

Contour fractal analysis of grains

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Abstract. Fractal analysis has been shown to be useful in image processing to characterise the shape and the grey-scale complexity in different applications spanning from electronic to medical engineering (*e.g.* [1]). Fractal analysis consists of several methods to assign a dimension and other fractal characteristics to a dataset describing geometric objects. Limited studies have been conducted on the application of fractal analysis to the classification of the shape characteristics of soil grains. The main objective of the work described in this paper is to obtain, from the results of systematic fractal analysis of artificial simple shapes, the characterization of the particle morphology at different scales. The long term objective of the research is to link the microscopic features of granular media with the mechanical behaviour observed in the laboratory and *in situ*.

1 Introduction

The shape of a particle can be expressed in term of three independent properties: overall shape, angularity, and roughness (surface texture) [2]. These properties correspond to a three-tiered hierarchy of observational scales and their characterization may lead to a better understanding of the behaviour of granular materials. Data analysis performed by [3] on a wide range of experimental data for granular materials, shows that an increased shape irregularity leads to more open particle configurations, therefore to an increase of maximum and minimum void ratios, respectively e_{\max} and e_{\min} . Irregularity causes also a decrease of stiffness, an increase of one-dimensional compressibility, an increase of its sensitivity to the state of stress, and an increase of critical state friction angle, φ_{cs} .

Overall shape, angularity and roughness are essentially independent properties, because one can vary widely without necessarily affecting the other two.

While many well established definitions of overall shape have been proposed in the literature, [2,4,5,6,7] the definition of angularity and roughness are still vague. Due to its fractal nature, roughness implies that there is no characteristic scale on the surface itself [3].

To be quantified, both angularity and roughness require more sophisticated techniques than overall shape, such as the acquisition of images by microscopy and computer based image analysis [8,9].

The challenge of this work is to find a quantitative approach to the characterisation of particle shape at different scales using the fractal analysis method.

2 Fractal analysis method

The fractal analysis contour method consists in an automatic computation of the particle perimeter on a 2D image, using sets of successive sticks of the same length. As the length of the sticks decreases, the computed perimeter tends to its “true” value. The length of the boundary of the particle is not strictly “true” because it depends on the image resolution adopted. This method is called fractal due to self-similar length of the sticks across the different scales.

Starting by the particle 2D image, the method consists in the following steps:

- extraction of the coordinates of the particle boundary (pixels);
- computation of the particle equivalent diameter, D , defined as the diameter of the circle having the same area as the particle;
- definition of the relative stick lengths as a fractions of the particle equivalent diameter D ;
- measurement of the particle perimeter starting from the first coordinate of the particle contour, counting the number of sticks to run all the boundary, for each stick length;
- normalisation of the computed perimeter by the equivalent diameter;
- plotting the measured relative perimeters as a function of the stick lengths.

The hypothesis is that the relatively large stick lengths give information about the overall shape of the particle, the intermediate stick lengths about its angularity and the small sticks about its roughness.

In the following, the method outlined above is applied first to artificial simple shapes, in order to study the effect of overall shape, angularity, and roughness on the pattern of the relative perimeter – relative stick length

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plot. Smooth shapes are adopted to study the influence of the overall shape (from isometric to elongated) and the angularity (from circles to squares). These are rendered increasingly rough by adopting regular zig-zagged profiles of different magnitude and frequency, to investigate the role of roughness. Finally, the method is used to describe the contour of a real particle of a Light Expanded Clay Aggregate (LECA).

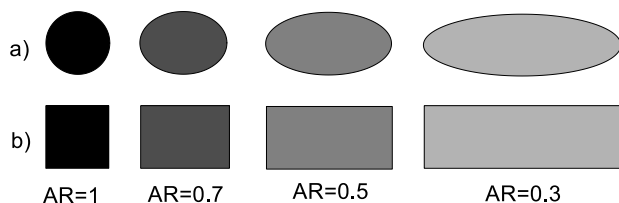


Fig. 1. a) family of circles and b) family of rectangles with AR equal to 1, 0.7, 0.5, and 0.3.

3 Simple shapes

3.1 Smooth particles

In order to study the effect of overall shape and angularity, four ellipses (Figure 1.a) and four rectangles (Figure 1.b) with different elongations are considered first. The elongation of each shape is quantified by the Bounding Box Ratio (BBR) defined as the ratio between the sizes of the rectangle circumscribed to the shape. BBR is a dimensionless parameter ranging between 0 (extremely elongated shapes, such as *e.g.*, lines) and 1 (isometric shapes, such as *e.g.*, circles or squares). The stick lengths adopted in the calculations are fractions of the shape equivalent diameter $D=2(\pi A_p)^{0.5}$, where A_p is the area of the shape. Each stick length is used as the unit to compute the perimeter of the shape. Figure 2 shows the computed values of the relative perimeter, p/D , as a function of the logarithm of the relative stick length, b/D , for: (a) the family of ellipses, and (b) the family of rectangles. As expected, the computed perimeter increases with decreasing stick length.

The stick value b in which the asymptotic perimeter is reached depends by the particle curvature. The black stars in Figure 2 show the point at which the asymptotic value of the perimeter is reached within a tolerance of 1%. For the ellipses (Figure 2.a) these points correspond to a relative stick length ranging between 0.13-0.24 and increasing with BBR . For the rectangles (Figure 2.b) these points correspond to a relative stick length of 0.03, essentially independent from BBR . This indicates that the value of the relative stick length at which the asymptotic perimeter is reached depends on the shape angularity: the rectangles are more angular than the ellipses and their angularity is independent of BBR , while the curvature of the ellipses, somehow related to their angularity, increases with decreasing BBR .

The asymptotic value of the relative perimeter contains information about the shape isometry: the circle, that is the most isometric shape, is characterized by the minimum attainable asymptotic value of the relative

perimeter, that is $p/D = \pi = 3.14$, and this increases with increasing elongation of the shape.

The oscillations in Figure 2, at large values of the relative stick length, are related to the changes of curvature of the shape along its boundary and/or to its angularity, because for angular shapes or shapes where there are strong variations of curvature along the boundary, the computed relative perimeter depends significantly on the position of the first point along the boundary. In fact, no oscillations occur in circular shapes, as every start point is equivalent to another. Information about macro and medium-scale shape descriptor can be obtained from relative stick lengths between 0.1 and 1.

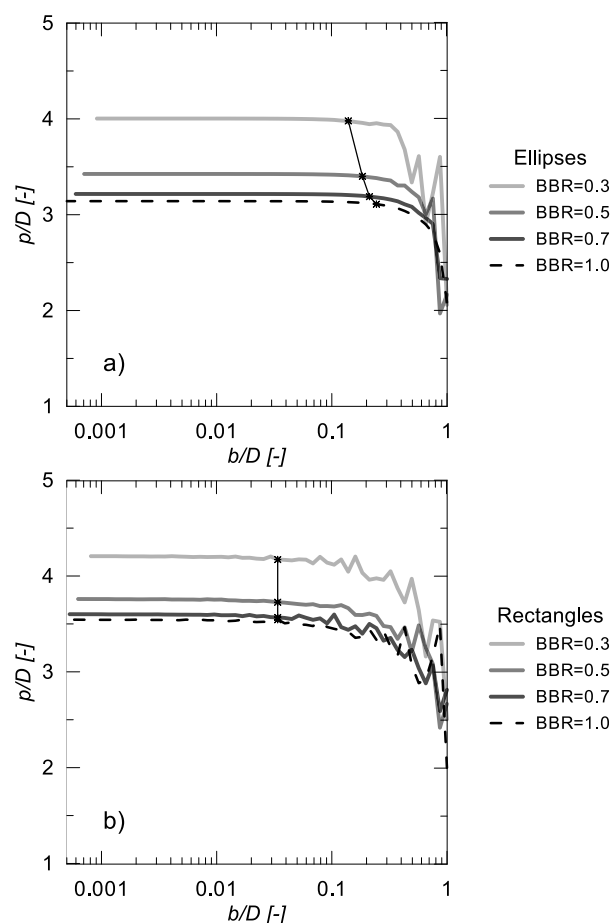


Fig. 2. Fractal analysis results for a) ellipses and b) rectangles

3.2 Rough particles

In order to study the effects of roughness, a family of circles with artificial roughness are examined in this section. The artificial roughness was generated by translating each even point externally and each odd point internally, to obtain a saw tooth boundary. Three different frequencies of roughness with three different amplitudes were considered. The frequency is related to the number of points included in the shape (100, 500 and 1000): the larger the number of points the smaller the distance between two following asperities. The amplitudes of the saw teeth considered in this study were of 0.2%, 0.6% and 1.0% of the semi major side of the

bounding box. Figure 3 shows an example of the saw tooth boundaries. In Figure 3.a, the amplitude is fixed to 0.6% and the number of points varies from 100 to 1000 points. In Figure 3.b the number of points is fixed to 500 and the amplitude varies from 0.2% to 1.0%.

Both the frequency and the amplitude of roughness affect the trend of the results of the fractal analysis. Figure 4.a shows the effect of the amplitude of the artificial roughness on the resulting curve; in this case the frequency was fixed at 500 points. The slope of the curves, for $b/D < 0.04$, increases with roughness amplitude. The effect of frequency (number of points), is shown in Figure 4.b where the amplitude is fixed at 1.0% and the frequency takes values between 100, and 1000 points. The final value of the relative perimeter is affected by the frequency while the slope to attain it is constant. The asymptotic value of the perimeter is reached when the stick length is comparable with the image resolution, and, therefore, it depends on the number of points (or pixels) defining the particle shape. It is interesting to note in Figures 4 that, as b/D increases (>0.04), all the curves overlap. This means that roughness does not affect the morphology of the shape at the meso-macro scale.

4 Real particle analysis

The procedure described above is applied to describe the fractal characteristics of the contour of a LECA particle extracted from a 2D image of the grain. The procedure followed is:

- elaboration of the Scanning Electron Microscopy (SEM) micrograph of the particle with MATLAB;
- segmentation of the image used to identify the contour of the particle;
- fractal analysis of the contour: definition of the stick lengths used to measure the grain perimeter;
- representation of the results in the dimensionless plane p/D vs b/D .

4.1. LECA material

LECA is an artificial granular material used in many civil engineering applications due to its very low unit weight. The reason of lightness is the high level of particle intra-porosity, that confers to its fragments a very irregular shape, sharp angularity and high roughness (e.g. [10]). Figure 5.a shows the SEM image of a LECA particle used for the fractal analysis. The particle dimensions are between the sieve range of 0.25-0.50 mm, and its Bounding Box Ratio is $BBR=0.45$.

4.2 Results and comparisons

Figure 5. b-d show the perimeter of the particle defined with decreasing stick lengths, corresponding to the stars in Figure 6. In step (b) the relative stick length is $b/D = 0.569$ and the evaluation of the particle perimeter is not accurate, but can still provide an indication about the overall shape of the particle (macro-scale). Step (c), corresponds to a relative stick length $b/D = 0.160$, and

provides information about the angularity or change of curvature of the boundary. Step (d), where $b/D = 0.006$, identifies the asymptotic value of the perimeter and provides information on the amplitude of roughness.

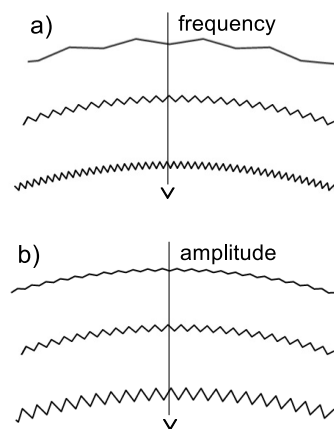


Fig. 3. Artificial roughness: a) varying frequency (100 to 1000 points) at constant amplitude (0.6%), and b) varying amplitude (0.2% to 1.0%) at constant frequency (500 points).

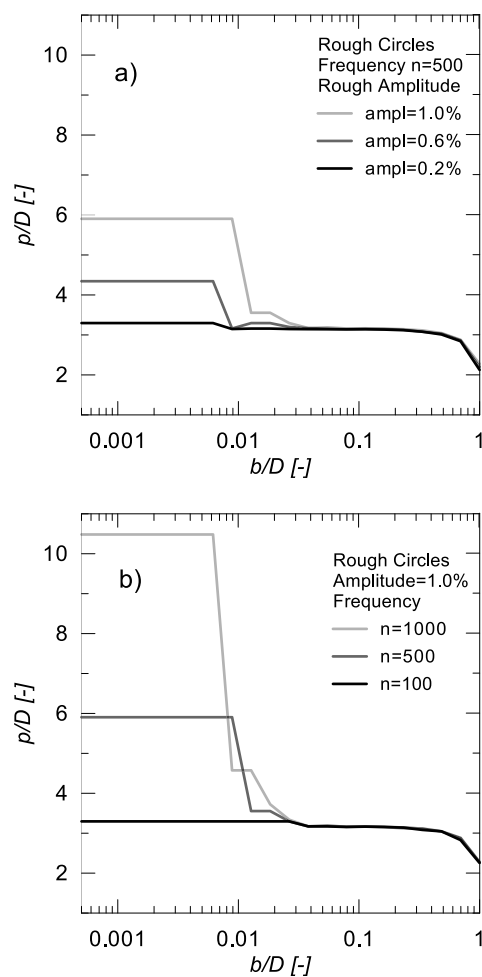


Fig. 4. Fractal analysis results for artificial rough circles. a) effects of amplitude. b) effects of frequency.

The fractal analysis performed on the particle of LECA (Figure 6) results in a more irregular trend than those

obtained for the simple shapes in Figure 4. Starting from values of b/D close to 1, the oscillations are more extended due to the angularities of the shape. A change in the slope of the curve is detectable for a value of $b/D \sim 0.1$ where there is a point of inflection in the curve. This may represent the attainment of the horizontal asymptote for the smooth shapes. From the results in Figure 6, the inflection point occurs close to point (c), and the corresponding b/D and p/D values provide information on the angularity and the elongation of the particle, as highlighted in section 3.1.

The final part of the curve is characterized by a longer path, with less constant slope compared with the artificial rough particles. In fact, the latter only include one artificial roughness, with a specific amplitude and frequency. In the case of the LECA particle, different families of roughness characterize the morphology with different amplitudes and frequencies, proper of the fractal nature of the particle roughness.

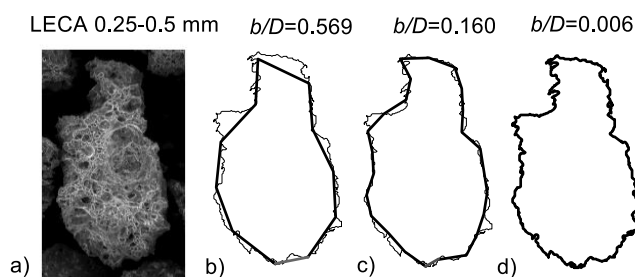


Fig. 5. LECA particle. a) original SEM image. b), c), d) different definitions of the particle perimeter using decreasing stick lengths.

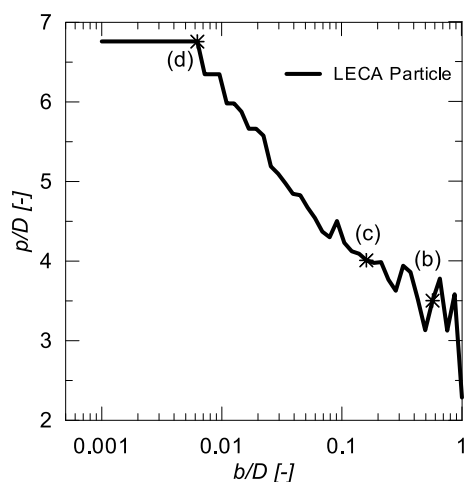


Fig. 6. Fractal analysis results on a LECA particle.

5 Conclusions

The fractal analysis of particle contour is a promising tool to study particles morphology at different scales. It consists of measuring the length of the perimeter of the particle using stick of decreasing length, and plotting the computed values against the logarithm of the stick lengths. Different shapes were examined to understand the effects of morphology at different scales, and the

results obtained from this simple shapes were used to guide the interpretation for a particle of LECA.

Each stick length conveys information on the associated morphological shape. Particular attention was paid to the characterisation of the particle's roughness, connected to the slope corresponding to the micro stick lengths ($b/D < 0.1$). The idea is that the slope of the curve relative to small stick length, can quantify the particle micro-roughness and the slope of the curve relative to big stick lengths, more than $b = 0,1D$ can quantify the macroscopic roughness. Further, the difference between the values of p/D , relative to sticks length maximum, of $b/D = 1.0$, and the one relative to the inflection point, $b/D \sim 0.1$, can quantify the macro isometry of the shape. Good analogies are between macro roughness and the literature parameter of regularity [5] and between the macro isometry value and the circularity [11]. The approach must be validated over a wider range of real particles.

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