

Micro-mechanical investigation of the effect of fine content on mechanical behavior of gap graded granular materials using DEM

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Abstract. In this paper, we present a micro-mechanical study of the effect of fine content on the behavior of gap graded granular samples by using numerical simulations performed with the Discrete Element Method. Different samples with fine content varied from 0% to 30% are simulated. The role of fine content in reinforcing the granular skeleton and in supporting the external deviatoric stress is then brought into the light.

1 Introduction

Granular materials are often used for construction of hydraulic structures such as dikes, levees, dams, etc. Under the action of the fluid flow, gap-graded or widely graded granular materials with an upwardly concave distribution are susceptible to internal erosion, during which fine particles can be detached and transported with the fluid flow through the pore space between coarse particles [1]. The loss of fine particles can impact the mechanical behavior of eroded materials, and then the stability of hydraulic structures. To predict this impact, it is important to deeply understand the effect of the fine content on the mechanical behavior of granular materials. The effect of fine content on the behavior of silty sands has been experimentally investigated by performing triaxial tests under drained conditions [2, 3] or undrained conditions [3, 4]. It was observed that fine content greatly influences the shear strength and the dilatancy of silty sands. Under undrained conditions, Thevanayagam et al. [4] found a threshold value $f_{c,th}$ for fine content f_c . When $f_c < f_{c,th}$, the collapse potential increases with an increase in fine content. The trend is opposite when $f_c > f_{c,th}$. From a micro-mechanical point of view, the dependence of macroscopic properties of widely graded materials on fine content can be attributed to a modification of their micro-structure. According to Thevanayagam et al. [4], for fine content $f_c < f_{c,th}$, contacts between coarse grains play the primary role in the micro-structure, while for $f_c > f_{c,th}$, contacts between fine grains are dominant. However, this is just a conjecture without any experimental verification. It is not clear yet how fine content reinforces granular skeleton and participates in supporting the external loading. It is worth mentioning that it is difficult to investigate experimentally the granular micro-structure. To overcome this

difficulty, numerical simulation of granular samples using the Discrete Element Method (DEM) can be performed to investigate the granular micro-structure.

In this paper, we use the DEM to simulate triaxial loading tests on gap graded granular samples with different fine contents. Firstly, simulated granular samples are presented and their macroscopic behavior is analyzed. The effect of fine content on the micro-structure is then investigated.

2 Simulated samples

YADE, an open-source framework for the DEM [5], is used for numerical simulations. Particles are spherical and the particle size distribution follows the idealized gap-graded curve shown in Figure 1. Each sample is composed of a coarse content and a fine content f_c which varies from 0% to 30% with a step of 5%. The gap ratio $G_r = D_{min}/d_{max}$ is defined as the ratio of the minimum diameter for the coarse content to the maximum diameter for the fine content. According to Chang and Zhang [6], internal instability due to seepage flow for a gap graded material might occur for a gap ratio $G_r \geq 3.0$. In our simulations, the gap ratio $G_r = 3$ is chosen. The number of coarse particles is chosen depending on fine content f_c such that the computation time is reasonable and the representativity of simulated samples is guaranteed.

The interaction between two particles i and j is described by two linear springs with stiffnesses k_n and k_s in the normal and tangential directions, respectively. The normal stiffness k_n is defined as $k_n = (k_i k_j)/(k_i + k_j)$ with k_i is the stiffness of particle i . In our simulations, we chose $k_i/d_i = 250$ MPa (d_i is the diameter of particle i) and $k_s = 0.5k_n$. The tangential interaction respects Coulomb's friction law with friction angle $\varphi = 35^\circ$.

Particles are first generated into a cube composed of six rigid walls. At this stage, each particle diameter is reduced by a factor of 2.0. Particles are then progressively

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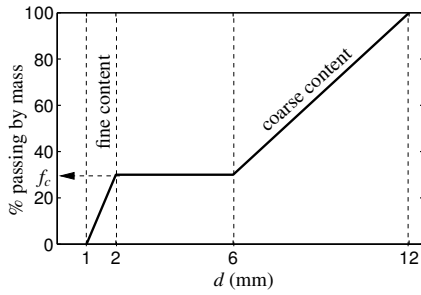


Figure 1. Gap-graded size distribution curve for the simulated samples.

expanded to reach the target size distribution. After that, the box dimensions are slowly reduced until the stresses σ_i , $i = 1, 2, 3$ reach the confinement stress of 100 kPa. During the compaction process, the friction between particles is set to 0 to obtain dense samples. At the end of this process, the friction is reset to its original value.

The density of a fines-coarses mixture can be described by the classical void ratio, e , and the *apparent* intergranular and interfine void ratios, e_c and e_f defined in Equation (1). The intergranular void ratio e_c is defined by considering that the fine grains are inactive so they are a part of voids between coarse grains. The interfine void ratio e_f is defined as the ratio of the effective volume of interfine voids to the solid volume of fine grains.

$$e_c = \frac{e + f_c/100}{1 - f_c/100}, \quad e_f = \frac{e}{f_c/100} \quad (1)$$

Figure 2 shows the void ratios e , e_c and e_f versus fine content f_c : the values of e and e_c are represented in the left axis, while the values of e_f is represented in the right axis. Fine grains fill voids between coarse grains so the classical void ratio e decreases with f_c . Lade et al. [7] experimentally found that the void ratio of sands decreases with fine content when $f_c < 40\%$. We can also see that the intergranular void ratio e_c increases with f_c since fine grains tend to intercalate between coarse grains. On the other hand, the fine fraction becomes denser with increasing fine content.

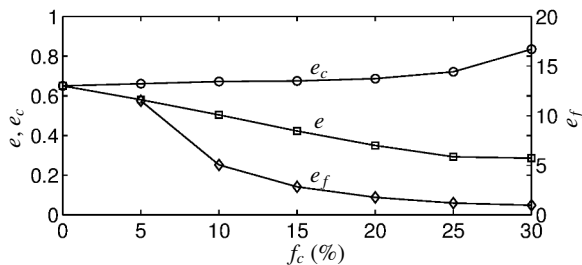


Figure 2. Void ratios e , e_c and e_f versus fine content f_c

3 Triaxial compression tests

Triaxial compression tests are performed on numerical samples. Figure 3 shows the stress ratio q/p (deviator stress $q = \sigma_1 - \sigma_3$ and mean stress $p = (\sigma_1 + \sigma_2 + \sigma_3)/3$

and volumetric strain ε_v versus axial strain ε_{11} for different fine contents. Stress ratio q/p at the peak and critical states are shown in Table 1. It is shown that the effect of fine content on the macroscopic behavior is not significant for $f_c < 20\%$. However, for $f_c \geq 20\%$, the shear strength and the dilatancy increases with f_c and we can see a marked softening behavior.

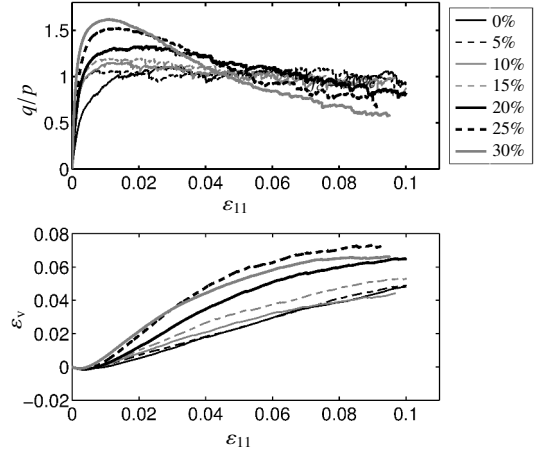


Figure 3. Stress ratio q/p and volumetric strain ε_v versus axial strain ε_{11} for different fine contents f_c .

Table 1. Stress ratios $(q/p)_{\text{peak}}$ and $(q/p)_{\text{critical}}$ at the peak and critical states for different fine contents f_c .

f_c (%)	0	5	10	15	20	25	30
$(q/p)_{\text{peak}}$	1.1	1.1	1.2	1.2	1.3	1.5	1.6
$(q/p)_{\text{critical}}$	0.9	0.9	1.0	0.9	0.8	0.7	0.6

4 Microscopic investigation

Contacts between particles in a gap graded sample can be split into three categories: coarse-coarse ($C-C$), coarse-fine ($C-F$) and fine-fine ($F-F$) contacts. In the following, we analyze the effect of fine content on coordination numbers defined for each category of contacts, on the anisotropy and on the stress transmission.

4.1 Coordination numbers

The coordination number, which is defined as the average number of contacts per particle, is often used to describe the density of a granular sample at the micro-scale. This definition seems to be not appropriate for a gap-graded sample for which the number of contacts on each coarse grain might be much higher than on each fine grain. For such a system, a coordination number for coarse fraction should be separated from the one defined for fine fraction as suggested in [8]. In this study, we define three coordination numbers \mathcal{N}_C^{C-C} , \mathcal{N}_C^{C-F} and \mathcal{N}_F which correspond to the respective average numbers of $C-C$ contacts per coarse particle, of $C-F$ contacts per coarse particle and of contacts (both $C-F$ and $F-F$) per fine particle.

Figure 4 shows \mathcal{N}_C^{C-C} , \mathcal{N}_C^{C-F} and \mathcal{N}_F versus fine content f_c at the initial, peak and critical states. It can be seen that for $f_c < 20\%$, coarse particles constitute the primary

skeleton with \mathcal{N}_C^{C-C} dominant with respect to \mathcal{N}_C^{C-F} and \mathcal{N}_F . This is one of the main reasons which explain why the macroscopic behavior is not significantly affected by fine content $f_c < 20\%$ as shown in Figure 3. For $f_c \geq 20\%$, fine particles come into contact with coarse particles, leading to a quick increase of \mathcal{N}_C^{C-F} with f_c . For the sample with $f_c = 30\%$ at the initial state, there is on average 45 fine particles around a coarse particle. Shire et al. [9] obtained a similar result for their simulations with higher gap ratio G_r . Note that \mathcal{N}_C^{C-C} at the initial state is not affected by $f_c \leq 30\%$, meaning that the initial primary skeleton formed by coarse particles is not altered by the fine fraction: fine particles actually surround and reinforce this primary skeleton.

The reinforcement of the primary skeleton by fine particles gives the ability to granular samples to dilate better so they can resist better the shear loading. On the other hand, the dilatancy leads to a loss of inter-particle contacts, and hence a reduction of the coordination numbers. For the sample with $f_c = 30\%$, \mathcal{N}_C^{C-C} at the peak state becomes lowest among various values of f_c . However, \mathcal{N}_C^{C-F} remains still highest (about 35 fine particles around a coarse particle), which compensates for the reduction of \mathcal{N}_C^{C-C} and allows this sample to have the best shear strength. After the peak state, \mathcal{N}_C^{C-F} reduces drastically. The destruction of the primary skeleton constituted by coarse particles and the drastic reduction in the reinforcement of the primary skeleton by fine particles explain why the sample with $f_c = 30\%$ shows a marked softening behavior (Figure 3).

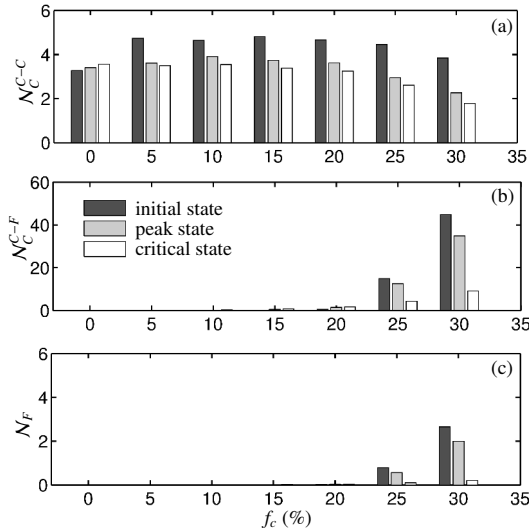


Figure 4. Coordination numbers (a) \mathcal{N}_C^{C-C} , (b) \mathcal{N}_C^{C-F} and (c) \mathcal{N}_F versus fine content f_c at the initial, peak and critical states.

4.2 Anisotropy

One of the particular features of a granular medium is its capability of orienting preferentially its contacts between particles in the major principal stress direction to resist the external deviatoric stress. This feature is called *the induced anisotropy*, which can be described in terms of the

distribution of contact orientation by the so-called *fabric tensor* \mathbf{H} . For a gap graded sample for which the particle size is very different from one to another, the particle size should be taken into account in the fabric tensor \mathbf{H} :

$$H_{ij} = \sum_k l^k n_i^k n_j^k \quad (2)$$

where superscript k runs over all contacts of the sample; l^k is the length of the branch vector joining the centers of two particles in contact k and \mathbf{n}^k is the unit normal vector at contact k . For a triaxial loading, \mathbf{H} is approximately diagonal and the two transversal components H_{22} and H_{33} are almost equal. In this case, the anisotropy can be quantified by the *anisotropy index* $H_d = (H_{11} - H_{33})/\text{Trace}(\mathbf{H})$.

Figure 5 shows the evolution of the anisotropy index H_d during the triaxial loading for different fine contents f_c . The effect of f_c on the evolution of H_d is not significant for $f_c < 20\%$ but it appears clearly for $f_c \geq 20\%$. At the peak state, the anisotropy index H_d is much higher for $f_c = 20\%$ than for $f_c < 20\%$. Beyond $f_c = 20\%$, the anisotropy at the peak state decreases with f_c although the shear strength increases with f_c as shown in Figure 3. This is quite surprising. However it can be explained by the fact that at a high fine content, the primary skeleton formed by coarse particles is so well reinforced by fine particles that the granular structure can resist the prescribed loading without resort to induce greatly the anisotropy. It is interesting to note that the softening behavior observed for high fine content (Figure 3) is accompanied by a clear decrease of the anisotropy index H_d after the peak state.

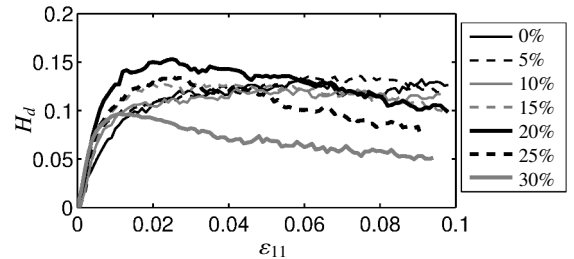


Figure 5. Anisotropy index H_d versus axial strain ϵ_{11} with different fine contents f_c .

4.3 Stress

Stress applied on the boundary of a granular sample is shared by inter-granular contacts. For a gap-graded sample, three categories of contacts are clearly distinguished. A question that arises here is the participation of each category of contacts in supporting the applied external stress. To answer to this question, we first link the macro-stress tensor $\boldsymbol{\sigma}$ to local information at contacts using the well-known averaging static operator [10]:

$$\sigma_{ij} = \frac{1}{V} \sum_k f_i^k l_j^k, \quad (3)$$

where V is the total volume of the sample; \mathbf{f}^k et \mathbf{l}^k are contact force vector and branch vector joining two centers of

particles in contact. We then decompose stress tensor σ into three parts σ^{C-C} , σ^{C-F} and σ^{F-F} which are the contribution of the respective categories of $C-C$, $C-F$ and $F-F$ contacts. Each part is defined by Equation (3) with superscript k running over the corresponding set of contacts. By doing so, the macroscopic deviator stress q can be decomposed as $q = q^{C-C} + q^{C-F} + q^{F-F}$. The contribution of each category of contacts to q can be calculated from the responding stress tensor, for example the contribution of $C-C$ contacts $q^{C-C} = \sigma_1^{C-C} - \sigma_3^{C-C}$.

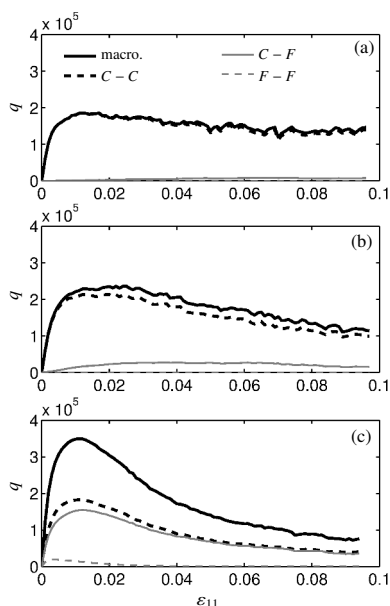


Figure 6. Contribution of each category of contacts $C-C$, $C-F$ and $F-F$ to the deviator stress q for different fine contents: a) $f_c = 10\%$, b) $f_c = 20\%$ and c) $f_c = 30\%$.

Figure 6 shows the macroscopic deviator stress q and its decomposition q^{C-C} , q^{C-F} and q^{F-F} versus axial strain ε_{11} for three fine contents $f_c = 10\%$, 20% and 30% . It can be seen that, for $f_c = 10\%$ and 20% , the applied deviatoric stress is essentially supported by contacts between coarse particles ($C-C$). However, for $f_c = 30\%$, contacts between fine and coarse particles ($C-F$) participate actively in supporting the applied deviatoric stress. For this high value of fine content, the contribution of $C-F$ contacts is almost equal to that of $C-C$ contacts. The contribution of contacts between fine particles ($F-F$) is small for the studied range of f_c . It is interesting to note that q^{C-C} is almost the same whatever the fine content. Combining with the results presented in Section 4.1, we can deduce that the skeleton formed by coarse particles remains almost unchanged and its capability of supporting the external deviatoric stress remains unchanged with increasing fine content. Fine particles which surround coarse

particles do not make coarse skeleton stronger. However, $C-F$ contacts take part in supporting the applied deviatoric stress. As shown in Figure 4, as fine content increases, there are more and more fine particles in contact with a coarse particle. As a result, the part of the deviatoric stress supported by $C-F$ contacts increase. This is the reason why the shear strength at high fine content is significantly higher than at low fine content as shown in Figure 3.

5 Conclusions

Despite an idealization of granular samples considered in the numerical simulation, this study has brought several insights into the micro-structure of gap-graded samples. For fine content $f_c < 20\%$, the skeleton formed by coarse particles is dominant and carries essentially the applied deviatoric stress. As a consequence, fine content does not have a significant effect on the macroscopic behavior. However, for $f_c \geq 20\%$, fine particles come into contact with coarse particles, reinforce the granular skeleton and take part in supporting the external stress, which leads to a higher shear strength. This influence of fine content on soil mechanical behavior highlights the necessity to consider the impact of suffusion process which mobilizes the fine particles and may induce instabilities on hydraulic earth structures.

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