature is less or equal the freeze-out temperature $T \leq T_{\rm frz}$, and 3) isoenergetic (iso-ε), which is entirely analogous to the isothermal one where the energy density plays the role of temperature. In the last two versions, the frozen cells are not excluded from the calculation after writing up them into a file and may influence the numerical solution, but this effect is nonessential.

The straightforward method to calculate observables is an application of the famous Cooper-Frye formulae [14]. Besides the "thermal" contribution calculated by it, resonance decays also have to be taken into account. However, below we shall use another approach allowing to speed up appreciably the numerical calculations. If the hypersurface is known, one may make the inverse transition from the fluid set to quasiparticles using the Monte-Carlo method – so called particlization. According to our algorithm, the number of particles in each cell is calculated following [15], while the 4-momentum is sampled according to [16]. The contribution of space-like cells is ignored.

To test our generator, the simulated distributions are compared to direct calculations of spectra, according to the Cooper-Frye formulae. We see a good agreement of the directly calculated spectra with the particlization results that allows us to apply the particlization procedure to calculation of observables.

Since our hydrodynamic equations do not include a separate equation for an electric charge current conservation, we use the symmetric EoS. Therefore, we need to estimate the particle fraction with the given electric charge among particles with the same mass (for example, the ratio of protons to the total number of nucleons). For the isochronous scenario, multiplying a number of nucleons by the isotopic factor Z/A we get dN/dy at $y = 0$ for protons essentially larger (closer to experiment) than for two others freeze-outs. So, an additional isotopic factor depends also on the freeze-out scenario. Thus, below we will compare the averaged experimental proton multiplicity with the calculated $(n_p + n_n)/2$ for the iso-ε and iso-T scenarios.

As to the pion yield, in all cases we take the isospin average $(n_{\pi^+} + n_{\pi^-} + n_{\pi^0})/3$. Our calculations show that such averaging is closer to the yield of n_{π^+}. One should note that the charge asymmetry is observed clearly in nuclear experiments in the NICA energy range. At energies of interest the ratio n_{π^-}/n_{π^+} exceeds remarkable 1. However, any version of the hadronic transport model HSD overestimates the pion multiplicity in the NICA energy range (see for example [17]). This overestimation of pions results in a too low kaon-to-pion ratio K^+/π^+. This problem is actively discussed in the last years and frequently associated with the signal of a possible quark-hadron phase transition.

In our analysis of central nucleus-nucleus collisions, by default, we use the impact parameter $b = 1$ fm at all energies under discussion since, as a rule, it gives results close to experiments.

3 Properties and results

3.1 The two-stage version

It is evident that the assumption on isochronous freeze-out is not realistic and can be used only for test calculations or as an intermediate stage. In realistic models where observable quantities are calculated on a "frozen" hypersurface, a constant temperature or the energy density scenarios are used. To construct a model having predictive power, one needs to choose a scenario and a method for calculating its parameters depending on collision energy.

Our calculations show that to reproduce the two-hump structure at $E_{\mathrm{lab}} = 40$ *A*GeV, one needs to take rather large values of the parameters $\varepsilon_{\mathrm{frz}}$ or T_{frz}. It is interesting that in contrast with the kinetic HSD model, the hybrid model has no problem with the description of proton spectra at large rapidity (see figure 1).

The results [18] obtained in the model with isochronous freeze-out demonstrate the point that to get a two-hump structure, the two-phase EoS is necessary while our model shows that the choice of the initial state and method/parameter of freeze-out plays not less important role. Such ambiguity between the choice of a proper initial state or EoS was also noted in [19]. Since the model [18] with the two-phase EoS predicts the two-hump structure at $E_{\mathrm{lab}} = 10.7$ *A*GeV as well, one may conclude that the method used there for constructing the initial state is too simplified.

In our hybrid model, the two-hump structure arises also if we take later time of transition to hydrodynamics. This confirms the conclusion about an important role of the initial state.

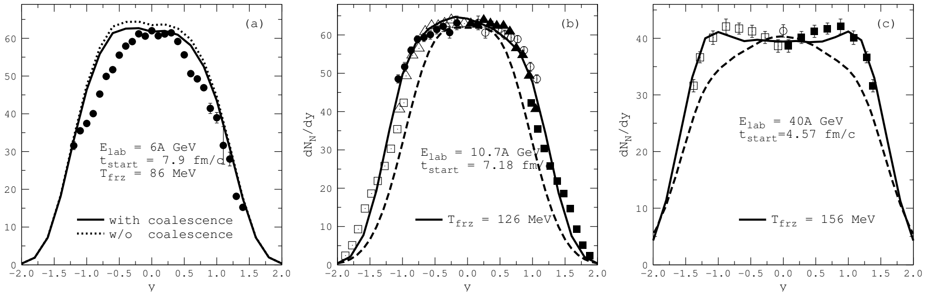

Figure 1. Ultimate proton spectra within the isothermal scenario at energies (*a*) $E_{\mathrm{lab}} = 6$ *A*GeV, (*b*) $E_{\mathrm{lab}} = 10.7$ *A*GeV and (*c*) $E_{\mathrm{lab}} = 40$ *A*GeV. The solid lines are our results, the ones dashed are the results with isochronous freeze-out from [18] (*b, c*). Experimental points for $E_{\mathrm{lab}} = 6$ *A*GeV are taken from [20], for $E_{\mathrm{lab}} = 10.7$ *A*GeV – [21] (triangles), [22] (squares) and [23] (circles), and for $E_{\mathrm{lab}} = 40$ *A*GeV – [24] (squares) and [25] (the circle).

To get the best description of the experiment, our model needs the freeze-out parameter depending on the collision energy. This dependence for freeze-out temperature T_{frz} can be obtained within the phenomenological statistical model of hadron and resonance production [26]. Thus, the solution is suggested to use as input data temperatures found from the data analysis within the statistical model. For all considered energies $E_{\mathrm{lab}} = 6, 10.7$, and 40 *A*GeV, a good agreement with the experiment for protons is achieved if we take respectively the values of $T_{\mathrm{frz}} = 86^{+13}_{-3}$, 124±7, and 156±11 MeV [26] (see figure 1).

The pion rapidity distribution height in the rapidity range $y \approx 0$ depends on both the freeze-out parameter and $t_{\rm start}$. However, we did not succeed in reproducing experimental pion spectra in either the isothermal or isoenergetic scenarios. The same behaviour is observed within the hybrid version of the UrQMD model with nonviscous hydrodynamics [27]. Our model differs from it only by the point that there the initial state is taken from the UrQMD model and hydrodynamics additionally includes the electric charge conservation. Hence the results should be quite similar. The hydro-UrQMD results [27] are also lower than experimental data for pion rapidity distributions.

As has been noted for the first time in [28] and clearly demonstrated in [27], the lack of pions is due to the absence in our model of dissipative effects which increase the entropy. In addition, one should remember that the SPS data are given for π^- pions, while the isospin averaged calculation results are closer to the number of less abundant π^+ pions.

Thus, our hybrid model has some difficulties with a simultaneous description of the AGS and SPS energy range. The model with isochronous freeze-out considered above [18] also has some problems in attempt to consider these both energies simultaneously because nobody succeeded in describing the shape of proton spectra within a single EoS.

3.2 The full version of the hybrid model

As is obvious, during the expansion at some time moment the system size becomes larger than the mean free path of particles and hydrodynamics breaks down. So the most justified approach has to include the posthydrodynamic rescattering. We realized this stage in our model and here we shortly mention main features of such full version of our code.

As a parameter of the back transition from fluids to particles, we choose the energy density in spirit of [27]. The particlization hypersurface is isochronous when the energy density of every cell becomes less the switching one.

Our results show that taking into account post-hydrodynamic rescattering decreases distributions both for protons and pions. The final rapidity spectra are worse than in the 2-stage version. The main reason for this is the isochronous transition since we obtain two-hump structure in proton distributions for $E_{\rm lab} = 10.7$ AGeV as in [18]. So we need more realistic procedure for switching to the kinetic description after the hydrodynamic expansion.

4 Conclusions

We propose the many-stage hybrid model to describe heavy-ion collisions in the energy range reachable at the heavy-ion collider NICA which is under construction in Dubna. The model can be realized in short and full versions.

The 2-stage model allows one to describe quantitatively the proton spectra within the isothermal freeze-out scenario if the temperatures $T_{\rm frz}$ are taken from the results of the statistical model processing the measured hadron yield. It is shown that within the hybrid model the parameters of the two-hump structure in the proton spectra may be obtained by either increasing the freeze-out temperature or energy density parameters or by transition to the hydrodynamical stage at a later time. The ideal hydrodynamics with the initial state calculated on the basis of the kinetic HSD model is not able to describe pion rapidity spectra simultaneously with those for protons. It is necessary to take into account the hadron matter viscosity at the hydrodynamic stage of the system evolution.

The importance of including dissipative effects in someway is demonstrated by all successful hydrodynamical models like 3-fluid hydro [7] or hydro+UrQMD [27].

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