The influence of the fractal particle size distribution on the mobility of dry granular materials

Luis E. Vallejo[1,*], *Jairo M.* Espitia[2], and *Bernardo* Caicedo[2]

[1]Department of Civil Engineering, University of Pittsburgh, Pittsburgh, USA
[2]Departamento de Ingenieria Civil, Universidad de los Andes, Bogota, Colombia.

Abstract. This study presents an experimental analysis on the influence of the particle size distribution (*psd*) on the mobility of dry granular materials. The *psd* obeys a power law of the form: $N(L>d)=kd^{-D_f}$, where N is the number of particles with diameter L greater than a given diameter d, k is a proportionality constant, and D_f is the fractal dimension of the *psd*. No laboratory or numerical study has been conducted to date analysing how a fractal *psd* influences the mobility of granular flows as in the case of rock avalanches. In this study, the flow characteristics of poly-dispersed granular materials that have a fractal *psd* were investigated in the laboratory. Granular mixtures having different fractal *psd* values were placed in a hollow cylinder. The cylinder was lifted and the distance of flow of the mixture was measured with respect to the original position of the cylinder. It was determined that the distance of flow of the mixtures was directly related to their fractal *psd* values. That is, the larger the distance of flow of the mixture, the larger is the fractal *psd* of the granular mixture tested. Thus, the fractal *psd* in dry granular mixtures seems to have a large influence on the easiness by which dry granular mixtures move in the field.

1 Introduction

Granular flows in the form of dry rock avalanches are widespread in nature and are among the most impressive, powerful and destructive phenomena with respect to their capacity to modify the natural landscape and devastate engineering structures. An important feature of these flows is their extremely high mobility (capable of moving up to tens of kilometers) on slopes of very small inclinations. Among the many explanations for the "hypermobility" of rock avalanches are the following [1]:

(1) Air cushion theory. It is suggested that a sheet of rock slide debris slides on a cushion of air, trapped when the slide is catapulted into an air trajectory by a ramp.

(2) Fluidization by trapped air. A similar air entrapped process is thought to cause a full or partial fluidization of the debris by means of upward flow of air.

(3) Fluidization by vapor. It has been demonstrated that rock avalanche movement expends sufficient energy to vaporize pore water. The pressure induced in the granular material by this vaporized water aids its mobility.

(4) Rock melting. It has been found that specimens of molten rock near the sliding surface of a rockslide in Switzerland. Frictional heat can be produced in exceptionally thick slide masses to melt igneous rock which in turn causes a corresponding reduction of the friction angle in experiments.

(5) Fluidization by dust dispersion. It has been proposed that during rock avalanche dense dispersion of rock dust acts as pore fluid among the larger clasts.

(6) Acoustic fluidization. Vibrations produced on the sliding surface by rapid movement over uneven ground could reduce the friction angle of the granular debris. Direct shear tests of sand conducted on a vibrating table have shown that such a phenomenon does exist.

(7) Lubrication by liquefied saturated soil. The high mobility of the Elm Slide was explained by the effects of mud, entrained by the rockslide from loose valley deposits, liquefied under the weight of the debris.

(8) Fragmentation and spreading. Studies of rock avalanches have indicated that the component material is pervasively fragmented with rock fragments of different sizes immersed in a matrix constituted of finely comminuted rock granules.

From field measurements, Crosta [2] have determined that the particle size distribution in various rockslide deposits is fractal in nature (Table 1).

An analysis of Table 1 indicates that the particle size distribution in the rock avalanches analyzed by Crosta et al. [2] is fractal in nature. The fractal dimension, D_f, of the particle size distribution (*psd*) of the materials forming part of rock avalanches varied in value between 1.90 and 3.50. According to Tyler [3] a particle size distribution with $D_f = 0$, reflects a distribution composed solely of particles of equal diameter; a D_f between 0 and 3 reflects a particle size distribution with a greater number of larger grains; while a $D_f > 3$ reflects a particle size distribution dominated by smaller grains. Thus, the different values of D_f reported in Table 1 seems to indicate different levels of fragmentation in the

* Corresponding author: vallejo@pitt.edu

material forming part of rock avalanches.

Table 1. Fractal dimension of size distribution, D_f, for rock avalanche deposits [2]

Location	Lithology	Fractal Dimension, D_f
Flims, Switzerland	limestone	1.90 – 2.95
Campo di Giove, Italy	limestone	2.65 – 2.72
Coal dumps, Canada	coal	2.62 – 2.89
Mount St. Helens, United States	volcanic	3.00 – 3.50
Thurwieser, Italy	dolostone	2.65 – 2.86
Mt. Cook, New Zealand	sandstone	2.73
Val Pola gabbro, Italy	diorite	2.20 - 310

The fractal dimension, D_f , shown in Table 1 was obtained by Crosta [2] using the following relationship developed by Turcotte [3]:

$$N(L>d)=kd^{-Df} \quad (1)$$

where, N is the number of particles with diameter L greater than a given diameter d, k is a proportionality constant, and D_f is the fractal dimension of the *psd*

No laboratory or numerical study has been conducted to date analysing how a fractal particle size distribution influences the mobility of granular flows as in the case of rock avalanches. In this study, the flow characteristics of poly-dispersed granular materials that have a fractal particle size distribution were investigated in the laboratory.

2 Laboratory program

2.1 Equipment and materials used

For the laboratory program a hollow Plexiglass cylinder measuring 51.5 mm in internal diameter, 61.5 mm in external diameter, and 136 mm in height was used for the angle of repose experiments for granular mixtures having a particle size distribution that was fractal in nature. The cylinder was placed on top of a sheet of sand paper with a coefficient of friction equal to 0.8. The interior volume of the cylinder was equal to 283,248.5 mm^3. This interior volume was filled with a mixture of rough granular materials. The average fractal dimension of the *profiles* of the rough granular materials was about 1.15. Fig. 1 shows a grain size distribution for the 4 different types of granular materials used in the experiments. Table 2 indicates the average diameter D_{50} of the materials used in the experiments. First, the granular materials made of uniform particles with D_{50}=5mm were placed in the interior volume of the hollow cylinder until they completely filled it. The cylinder filled with the uniform particles was then

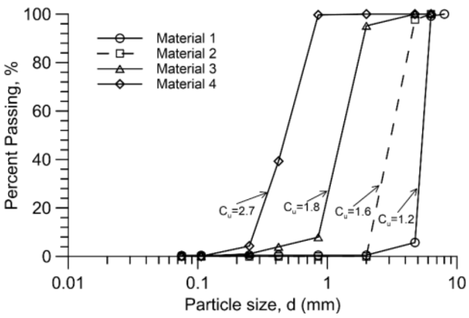

Fig.1. Particle size distribution of the granular materials used in the experiments

Table 2. Granular materials used in experiments

Granular Material	Average Diameter, D_{50} (mm)
Material 1	5.5
Material 2	3.4
Material 3	1.4
Material 4	0.5

lifted on top of the horizontally inclined sand paper. The lifting velocity of the cylinder was equal to 2 cm/sec. After release, the granular material formed a cone with sides inclined at the angle of repose of the granular material (for the uniform granular material with D_f = 0 this average angle, θ was equal to 46.5°). Besides the angle of repose, the lengths of spreading (L_1 and L_2) of the sand into the sand paper was also measured (Fig. 3). The angle of repose was calculated as the average of the two sides of the cone; the length of the spreading was also obtained as the average of the two spreading lengths measured on the two sides from the tip of the resulting cone (the average value of L for the sample with D_f = 0 was equal to 6.2 cm).

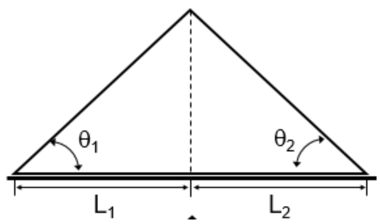

Fig.2. Angles of repose and spreading lengths in the angle of repose experiments.

In order to evaluate the influence of the fractal particle size distribution, D_f, on the values of the angle of repose and the length of spreading (L_1 and

L_2, Fig. 2), mixtures of the granular materials with the average grain diameters shown in Table 2 were prepared in the laboratory.

Six mixtures having different fractal particle size distribution were prepared in the laboratory. Mixture number 1 was the one made of uniform particles with D_{50}= 5.5 mm and D_f = 0. The particle size distributions with their respective fractal dimensions of the remaining five mixtures are shown in Fig. 3.

The fractal particle size distribution of the five mixtures was obtained by plotting on a log-log paper the cumulative number of the particles in the mixtures and the size of the particles (Fig. 4).

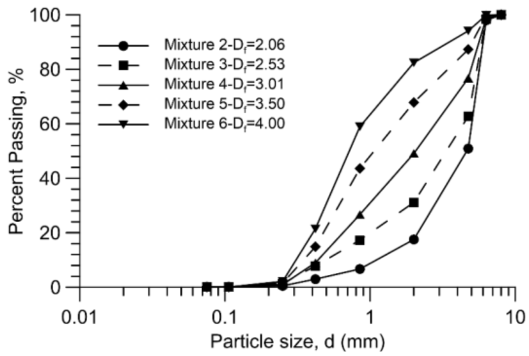

Fig.3. Particle size distribution of the five mixtures with different fractal dimensions of the five used to measure the angle of repose and advancement length (Fig. 2) in the laboratory experiments.

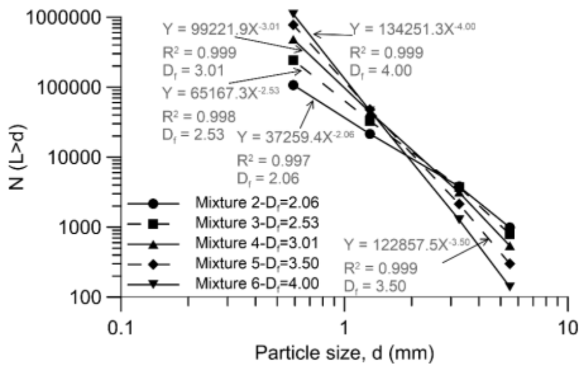

Fig.4. The number of particles, N, versus the particle size, d, method used to obtain the fractal dimension, D_f, of the mixtures with five different fractal particle size distribution shown in Fig. 3. In the resulting plots, $N = y$, and $X = d$, and the power is D_f as depicted by Eq. (1).

2.2 Angle of repose and length of advancement for the granular mixtures with varying fractal particle size distributions

Following the angle of repose experiments on the uniform material with D_{50} = 5.5 mm and D_f = 0, the same type of experiments were performed on the mixtures with different fractal particle size distributions (Figs. 3 and 4). The mixtures after filling the hollow cylinder were released on top of the sand paper sheet and the angles of repose (θ) of the two sides of the resulting cone were measured (Fig. 2). The average value of the two lengths of spreading (L_1 and L_2, Fig. 2) were also measured.

Figure 5 shows the angles of repose that the mixtures with D_f = 2.06 and D_f = 3.50 acquired when the mixtures were released on the horizontally inclined sand paper. The lifting velocity of the cylinder was equal to 2 cm/sec. Also, the average value of the two lengths of spreading (L_1 and L_2, Fig. 2) was measured and shown in Fig. 5.

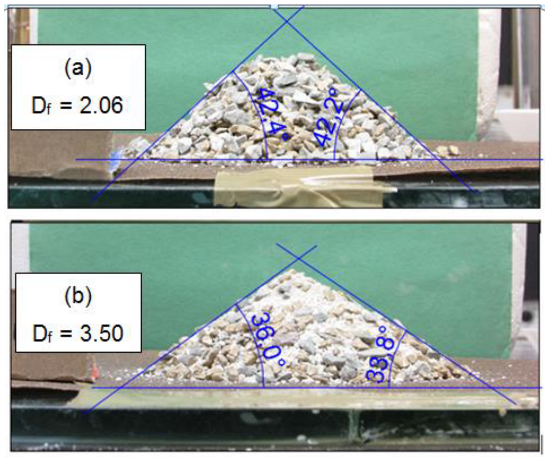

Fig.5. (a) The angles of repose of the mixture with D_f = 2.06. The average length of spreading, L = 6.02 cm. (b) The angle of repose of the mixture with D_f = 3.50. The average length of spreading L =7.1 cm.

Similar results as those shown in Fig. 5 were obtained for the mixtures with the other four fractal particle size distributions (Figs 3 and 4).

A summary of the laboratory results with respect to the average measured angles of repose, θ, the average length of advancement, L (Fig. 2) as a function of the fractal particle size distribution, D_f, are shown in Figures 6 and 7.

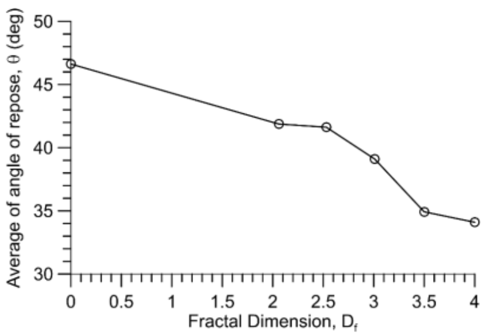

Fig.6. Relationship between the average angle of repose and the fractal dimension of the particle size distribution of the mixtures.

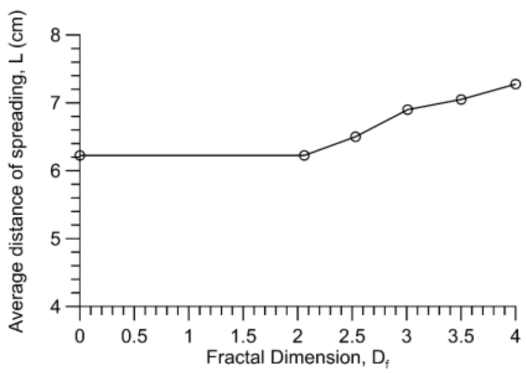

Fig.7. Relationship between the average distance of spreading, L, and the fractal dimension of the particle size distribution of the mixtures.

3 Analysis of the results

An analysis of the results shown in Figures 6 and 7 indicates that the fractal particle size distribution values (D_f) of the mixtures have a large influence on the angle of repose values (θ), and the average length of spreading (L) of the mixtures. It was determined that the higher were the fractal dimension values of the size distribution of the mixtures, the lower were the values of the angle of repose. Also, it was determined that the higher were the fractal dimension values of the size distribution of the mixtures, the higher were the values of the spreading distances of the mixtures.

According to Tyler [4] a granular material with particle size distribution with D_f = 0, reflects a distribution composed solely of particles of equal diameter (D_{50}= 5.5 mm, Table 2); a D_f between 0 and 3 reflects a particle size distribution with a greater number of larger grains (made mostly of particles with D_{50} equal to 5.5, and 3.4mm, Table 2); while a D_f > 3 reflects a particle size distribution dominated by smaller grains (mixtures made mostly of particles with D_{50} equal to 1.4 and 0.5 mm, Table 2). The laboratory results shown in Fig. 7 indicates that granular mixtures with high fractal particle size distribution values can spread to a large distance on ground slopes with zero inclination (behaving as if the frictional values between the particles is very low) [1].

A DEM study of flows of granular materials made of two different size particles indicated that the smaller size particles improved the fluidity of the mixture [5]. The same seems to occur in the flow of granular mixtures with a fractal particle size distribution. The presence of a great number of the small grain size particles in the mixtures (D_f > 3, Table 2) seems to contribute to the rotation of the particles and not to the shearing friction between during the flow of the particles (Fig. 8). The increase in number of the small grain size particles in the granular mixtures seems to cause not only a decrease in their angle of repose (θ), but also to increase the spreading distance (L) of the mixtures.

The analysis presented in this study focused on the effect of the fractal particle size distribution on the angle of repose and the length of spreading of dry granular mixtures. The effects of important parameters such as the average diameter of the grains, their shapes and degree of roughness on the angle of repose and spreading lengths were not considered in this study. The effect of these parameters could be important not only on the angle of repose but on the spreading of the granular mixtures [6,7].

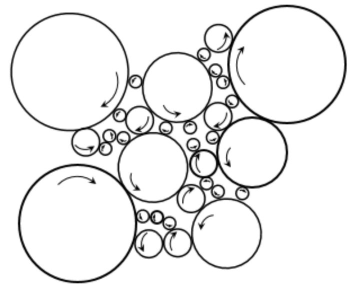

Fig.8. Relationship between the average distance of spreading, L, and the fractal dimension of the particle size distribution of the mixtures.

4 Conclusions

From the laboratory experiments on the angle of repose of granular mixtures with a fractal particle size distribution the following conclusions can be reached:

(1) The angle of repose of the mixtures decreased in value as the fractal dimension of the particle size distribution increased in value.

(2) The spreading of the mixtures on a horizontal plane increased in value as the fractal dimension of the particle size distribution increased in value.

(3) The mobility of the granular mixtures was improved by the presence of large number of smaller size particles in the mixture. The presence of these small size particles promoted the rotation of the particles in the mixtures.

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