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Form factors and related quantities in clothed-particle representation

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Abstract. We show new applications of the notion of clothed particles in quantum field theory. Its realization by means of the clothing procedure put forward by Greenberg and Schweber allows one to express the total Hamiltonian H and other generators of the Poincaré group for a given system of interacting fields through the creation (annihilation) operators for the so-called clothed particles with physical (observed) properties. Here such a clothed particle representation is used to calculate the matrix elements (shortly, form factors) of the corresponding Nöther current operators sandwiched between the H eigenstates. Our calculations are performed with help of an iterative technique suggested by us earlier when constructing the $NN \to \pi NN$ transition operators. As an illustration, we outline some application of our approach in the spinor quantum electrodynamics.

1 Introduction

The method of unitary clothing transformations (UCTs) ([1]–[5]) allows us to develop an alternate approach for calculating the matrix elements $F^\mu(p', p) \equiv \langle p'; out|J^\mu(0)|p; in\rangle$ of the Nöther current density operator $J^\mu(0)$ sandwiched between the $in(out)$ states of interacting fields. Since every UCT $W(\alpha_c) = W(\alpha) = \exp R$, $R = -R^\dagger$ connects a primary set α of creation (annihilation) operators in the bare-particle representation (BPR) with new operators α_c in the clothed-particle representation (CPR) via similarity transformation $\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c)$, we consider the expansion

$$J^\mu(0) \quad = \quad W J_c^\mu(0) W^\dagger = J_c^\mu(0) + [R, J_c^\mu(0)] + \frac{1}{2}[R, [R, J_c^\mu(0)]] + \dots, \tag{1}$$

where $J_c^\mu(0)$ is the initial current in which the bare operators are replaced by their clothed counterparts. Along this guideline we have to handle R-commutators $[R, [R, \dots [R, J]]] \equiv J^{[n]}$ with n brackets. As an illustration of the recursive procedure proposed in [3] for calculations with similar multiple commutators, we will show our results in case of the electromagnetic current operator of meson-nucleon system

$$J^\mu(0) = J_N^\mu(0) + J_\pi^\mu(0) + J_\rho^\mu(0) + J_\omega^\mu(0) + \dots \tag{2}$$

and in quantum electrodynamics (QED) with

$$J^\mu(0) = J_e^\mu(0) = e\frac{1}{2}[\overline{\psi}(0), \gamma^\mu \psi(0)] = e : \overline{\psi}(0)\gamma^\mu \psi(0) :, \tag{3}$$

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where $\psi(0)$ is the electron-positron field $\psi(x)$ at $x = (0, \vec{0})$. In both cases the matrix elements of interest between the one-fermion states $|p; in\rangle = |p; out\rangle = b_c^\dagger(p)|\Omega\rangle$ with the 4-momentum $p = (p_0, \vec{p})$, $p_0 = \sqrt{\vec{p}^2 + m^2}$ are expressed through the Dirac $F_1(q^2)$ and Pauli $F_2(q^2)$ form factors (FFs) that depend on the 'transferred' 4-momentum $q = p' - p$ squared, viz.,

$$F^\mu(p', p) = e\bar{u}(p')\{F_1(q^2)\gamma^\mu + i\sigma^{\mu\nu}F_2(q^2)(p' - p)_\nu\}u(p). \quad (4)$$

Other notations embody the one-clothed fermion (electron, nucleon) creation operator $b_c^\dagger(p)$, fermion mass m and physical vacuum state $|\Omega\rangle$. All polarization indices are implied.

2 The UCT Method in Action

A key point of the clothing procedure in question is to remove the so-called bad terms from the total Hamiltonian for interacting particles

$$H \equiv H(\alpha) = H_F(\alpha) + H_I(\alpha) = W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c) \equiv K(\alpha_c).$$

By definition, such terms prevent the physical vacuum $|\Omega\rangle$ (the H lowest energy eigenstate) and the one-clothed-particle states $|n\rangle_c = a_c^\dagger(n)|\Omega\rangle$ to be the H eigenvectors for all necessary quantum numbers n included. Bad terms occur every time when any normally ordered product

$$a^\dagger(1')a^\dagger(2')...a^\dagger(n'_C)a(n_A)...a(2)a(1)$$

of class [C.A] embodies, at least, one substructure $\in [k.0]$ ($k = 1, 2...$) or/and $[k.1]$ ($k = 2, 3, ...$). In this context all primary Yukawa-type (trilinear) couplings should be eliminated from interaction $V(\alpha)$ that enters

$$H_I(\alpha) = V(\alpha) + \text{mass and vertex counterterms}.$$

It results in the form

$$H = K_F(\alpha_c) + K_I(\alpha_c) = K, \quad (5)$$

where free part $K_F(\alpha_c) = H_F(\alpha_c)$ while operator $K_I(\alpha_c)$ contains interactions between clothed particles. By construction, the latter has the property

$$K_I(\alpha_c)|\Omega\rangle = K_I(\alpha_c)|n\rangle_c \equiv 0.$$

For a boson-fermion (meson-nucleon, photon-electron) system
we have the decomposition

$$\begin{aligned} K_I(\alpha_c) = K(ff \to ff) + K(\bar{f}\bar{f} \to \bar{f}\bar{f}) + K(f\bar{f} \to f\bar{f}) \\ + K(bf \to bf) + K(b\bar{f} \to b\bar{f}) + K(f\bar{f} \leftrightarrow bb') \\ + K(ff \leftrightarrow bff) + K(f\bar{f} \leftrightarrow 3b) + K(3f \to 3f) + \cdots, \quad (6) \end{aligned}$$

where separate contributions are responsible for different physical processes so, for instance, operators $K(\gamma e \to \gamma e)$, $K(e\ e \leftrightarrow \gamma e\ e)$ and $K(3N \to 3N)$ can be used in describing the Compton scattering on electrons, electron-electron bremsstrahlung and modeling three-nucleon forces, respectively.
In particular, the fermion-fermion interaction operator in the CPR can be written as

$$K(ff \to ff) = \sum_b K_b(ff \to ff),$$

$$K_b(ff \to ff) = \int \sum_\mu d\vec{p}_1{}' d\vec{p}_2{}' d\vec{p}_1 d\vec{p}_2\, V_b(1', 2'; 1, 2) b_c^\dagger(1')b_c^\dagger(2')b_c(1)b_c(2),$$

where the symbol $\sum_\mu$ denotes the summation over fermion spin projections, $1 = \{\vec{p}_1, \mu_1\}$, etc.

3 The S-Matrix in SPR

Of interest is how the S–matrix for given interactions between "bare" particles can be expressed in terms of a new family of clothed-particle interactions ("quasipotentials")? As we have seen ([5] and references therein), such a reduction becomes possible provided that

$$\lim_{t\to\pm\infty} W_D(t) = 1, \tag{7}$$

this supplementary constraint imposed on UCTs in the Dirac (D) picture at the distant past and future. It leads to important links between $in(out)$ states and clothed-particle ones. As well known, the former, being exact eigenstates of a total field Hamiltonian, are key elements of the collision theory in the Heisenberg (H) picture, e.g. within such a nonperturbative formalism by Lehmann, Symanzik and Zimmermann (LSZ), when evaluating the S–matrix $S_{fi} = \langle f; out \mid i; in\rangle$ with many-particle states

$$|p_1 \ldots p_n;\ in\,(out)\rangle = a^{\dagger}_{in\,(out)}(p_1)\ \ldots a^{\dagger}_{in\,(out)}(p_n)\,|\Omega\rangle. \tag{8}$$

Also let me recall that the creation (destruction) in(out) operators $a^{\dagger}_{in(out)}$ ($a_{in(out)}$) meet the same canonical commutation relations that $a^{\dagger}_c$ (a_c) do. We find for one-particle states,

$$|p;\ in\,(out)\rangle = a^{\dagger}_{in\,(out)}(p)|\Omega\rangle = a^{\dagger}_c(p)|\Omega\rangle \equiv|\ p;\ c\rangle. \tag{9}$$

Of course, (9) does not mean that $a^{\dagger}_c(p) = a^{\dagger}_{in\,(out)}(p)$. In this context, note links

$$|p_1\, p_2;\ in\rangle \equiv a^{\dagger}_{in}(p_1)a^{\dagger}_{in}(p_2)|\Omega\rangle = \Omega^{(+)}(p^0_1 + p^0_2)a^{\dagger}_c(p_1)a^{\dagger}_c(p_2|\Omega\rangle \tag{10}$$

and

$$|p_1\, p_2;\ out\rangle \equiv a^{\dagger}_{out}(p_1)a^{\dagger}_{out}(p_2)|\Omega\rangle = \Omega^{(-)}(p^0_1 + p^0_2)a^{\dagger}_c(p_1)a^{\dagger}_c(p_2|\Omega\rangle \tag{11}$$

with $\Omega^{(\pm)}(E) = \pm\imath \lim_{\epsilon\to+0} \epsilon G(E \pm \imath\epsilon)$, where $G(z) = (z - H)^{-1}$ is the Hamiltonian resolvent. Just such a transition has been used by us when describing properties of nucleon-nucleon collisions.
Now we employ such links to evaluate the matrix elements $\langle out|J|in\rangle$ of a current (source) J between initial and final states of leptons and hadrons. So for the electron-hadron scattering $e + h \to e' + h'$ one has to evaluate its amplitude

$$\langle e'h'; out|eh; in\rangle = 2\pi i\frac{m_e}{\sqrt{EE'}}e_0\bar{u}_e(k')\gamma^{\mu}u_e(k)\frac{\langle h'|J_{\rho}(0)|h\rangle}{q^2}\delta(p' + k' - p - k), \tag{12}$$

where m_e is the mass of the physical electron, $q^2 = (k' - k)^2 = (p' - p)^2$, $u_e(k) \equiv u_e(k\sigma)$ the Dirac spinor for the electron with 4-momentum k and polarization σ.
In case of the one-hadron state $|h\rangle = |p\rangle = b^{\dagger}_c(p)|\Omega\rangle$ (other quantum numbers are suppressed) we use the expansion (1) so by introducing the contraction $J^{\mu}_c(0)l_{\mu} = J_c(0)$ with an arbitrary "polarization" 4-vector l we get the matrix elements

$$M_{OPEA} = \langle p'|J_c(0)|p\rangle \tag{13}$$
$$M^{[1]} = \langle p'|[R, J_c(0)]|p\rangle \tag{14}$$
$$M^{[2]} = \frac{1}{2!}\langle p'|[R, [R, J_c(0)]]|p\rangle \tag{15}$$

of successive R - commutators. Here the abbreviation OPEA is reminiscent of One-Photon-Exchange Approximation.

4 *R*-commutators in CPR

We have seen [3] how the commutators

$$[R, [R, \ldots [R, V]]] \equiv V^{[n]} \tag{16}$$

with n brackets ($n = 1, 2, \ldots$) and couplings V of the type $V = f * m + H.c.$, where $f * m$ is a polynomial composed of products of fermion and meson operators, become a spring of interactions between the clothed fermions and mesons, the mass and vertex contributions, etc. Let us consider the πN pseudoscalar coupling

$$V = \int \frac{d\vec{k}}{\omega_k} \hat{V}(k) a(k) + H.c., \quad R = \int \frac{d\vec{k}}{\omega_k} \hat{R}(k) a(k) - H.c., \tag{17}$$

where the generator R is determined by the condition $[H_F, R] = V^1$. Subsequent calculations are essentially simplified after introducing the following quantities $\hat{V}(k) =: F^\dagger V(k) F :$ and $\hat{R}(k) =: F^\dagger R(k) F :$ with the compact notation

$$F^\dagger X F \equiv \sum_{\mu'} \int \frac{d\vec{p}'}{E_{p'}} \sum_{\mu} \int \frac{d\vec{p}}{E_p} F^\dagger_{\epsilon'}(p'\mu') X_{\epsilon'\epsilon}(p'\mu', p\mu) F_\epsilon(p\mu) \tag{18}$$

for any 2×2 matrix

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$$

and the column

$$F(p\mu) = \begin{bmatrix} F_1(p\mu) \\ F_2(p\mu) \end{bmatrix} = \begin{bmatrix} b(p\mu) \\ d^\dagger(p\mu) \end{bmatrix}.$$

In our case the matrices $V(k)$ and $R(k)$ can be written as

$$V_{\epsilon'\epsilon}(p'\mu', p\mu; \vec{k}) = \frac{igm}{\sqrt{2(2\pi)^3}} v_{\epsilon'\epsilon}(p'\mu', p\mu) \delta(\vec{k} + (-1)^{\epsilon'} \vec{p}' - (-1)^{\epsilon} \vec{p}) \tag{19}$$

$$v_{\epsilon'\epsilon}(p'\mu', p\mu) = \begin{bmatrix} \bar{u}(p'\mu')\gamma_5 u(p\mu) & \bar{u}(p'\mu')\gamma_5 v(p\mu) \\ \bar{v}(p'\mu')\gamma_5 u(p\mu) & \bar{v}(p'\mu')\gamma_5 v(p\mu) \end{bmatrix} \tag{20}$$

$$R_{\epsilon'\epsilon}(p'\mu', p\mu; \vec{k}) = -\frac{V_{\epsilon'\epsilon}(p'\mu', p\mu; \vec{k})}{\omega_{\vec{k}} + (-1)^{\epsilon'} E_{\vec{p}'} - (-1)^{\epsilon} E_{\vec{p}}} \tag{21}$$

with the sign-energy indices ($\epsilon', \epsilon = 1, 2$) and all necessary algebraic operations are based upon the covariant commutation rules

$$\{F_{\epsilon'}(p'\mu'), F^\dagger_\epsilon(p, \mu)\} = p_0 \delta(\vec{p}' - \vec{p}) \delta_{\mu'\mu} \delta_{\epsilon'\epsilon}. \tag{22}$$

In practical calculations it is convenient to use

$$V^{[n]} = \sum_{s=0}^{n} \frac{n!}{s!(n-s)!} f^{[n-s]} * m^{[s]} + H.c. \tag{23}$$

[1] In this case $m(k) = a_c(k)$ is the one-clothed-meson destruction operator and henceforth the label c will be omitted

and the recurrence relation

$$V^{[n]} = [R, V^{[n-1]}] = [\mathcal{R} - \mathcal{R}^{\dagger}, V^{[n-1]}] = [\mathcal{R}, V^{[n-1]}] + H.c. \tag{24}$$

as a test and a subsequent step when calculating more complex brackets.
Doing so, we find step by step

$$V^{[1]} = \int \frac{d\vec{k}}{\omega_k} \left[\{f(k)\}^{[1]} m(k) + f(k)\{m(k)\}^{[1]} \right] + H.c., \tag{25}$$

$$V^{[2]} = \int \frac{d\vec{k}}{\omega_k} \left[\{f(k)\}^{[2]} m(k) + 2\{f(k)\}^{[1]}\{m(k)\}^{[1]} f(k)\{m(k)\}^{[2]} \right] + H.c. \tag{26}$$

or

$$[R, V] = \int \frac{d\vec{k}_1}{\omega_{k_1}} \hat{V}(k_1)\hat{R}^{\dagger}(k_1) + terms\ quadratic\ in\ a^{\dagger}(a) + H.c. \tag{27}$$

$$[R, [R, V]] = \int \frac{d\vec{k}_1}{\omega_{k_1}} \int \frac{d\vec{k}_2}{\omega_{k_2}} \{\hat{C}_{10}(k_1, k_2) a^{\dagger}(k_1) + \hat{C}_{01}(k_1, k_2) a(k_1)\} + terms\ cubic\ in\ a^{\dagger}(a) + H.c. \tag{28}$$

to get the g^4-order interactions between the clothed fermions that enter the commutator

$$V^{[3]}_{ferm} = [R, V^{[2]}]_{ferm} = \int \frac{d\vec{k}_1}{\omega_{k_1}} \int \frac{d\vec{k}_2}{\omega_{k_2}} \hat{R}(k_1)\{\hat{C}_{10}(k_1, k_2) + \hat{C}^{\dagger}_{01}(k_1, k_2)\} + H.c. \tag{29}$$

along with the fermion mass renormalization terms in the same order. Here

$$\hat{C}_{10}(k_1, k_2) = -2[\hat{R}^{\dagger}(k_1), \hat{V}(k_2)]\hat{R}^{\dagger}(k_2) - \hat{V}(k_2)[\hat{R}^{\dagger}(k_1), \hat{R}^{\dagger}(k_2)] \tag{30}$$

$$\begin{aligned} \hat{C}_{01}(k_1, k_2) &= [\hat{R}(k_2), \hat{V}(k_1)]\hat{R}^{\dagger}(k_2) - [\hat{R}^{\dagger}(k_2), \hat{V}(k_1)]\hat{R}(k_2) \\ &- [\hat{R}(k_2), [\hat{R}^{\dagger}(k_2), \hat{V}(k_1)]] + 2[\hat{R}(k_1), \hat{V}(k_2)]\hat{R}^{\dagger}(k_2) \\ &+ \hat{V}(k_2)[\hat{R}(k_1), \hat{R}^{\dagger}(k_2)]. \end{aligned} \tag{31}$$

5 Calculations of typical matrix elements

Certainly, these ideas can be realized when calculating the meson and nucleon FFs for the density (2), where, e.g., the primary meson and nucleon currents are given by $J^{\mu}_{\pi}(\vec{x}) =: \vec{\pi}(\vec{x}) \times \partial^{\mu}\vec{\pi}(\vec{x})$: and $J^{\mu}_{N}(\vec{x}) =: \overline{\psi}(\vec{x})\gamma^{\mu}(1 \pm \tau_3)/2\psi(\vec{x})$:, and the electron (positron) FF in QED for the density (3). Let us remind that in the latter $\psi(0)$ is the electron-positron field in the Schrödinger S picture, while in the nucleon current density operator the field $\psi(0)$ has both nucleon and antinucleon components.
At this point, we would like to outline some application of our approach in QED. It is the case when the relevant R commutators should be calculated assuming that $V = V_{\gamma} + H.c.$, $V_{\gamma} = \int \frac{d\vec{k}}{k^0} : F^{\dagger} V(k,\sigma) F : c(k,\sigma)$ with the contraction $V(k,\sigma) = V_{\rho}(k) e^{\rho}(k,\sigma)$ where the c-number matrices $V_{\rho}(k)$ repeat the structure (19) after the replacement in the latter of ig by e and the matrix γ_5 by Dirac matrices γ_{ρ}. The corresponding generator is determined as it follows, viz., $R = R_{\gamma} - R^{\dagger}_{\gamma}$, $R_{\gamma} = \int \frac{d\vec{k}}{k^0} : F^{\dagger} R(k,\sigma) F : c(k,\sigma)$ with the contraction $R(k,\sigma) = R_{\rho}(k) e^{\rho}(k,\sigma)$ and matrix $R_{\rho}(k)$ composed of elements

$$R^{\rho}_{ij}(k) = -\frac{V^{\rho}_{ij}(k)}{(-1)^i E_{\vec{p}'} - (-1)^j E_{\vec{p}} + k^0} \quad (i, j = 1, 2). \tag{32}$$

Photon destruction operators $c(k,\sigma)$ enter the Fourier expansion of the e.m. field

$$A^{\rho}(\vec{x}) = \int \frac{d\vec{k}}{\sqrt{2(2\pi)^3 k^0}} \sum_{\sigma} \left[e^{\rho}(k,\sigma)c(k,\sigma) + e^{\rho^*}(k_-,\sigma)c^{\dagger}(k_-,\sigma) \right] \exp(i\vec{k}\vec{x}), \tag{33}$$

together with photon polarization vectors $e^{\rho}(k,\sigma)$ that form the projection operator

$$\Pi_{\rho_1\rho_2}(k) = -g_{\rho_1\rho_2} + \frac{q^0(q_{\rho_1}n_{\rho_2} + q_{\rho_2}n_{\rho_1}) - q_{\rho_1}q_{\rho_2} - q^2 n_{\rho_1}n_{\rho_2}}{\vec{k}^2},$$

where $n^{\mu} = (1,0,0,0)$ is a fixed time-like vector, $q = (q_0,\vec{k})$, $q^2 = q_0^2 - \vec{k}^2$, but here q_0 being generally arbitrary is chosen to be equal to difference $q^0 = E_{\vec{p}'} - E_{\vec{p}}$ if once it appeared $\vec{k} = \vec{p}' - \vec{p}$ for two on-mass-shell fermions (electrons).
At last, the electron-positron current density in the CPR is given by $J_c(0) = N : F^{\dagger} jF : ((2\pi)^3 N = em_e)$ with the 2×2 matrix

$$j_(p'\mu', p\mu) \quad = \quad \begin{bmatrix} \bar{u}(p'\mu')\hat{l}u(p\mu) & \bar{u}(p'\mu')\hat{l}v(p\mu) \\ \bar{v}(p'\mu')\hat{l}u(p\mu) & \bar{v}(p'\mu')\hat{l}v(p\mu) \end{bmatrix}. \tag{34}$$

Then, for example, we have

$$M^{[2]} = \frac{1}{2!}\langle p'|[R,[R,J]|p\rangle, \tag{35}$$

$$J^{[1]} \equiv [R,J] = N \int \frac{d\vec{k}}{k^0} F^{\dagger}[R(k,\sigma), j]Fc(k,\sigma) + H.c. \tag{36}$$

so $J^{[2]} = [R,[R,J]] = [R,J^{[1]}]$ and the initial task reduces to finding the expectation value $\langle\Omega|b(p')[R,J^{[1]}]b^{\dagger}(p)|\Omega\rangle$.
Detailed calculations by these formulas and comparison with well known results are in preparation to be published somewhere else.

In our opinion, the approach exposed here can be helpful when calculating higher-order radiative corrections in mesodynamics, QED and other field models. In this context, our current explorations within the formalism of clothed particles are concentrated upon the three directions: i) test calculations of the Dirac and Pauli FFs in the e^2- and e^4 - orders to compared the standard ones; ii) calculations of the nucleon FFs in the g^2- and g^4- orders being aware of a decisive role of the normal ordering operation for separating the $b^{\dagger}b$ - contributions from the operators of fermion type $F^{\dagger}XFF^{\dagger}YF$; relying on our experience in evaluating the dipole magnetic and electric quadrupole deuteron moments in the CPR (see [5] and references therein) a further development of the theory of electromagnetic interactions with nuclei, in particular, including the construction of a new family of meson exchange two-body currents.

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