Granular mechanics of normally consolidated fine soils

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Abstract. In this paper, duality is demonstrated to be one of the inherent properties of granular packings, by mapping the stress-strain curve into the diagram that relates the pore ratio and the localization of the contact point. In this way, it is demonstrated that critical state is not related to the maximum void ratio, but to a unique value related to two different angles of packing, one limiting the domain of the dense state, and other limiting the domain of the loose state. As a consequence, packings can be dilative or contractive, as mutually exclusive states, except by the critical state point, where equations for both granular packings are equally valid. Further analysis shows that stresses, in a dilative packing, are transmitted by chains of contact forces, and, in a contractive packing, by shear forces. So that, stresses, for the first case, depend on the initial void ratio, and, for the second case, are independent. As it is known, normally consolidated and lightly overconsolidated fine soils are in loose state, and, hence, their strength is constant, because it does not depend on their initial void ratio; except at the critical state, for which, the consolidated-drained angle of friction is related to the plasticity index or the liquid limit. In this fashion, experimental results reported by several authors around the world are confronted with the theory, showing a good agreement.

1 Introduction

Clays behaves in different way when submitted to a shearing stress, according to the degree of overconsolidation. Normally consolidated or lightly overconsolidated saturated clays are observed to show gradual increase in shear stress as the shear strain increases until an approximately constant shear stress, $\tau=\tau_{cs}$, is reached, and to compress until an approximate voluminosity, $\upsilon=\upsilon_{cs}$, is attained. Heavily overconsolidated clays are observed to show a rapid increase in shear stress reaching a peak value, τ_p, at low shear strains, and then show a decrease in shear stress with increasing shear strain, ultimately attaining the value $\tau=\tau_{cs}$. These clays are observed to compress initially and then to expand until a voluminosity $\upsilon=\upsilon_{cs}$ is attained. The quantities τ_{cs} and υ_{cs} define the critical condition, at which the shear stress and the voluminosity do not change any more. The behavior of the first soil is called "contractancy", and that of the second was named by Reynolds "dilatancy" [1], [2]. In this paper, only normally consolidated and lightly overconsolidated clays are studied.

As occurs with sands, this peculiar behavior of clays is a consequence of their particulate nature. So that, normally consolidated and lightly overconsolidated clays may be analyzed by means of the theory of granular packings, having account of the second order of their structure, for what υ_1 is the packing of clay particles and υ_2, the packing of clusters [3]:

$$\upsilon = \upsilon_1 \upsilon_2 \tag{1}$$

As it has been shown previously, the minimum voluminosity of the packing of particles remains constant for a geological formation, and, hence, the packing of the second order governs the mechanical behavior of clays.

2 Mechanics of granular packings

The strain of the basic cell whose vertexes are the center of the contact spheres, considered for the moment to be rigid, may be described in terms of the azimuth angles of assemblage, regarding the principle of multiplicity of packings. In this path, it has been established that the rhombohedron is the appropriate lattice for studying the clay behavior [3].

2.1 Distortional packing

Because its kinematics is ease to understand, the distortional or shearing packing of contact spheres is to be described in some detail (Fig.1). Once the relationship between the voluminosity, υ, and the azimuth angle β of the line of contact is established [3], the change of volume with the shearing strain is obtained as its derivative:

$$\frac{d\upsilon}{d\beta} = \frac{6\chi}{\pi}\sin\beta \tag{2}$$

According to this equation, there is not change of volume during pure shearing when β is zero, that is, when the packing is at the loosest state. If β is positive,

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the packing dilates, and, if β is negative, it compresses. This model, proposed by several authors, seems to be appropriate to describe the behavior of a soil assuming the friction between grains as the friction angle at the critical state, φ_{cs}. The static balance on the two vertical spheres, in contact gives the friction angle of the soil as: $\varphi = \varphi_{cs} + \beta$ [1]. But this equation does not match the experimental data. This is due to the following mistakes: first, the static equilibrium is established along the inclined chain of forces only; and second, the critical state is supposed to be related to the loosest state.

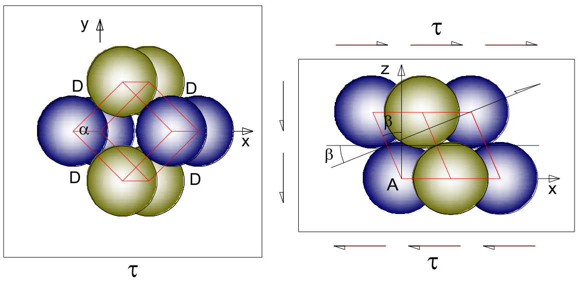

Fig. 1. Dilative shearing packing: a) plan view, and b) section showing the mechanical elements

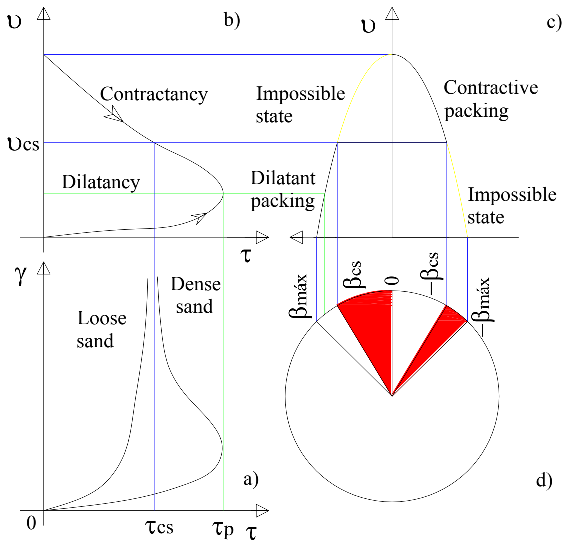

Fig.2. Mapping of the soil shear strength and the parameters of contact packing

The first mistake is overcome by including the horizontal forces of contact, regarding the law of reciprocity of the shear stress. To overcome the second mistake, it is necessary to map the stress-strain plot onto the lower contact sphere of the basic cell, through the Casagrande´s diagram, and the volumetric strain plot (Fig. 2). From this mapping, it is seen that the voluminosity at the critical state is not related to the maximum voluminosity, but with two points of contact, defined by the angles $|\beta_{cs}|$, one for the dilative branch and other for the contractive branch, both related to the same voluminosity, υ_{cs}.

Further analysis of the υ-sinβ diagram shows that, on the subtended elliptical arc, there are four sections, separated by five points: two of them associated to the densest states, two defined by the critical state, and one related to the loosest state. Of these four sections, only two are physically possible: one for the dilative packing and one for the contractive packing, both of them connected by the voluminosity at the critical state, υ_{cs}. In synthesis: for a dilative packing, contact is possible if $\beta_{cs} \leq \beta \leq 45°$ and is impossible if $0 \leq \beta \leq \beta_{cs}$; and, for a contractive packing, contact is possible if $-\beta_{cs} \leq \beta \leq 0$ and is impossible if $-45° \leq \beta \leq -\beta_{cs}$.

2.2 Elongational packing

The above analysis also applies to triaxial compression test. In this case, the basic cell must meet the conditions of axial symmetry and large elongation. This happens when the principal diagonal of the rhombohedron is vertical, and its eight spheres occupy four horizontal layers [3]. The change of volume may be related to the axial shearing strain through the equation:

$$\frac{d\upsilon_{\phi}}{d\theta} = \frac{9\sqrt{3}}{\pi}\sin\theta\left(3\cos^2\theta - 1\right) \tag{3}$$

where θ is the azimuth angle of the line of contact. Equation (3) highlights the dual character of the elongational packing. For a dense or dilative packing, contact is possible if $35.26° \leq \theta \leq \theta_{cs1}$ and is impossible if $\theta_{cs1} \leq \theta \leq 54.74°$. For a loose or contractive packing, contact is possible if $54.74° \leq \theta \leq \theta_{cs2}$ and is impossible if $\theta_{cs2} \leq \theta \leq 70.53°$; where θ_{cs1} and θ_{cs2} are the angles corresponding to the critical state voluminosity, υ_{cs}.

3 Shear strength of a clay sample

In the classic Casagrande´s diagram, curves refer to the initial loosest and densest states of a soil sample. But the complete description of all the initial states of a sample submitted to the shear test requires an infinite family of curves. In figure 3.a, it has been plotted the curves corresponding to ten initial states of a sample; five for the dense state and five for the loose state, separated each other by the line $\upsilon = \upsilon_{cs}$. It can be seen that the shear stress of each dense state is associated with a peak value, after which the response curve decreases asymptotically to the critical stress; whereas for the loose state, the maximum shear stress is the critical strength, τ_{cs}. In figure 3.b, the graph of the initial voluminosities against the respective maximum values of the shear strength is shown. There is observed that the voluminosity at the critical state separates two regions: one where the strength of the soil is a curve and is always greater than the critical strength; and other where the strength is constant and equal to the critical strength. That is the reason by which the shear strength of clays has been successfully correlated equally with the liquid limit, the plasticity index, the saturated natural water content, among others. Particularly, in soil mechanics, it has extensively used the plasticity index and the liquid limit. It can be concluded that shear strength of a clay sample, such as the consolidated-drained friction angle, is a constant.

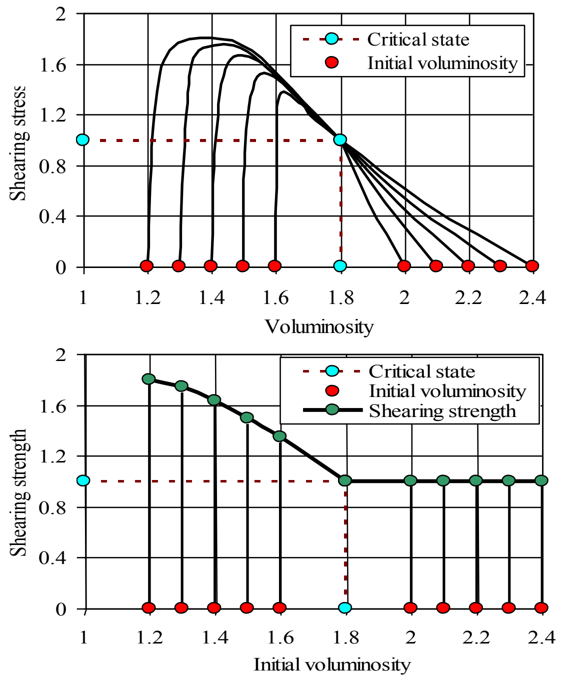

Fig. 3. Illustrative results on clay sample: a) Generalized Casagrande´s diagram, b) The shear strength as a function of the initial voluminosity of a soil sample, showing the critical initial voluminosity that separates the shearing strengths for a dilative and contractive packing respectively.

4 Shear strength of a clay formation

For clay soil samples coming from the same geological deposit, soil engineers have noticed that the shear strength exhibit a correlation with the voids contents of the constituent layers. This means that the shear strength is constant for any sample of the same layer, but is different for the samples coming from different layers of the same geological formation. As demonstrated above, it is not possible to establish a relationship between the saturated water content and the friction angle because, this is independent of the voluminosity for normally consolidated clays, except by one point: the critical state, where equations of dilative and contractive states are completely valid. Therefore, for the critical state, the contractive soil can be described by using the hypothesis and methods of dilative soils. So this, the entire geological formation is described by the equation [3]:

$$w = \frac{1}{G}\left[\frac{9\sqrt{3}}{\pi}\chi_1 \sin^2\theta_{cs}\cos\theta_{cs} + \chi_2\right] \tag{4}$$

5 Elementary granular statics

Structural cinematic parameters of soil are usually not known. For this reason, it is necessary to establish a relationship between them and any known parameter. In this path, simplest stress states can be derived, such as the geostatic and the infinite mass slopes. For dense soils, the transmission of the stresses happens as a network of chains of contact forces. These properties can be evidenced, for example, in photographs taken on the two - dimensional specimens of birefringent packing of disks regularly or randomly distributed [4]. They can also be obtained through simulations based on molecular dynamics. It has been shown [5] that stresses obtained from chains of contact forces between grains, assumed as linear in an infinite soil slope, leads to the following relationships:

$$\frac{\tan^2\theta_{cs}}{n-1} = K_0 = \frac{1}{1+n\tan^2\varphi_{cs}} \tag{5}$$

where n represents the dimension of the packing, φ_{cs}, the consolidated and drained angle of internal friction, and K_0, the coefficient of lateral pressure at rest. Substituting (5) in (4), a general equation for the clay formation is attained. Particularly, the saturated water content can be replaced by the liquid limit, w_l, or by the plasticity index, $I_p = w_l - w_p$, where w_p, is the plastic limit. Besides, it must be noticed that the general equation is bounded as follows: $\varphi_\mu \le \varphi_{cs} \le \varphi$, where φ_μ is the particle-to-particle shear strength, that plays the same roll respect to φ_{cs} as done by φ_{cs} respect to φ. For instance, recognizing that the plastic limit remains constant for a geological formation, the following equation holds:

$$I_p = \frac{9\sqrt{3}\chi_{1p}}{\pi G}\left[\cos^2\varphi_{cs}\sqrt{3-2\cos^2\varphi_{cs}} - \chi_{2p}\right] \tag{6}$$

6 Comparison with experimental data

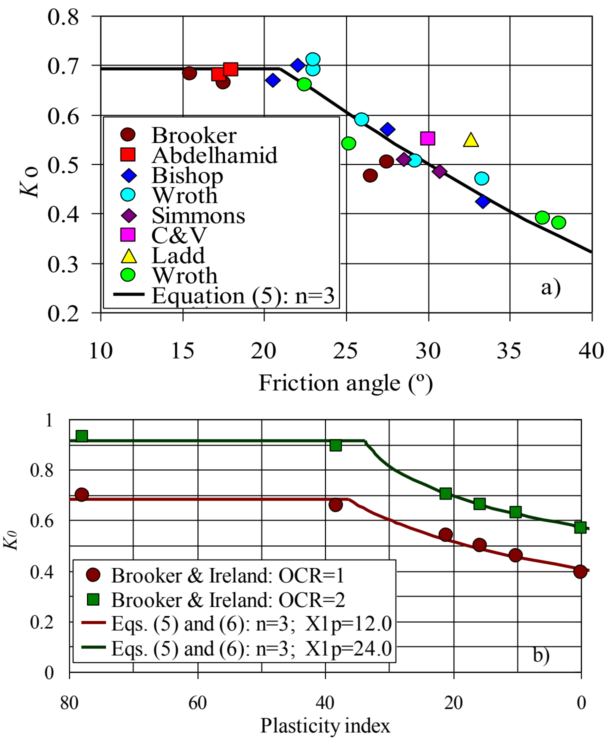

Fig. 4. Relationship between the coefficient of lateral pressure, K_0, and: a) the consolidated-drained friction angle at the critical state for different clay samples, as compiled by Holtz and Kovacs [6], b) the plasticity index for two clay samples [7],

Equations (5) and (6) may be confronted with the experimental data reported by several authors around the world. In figure 4.a, the K_0 values are compared with the

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consolidated-drained friction angle, φ_{cs} for clays of diverse origin. In this case, the curve has two sections, one that is independent of φ_{cs} and comes from the particle-to-particle friction angle, φ_{μ}, and another that corresponds to the critical state. In figure 4.b, K_0 values are related to the plasticity index, I_p, for two clay samples whose overconsolidation ratios (OCR) are 1.0 and 2.0. It is seen that the curve shows the same duality, with $I_p = 35$ as common point, approximately. In figures 5 and 6, the data reported by several authors, linking the friction angle at the critical state with the plasticity index and the liquid limit are presented to show the good correspondence with equations (6) and (5), respectively.

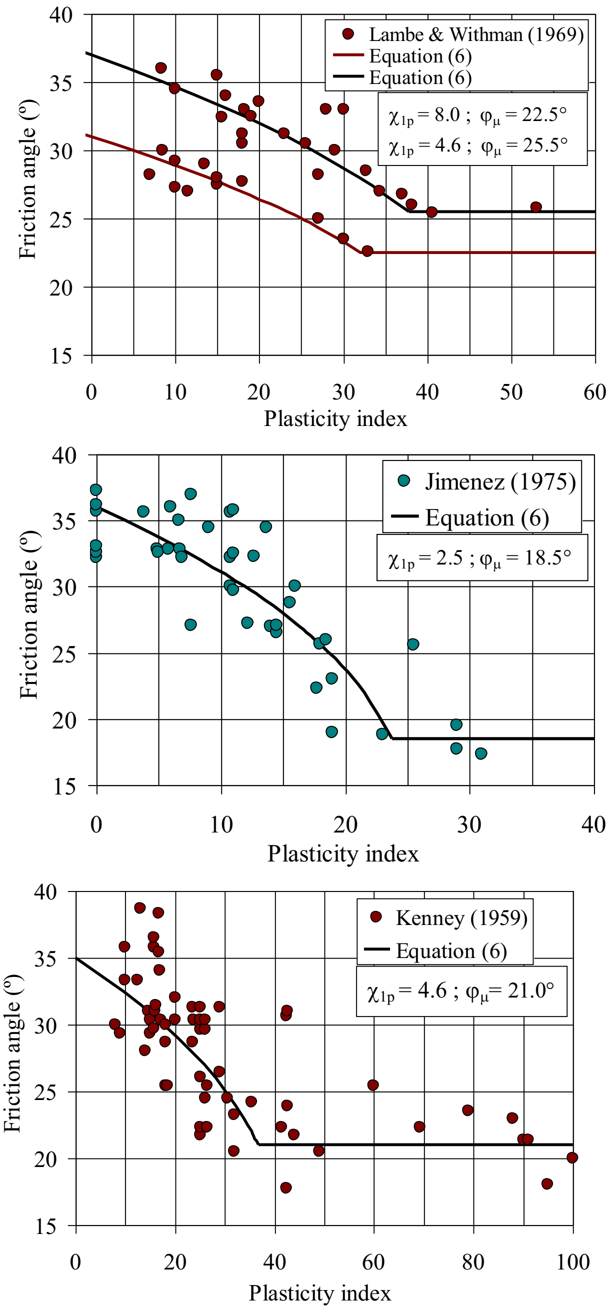

Fig. 5. Comparison of equation (6) with the experimental data reported by some authors: a) Marine clays [2], b) Spanish compacted clays [8], c) Several clays samples, as compiled by Kenney [9]

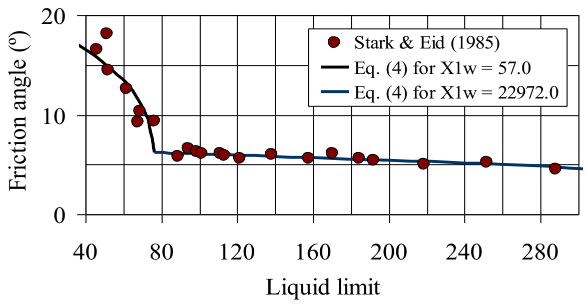

Fig. 6. Comparison of equation (4) with the experimental data reported by Stark and Eid [10] for a clay sample, showing the critical state and the residual shear strengths

7 Conclusions

Normally consolidated and lightly overconsolidated clay can be modeled as contractive packings. Mapping of the Casagrande´s diagram into the circumference of contact shows that the voluminosity at the critical state does not correspond to the loosest state of a packing. The shear strength of contractive packings is constant and independent of the initial water content. Loose and dense states are exclusive one from the other, except at the critical state. For this reason, clays belonging to the same geological formation can be studied by using the equation of the dilative packing at the critical state. Within this context, the theory of linear chains of contact forces can be used to obtain a relationship between the consolidated-drained friction angle and the packing angle, as well as the coefficient of lateral stress at rest. The experimental data for specific geological formations reported by several authors fits very well the theoretical curve. Even for broad regions, the correlation is good.

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