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On possible contribution of partons' orbital momentum to the proton spin

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Abstract. Virtual parton excitations of proton achieve status of real particles after transition into the infinite momentum frame. In particular, QCD evolution of them should respect the conservation laws which in turn should be a necessary consequence of symmetry properties. I argue that these simple reasons may cast light on the so-called proton spin puzzle and upon an orbital momentum contribution of quarks and gluons to the proton spin. An already fulfilled experiment with a hint on possible observation of orbital excitations of gluons is pointed out.

1 Introduction

The relativistic covariance of the Dirac equation means that a fermion seen from a moving frame acquires an admixture of its antiparticles, exactly as the electric field acquires the magnetic field component if you look at it from a moving, $v \neq 0$, frame. For example, the Dirac field of a quark polarized at rest along z-axis is of the form [1]

$$\sqrt{\frac{E+m}{2m}}\left(1, 0, 0, \frac{p}{E+m}\right)^T$$

in a frame moving along z. It is easy to see that the electric charge and z-component of spin are conserved in the moving frame despite an increase of total intensity of the $q - \bar{q}$ field for a moving observer. Besides valence quarks, there are virtual quarks and gluons in proton, which may live very long in a fast moving frame, even forever if $v \to 1$. They describe virtual excitations of nucleon, which may be considered as real ones in the infinite momentum frame (IMF). Just as excited states of hydrogen atom are described with taking into account corresponding physical symmetries, almost real excitations of nucleon may be naturally treated in the same way.

E. Fermi was the first who understood that electric field surrounding a charged particle and, as we understand it now, corresponding to virtual photons turns into real electromagnetic waves after transition into the high-speed reference frame [2]. Therefore, it was already shown by Fermi and developed further by Weizsäcker [3] and Williams [4] that physical picture of the world may undergo qualitative changes after transition into IMF[1]. On this evidence, it is not strange that the constituent

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[1] For a modern exposition of the subject see [5]. It is even more relevant to our study an investigation of gluon production in ultrarelativistic collisions of two nuclei considered as sources of non-Abelian Weizsäcker-Williams fields, see [6, 7].

quark model defined in the proton rest frame is not compatible with the helicity content for the current quarks and gluons defined in the above mentioned limit. Furthermore, the total contribution of quark helicities to the proton spin is small and consistent with zero within the range of experimental errors according to the EMC measurement of polarized structure functions [8]. Therefore it is natural to suggest on basis of the angular momentum conservation reason that the deficiency, which persists till now, is hidden in orbital angular momentum of quarks and gluons.

Now let us try to understand why gluons should indeed carry nonzero orbital momentum in the experiments of the EMC type.

2 Holographic principle and gluons

Three-dimensional nucleon turns into a two-dimensional object after transition into IMF. As it was explained above, nucleon's content may appear very different in 3-D and 2-D pictures as well as an appearance of a holographic plate differs from an image of a physical object embodied in it. Nevertheless, total information about nucleon spin should be the same in the both. We may drastically simplify the color-Maxwell equations,

$$\partial_\mu F^{\mu\nu} = gJ^\nu, \tag{1}$$

if take into account quasi-free character of knockout of partons from nucleon in deep inelastic scattering processes. This means that $g - q$ and $g - g$ interactions are negligibly rare, at least for very short intervals of time, and the color-Maxwell current may be omitted in the right hand side of (1). Eventually we obtain 8 color-Maxwell equations, one for each color, which coincides with the Maxwell equations for the free electromagnetic field.

To be explicit, let us introduce the color vector potential A^μ in a form as follows (hereafter the color index attributed to A^μ is omitted):

$$\mathbf{A} = \mathbf{e}_x u(x, y, z, t),$$

which satisfies to the wave equation,

$$u_{xx} + u_{yy} + u_{zz} - u_{tt} = 0.$$

After separation of time,

$$u(x, y, z, t) = e^{-i\omega t} v(x, y, z, \omega),$$

we obtain the Helmholtz equation with the wavenumber $k = \omega$,

$$v_{xx} + v_{yy} + v_{zz} + k^2 v = 0.$$

Now we use an ansatz

$$v(x, y, z, \omega) = e^{ikz} w(x, y, z, \omega)$$

with the view of taking advantage of a special role of z-direction and a high value of gluons' momentum in our picture. Because w depends on z much weaker than v, we neglect the term w_{xx} in the exact equation and arrive to

$$w_{xx} + w_{yy} + 2ikw_z = 0. \tag{2}$$

The well-known solution of (2)

$$w = (1/z)e^{ik(x^2+y^2)/2z}$$

may be transformed into a solution describing gluons confined in a small transverse region using a shift along z axis by a complex constant

$$z \to z_* = z - i\varepsilon, \qquad \varepsilon > 0.$$

Thus we acquire the solution of (2) in the form

$$g = (1/z_*)e^{ik(x^2+y^2)/2z_*} = (1/z_*)e^{ikz\psi}e^{-\rho^2/2\Delta^2(z_*)}, \tag{3}$$

where $\rho = \sqrt{x^2+y^2}$, $\psi = (\rho/|z_*|)^2/2$, $\Delta(z_*) = \sqrt{z^2+\varepsilon^2}/\sqrt{k\varepsilon}$. We are inside 2-D nucleon at $z = 0$, where $\Delta_0 \equiv \Delta(-i\varepsilon) = \sqrt{\varepsilon/k}$ and $\psi = \rho^2/2\varepsilon^2$. It is evident that localization corresponding to the gluon confinement is strong if $\Delta(-i\varepsilon)/\lambda = \sqrt{k\varepsilon} \gg 1$. According to the modern beliefs based on QCD calculations, gluon locations are classified according to the portion x of the total (nucleon) momentum it carries (see, e.g. [9]). Thus, it is usually suggested that for $0.1 < x < 1$ gluon is inside a valence quark, and it is centrally symmetric distributed inside nucleon at smaller x (with increasing Δ if x decrease).

Function (3) corresponds to the normal state of the two-dimensional harmonic oscillator. It describes the unexcited (ground) state of virtual gluons in nucleon. Besides, there are many excited virtual states which may live long if nucleon moves with a high velocity. For example, there are solutions of (2) describing excitations of two independent oscillators directed along x and y axes [10], which do not carry orbital momentum:

$$h = z_*^{-(n+m)/2}A_{nm}H_n(\mu x/\sqrt{z_*})H_m(\mu y/\sqrt{z_*})g,$$

where H_n, H_m are the Hermite polynomials, and $\mu = e^{i\pi/4}\sqrt{k/2}$, $A_{nm} = (-\mu)^{n+m}$. Of course, solutions of this type can hardly be used for description of virtual excitations of gluons because they do not reflect a true symmetry of the problem.

In the cylindrical coordinate system, equation (2) has solutions inside 2-D nucleon [11]

$$w_{p,m}(\rho,\phi,0) = \sqrt{\frac{2^{|m|+1}p!}{\pi(p+|m|)!}}\rho^{|m|}e^{-\rho^2}L_p^{|m|}(2\rho^2), \tag{4}$$

where $L_p^{|m|}$ for natural m are the generalized Laguerre polynomials. For short, we have changed notation $\rho \to \rho/\Delta_0$. The solutions (4) are known as Laguerre-Gauss (LG) modes which carry orbital angular momentum m directed along z-axis, and index p enumerates different radial excitation levels of virtual gluons. Thus, we may conclude that the virtual excitations of the gluon field should have nonzero orbital momentum by only symmetry reasons concerning its geometrical form.

3 A possibility to observe gluons' orbital excitations

The virtual electromagnetic excitations considered by Fermi turn into the real bremsstrahlung field if a fast electron carrying them undergo a blow [5]. Similarly, we may expect that hard collisions of nucleons may reveal the virtual orbital excitations of the gluon field. In experiments with a large momentum transfer, external impact may transform only the transverse components of virtual gluons into real ones and excitations of such a type may appear as narrow baryon resonances. Results of experiment [12, 13] fulfilled already in 1993 give a possible candidate for the phenomenon. A detailed analysis of the papers [12, 13] may be found in [14]. Below we sketch a probable theoretical approach to description of these data.

It is possible to find the dynamical symmetry lying behind the radial nodes of LG modes and construct generalized coherent states based on corresponding algebra for fixed m (see [11]). It may be shown that the radial nodes ladder operators,

$$\hat{\mathcal{P}}_+ |p,m\rangle = \mathcal{P}_+ |p+1,m\rangle, \qquad \hat{\mathcal{P}}_- |p,m\rangle = \mathcal{P}_- |p-1,m\rangle,$$

where $\hat{\mathcal{P}}_\pm = \hat{Q} \mp \left(\frac{|m|}{2} - \rho^2\right)$, $\hat{Q} = \frac{1}{2}\rho\partial_\rho + 2\rho^2 - (\hat{p} + |m| + 1)$ are the raising and lowering operators with eigenvalues $\mathcal{P}_+ = \sqrt{(p+1)(p+|m|+1)}$ and $\mathcal{P}_- = \sqrt{p(p+|m|)}$ correspondingly, and

$$\hat{p} = -\frac{1}{8\rho}\partial_\rho(\rho\partial_\rho) + \left(\frac{|m|^2}{8\rho^2} + \frac{\rho^2}{2} - \frac{|m|+1}{2}\right).$$

The ladder operators obey the $su(1,1)$ Lie algebra [15]

$$[\hat{\mathcal{P}}_+, \hat{\mathcal{P}}_-] = -2\hat{\mathcal{P}}_0, \qquad [\hat{\mathcal{P}}_0, \hat{\mathcal{P}}_\pm] = \pm\hat{\mathcal{P}}_\pm,$$

where $\hat{\mathcal{P}}_0 = (2\hat{p} + |m| + 1)/2$. The radial nodes at fixed m arise now as eigenvectors of the Casimir operator $\hat{C} = \hat{\mathcal{P}}_0^2 - 1/2(\hat{\mathcal{P}}_+\hat{\mathcal{P}}_- + \hat{\mathcal{P}}_-\hat{\mathcal{P}}_+)$: $\hat{C}|p,m\rangle = (m^2-1)/4\,|p,m\rangle$. The generalized coherent states corresponding to the $su(1,1)$ Lie group result from the displacement of the ground state $|0,m\rangle$:

$$|\alpha\rangle_m = e^{\alpha\hat{\mathcal{P}}_+ - \alpha^*\hat{\mathcal{P}}_-}|0,m\rangle = (1-|\alpha|^2)^{\frac{|m|+1}{2}} \sum_{p=0}^{\infty} \sqrt{\frac{(p+|m|)!}{|m|!p!}}\alpha^p |p,m\rangle, \tag{5}$$

where $\alpha = \tanh(\xi/2)e^{i\theta}$ is a complex number defined in terms of the two real quantities ξ and θ. The position representation of this coherent states in the two-dimensional world has the form [11]

$$\langle \rho, \phi, 0 \mid \alpha\rangle_m = \sqrt{\frac{2^{|m|+1}}{\pi\,|m|!}}\left(\frac{1-\alpha^*}{1-\alpha}\right)^{\frac{|m|+1}{2}} e^{im\phi - \frac{1+\alpha}{1-\alpha}\rho^2}\rho^{|m|}.$$

The factor $e^{im\phi}$ in the last expression says us that gluons in $su(1,1)$ coherent state have well-defined orbital angular momentum value of m per gluon. A similar formula with the same interpretation exists for photons in the coherent laser beams [16].

4 Quark confinement and the proton spin puzzle

We established in section 2 that the confinement of gluons in restricted domains and the cylindrical symmetry are sufficient grounds for their virtual excited states carry nonzero orbital momenta. Here we consider a general model which demonstrates that the confinement stimulates also quarks to change their projection of spin with the simultaneous alteration of their orbital angular momentum. This observation may be considered as a solution to the proton spin puzzle.

As the first step, let us establish whether free quarks can be confined by exponent in small transverse regions by analogy with (4). Using the fact that γ-matrices transforms as vectors under transformations of coordinates [17], $\gamma^s = \gamma^i \partial x^s/\partial x^i$, it is possible to obtain the Dirac equation in the cylindrical coordinate system:

$$\left[i\left(\gamma^\rho\frac{\partial}{\partial\rho} + \gamma^\varphi\frac{\partial}{\partial\varphi} + \gamma^z\frac{\partial}{\partial z}\right) + \gamma^0 E - M\right]\psi(\rho,\varphi,z;E) = 0, \tag{6}$$

where

$$\gamma^\rho = \begin{pmatrix} 0 & \sigma^\rho \\ -\sigma^\rho & 0 \end{pmatrix}, \qquad \gamma^\varphi = \begin{pmatrix} 0 & \sigma^\varphi \\ -\sigma^\varphi & 0 \end{pmatrix}, \qquad \gamma^z = \begin{pmatrix} 0 & \sigma^z \\ -\sigma^z & 0 \end{pmatrix} \qquad \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

and

$$\sigma^\rho = \begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}, \qquad \sigma^\varphi = \begin{pmatrix} 0 & -ie^{-i\varphi} \\ ie^{i\varphi} & 0 \end{pmatrix}, \qquad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Then substitution $\psi = (u(\rho,\varphi), v(\rho,\varphi))^T e^{ip_z z}$ separates quarks and antiquarks and introduces the longitudinal component of momentum in explicit form. The substitution makes it possible to change from Dirac's equation to the system of two equations for quarks and antiquarks as follows:

$$(E - M_q)u + \left[i\left(\sigma^\rho \frac{\partial}{\partial\rho} + \frac{\sigma^\varphi}{\rho}\frac{\partial}{\partial\varphi}\right) - \sigma^z p_z\right] v = 0, \tag{7}$$

$$(E + M_{\bar{q}})v + \left[i\left(\sigma^\rho \frac{\partial}{\partial\rho} + \frac{\sigma^\varphi}{\rho}\frac{\partial}{\partial\varphi}\right) - \sigma^z p_z\right] u = 0. \tag{8}$$

Using (8), it is possible to express v in terms of u and substitute it in (7). As a result, we obtain

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right)u + p_\perp^2 u = 0, \tag{9}$$

where we have used the commutation relations $[\sigma^\alpha, \sigma^\beta] = 2i\varepsilon^{\alpha\beta\delta}\sigma^\delta$ with ranking (ρ, φ, z). Now we can perform a separation of variables by means of the substitution $u(\rho,\varphi) = w(\rho)e^{im\varphi}$ and acquire

$$\rho^2 \frac{d^2 w}{d\rho^2} + \rho\frac{dw}{d\rho} - m^2 w + p_\perp^2 \rho^2 w = 0, \tag{10}$$

where $m = 0, \pm 1, \pm 2, ...$ due to the single-valuedness of $u(\rho,\varphi)$. Equation (10) is Bessel's differential equation which gives the Bessel functions of the first kind $J_m(p_\perp\rho)$ as its solutions. The wave functions of quarks $u(\rho,\varphi)$ are the eigenstates of z component of orbital momentum operator $L_z = -i\partial/\partial\varphi$, which are localized in the vicinity of the point $\rho = 0$. However, this localization is insufficient for description of the confinement of quarks. Indeed, it is well known that $J_m(r)$ at long distances, $r \equiv p_\perp\rho \gg 1$, are oscillating functions damping as $1/\sqrt{r}$.

Here we consider a model describing the quark confinement using external static potential $V(\rho)$, which is chosen thereby to be cylindrically symmetric. For this purpose, let us insert in the Dirac equation (6) the expression $E - V(\rho)$ instead of E. Then equations (7) and (8) will undergo the corresponding change too, and instead of (9) we find

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right)u + [(E-V)^2 - (p_z^2 + M_q^2)]u - i\sigma^\rho \frac{dV/d\rho}{E - V + M_q}\left[i\left(\sigma^\rho \frac{\partial}{\partial\rho} + \frac{\sigma^\varphi}{\rho}\frac{\partial}{\partial\varphi}\right) - \sigma^z p_z\right]u = 0. \tag{11}$$

Now let us pay attention to the presence of the term $\sigma^\rho\sigma^z = \sigma^\varphi$ in (11). It destroys the initial polarization of quarks. Indeed, even initially the quark field was totaly polarized along z axis,

$$u(\rho,\varphi) = \begin{pmatrix} u_1 \\ 0 \end{pmatrix},$$

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it obtains an admixture of states with the opposite polarization due to σ^φ term because

$$i\sigma^\varphi u(\rho,\varphi) = \begin{pmatrix} 0 \\ -u_1 e^{i\varphi} \end{pmatrix}.$$

It is also evident that the total angular momentum J_z, is conserved,

$$J_z = s_z + L_z = \frac{1}{2},$$

as it should be because of the cylindrically symmetric ambient conditions.

Of course, formula (11) says more than only about the influence of the confinement on the quark spin. Any external potential changes z projection of spin with the intensity proportional to the force $dV/d\rho$.

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