

Effect of contact anisotropy on the crushing strength of aggregates

Aurélien Neveu^{1,*}, Riccardo Artoni^{1,**}, Yannick Descantes^{1,***}, and Patrick Richard^{1,****}

¹LUNAM Université, IFSTTAR, MAST, GPEM, F-44340 Bouguenais, France

Abstract. This work deals with the effect of the contact orientation distribution on the crushing of granular materials. At first, a simple drop weight experiment was designed in order to study the effect of the location of three contact edges on the fracture pattern and the strength of a model cylindrical particle. The sample was placed on two bottom contact edges symmetrically distributed with reference to the vertical symmetry plane of the particle and subjected to an impact at the top. Angle α between the plane connecting a bottom contact edge to the centerline of the cylinder and a vertical plane was varied. The energy required to fracture the particle was shown to be an increasing function of angle α . Peculiar crack patterns were also observed. Then, we present a discrete model of grain fracture [1] and employ it for a numerical analysis of the problem. The cylindrical particle is discretized by means of a space filling Voronoi tessellation, and submitted to a compression test for different values of angle α . In agreement with experiments, simulations predict a strong effect of the contact orientation on the strength of the particle as well as similar fracture patterns. The proposed numerical model is therefore an interesting tool for understanding fracture mechanisms with the purpose of optimizing the crushing process.

1 Introduction

Studying the fragmentation of cohesive materials is of importance for a wide range of natural and industrial processes. In the production of aggregates, rock blocks are crushed and the resulting fragments are required to meet high standards namely in terms of size and shape. Successive crushing steps are usually carried out to achieve the requested aggregate characteristics, leading to a waste of good quality raw materials and a high energy cost that could both be mitigated upon improving the crushing efficiency.

An important aspect of the crushing process is that loads are applied on a collection of particles, and therefore stresses are transmitted through particle contacts. In this perspective, some authors have proposed failure criteria for tensile [2] and plane shear [3] fracture modes taking into account the effect of the coordination number. However, the effect of anisotropy of the contact points location on the strength of the particle has not been deeply investigated.

In order to understand fracture mechanisms with the purpose of optimizing the crushing process, discrete numerical simulation is an important tool. In this work, we first examine from simple drop weight crushing tests how the contact orientation distribution on a cylindrical particle affects the particle strength and the fracture pattern. Then we employ a discrete model of cohesive material to

the study of the effect of contact anisotropy on the shear strength. First the model is introduced, then numerical simulations of compression of cylindrical particles are presented.

2 Experiments: Drop weight tests

In order to gather information on the effect of contact distribution on particle fracture and strength, a simple experiment was designed. Cylindrical mortar samples ($D = 50$ mm, $H = 25$ mm) were tested using a drop weight setup which allows varying the location of contacts at the bottom (see Fig. 1a). A 2kg mass held by an electro-magnet is dropped from variable height. The shock energy transmitted to the sample is assessed from the dropping height whereas a high speed video camera records the sample behaviour. For different values of the angle α , the probability of breaking the sample is displayed in Figure 1b as a function of the input energy. 20 to 30 tests were performed for each energy value. The figure shows that the energy required to crush the particle is an increasing function of angle α , at least in the range considered. This is even clearer when looking at the inset in Figure 1b, where the interpolated shock energy required to achieve 50% chance of breaking a sample is plotted as a function of angle α . The effect of α is very strong, given that the energy required for crushing at $\alpha = \pi/3$ is approximately four times larger than that at $\alpha = 0$.

The fracture pattern of the sample for two contact configurations ($\alpha = 0$ and $\alpha = \pi/6$) is also shown in Fig. 1c-d. It is clear that in the $\alpha = 0$ case, a classical fracture pat-

*e-mail: aurelien.neveu@ifsttar.fr
**e-mail: riccardo.artoni@ifsttar.fr
***e-mail: yannick.descantes@ifsttar.fr
****e-mail: patrick.richard@ifsttar.fr

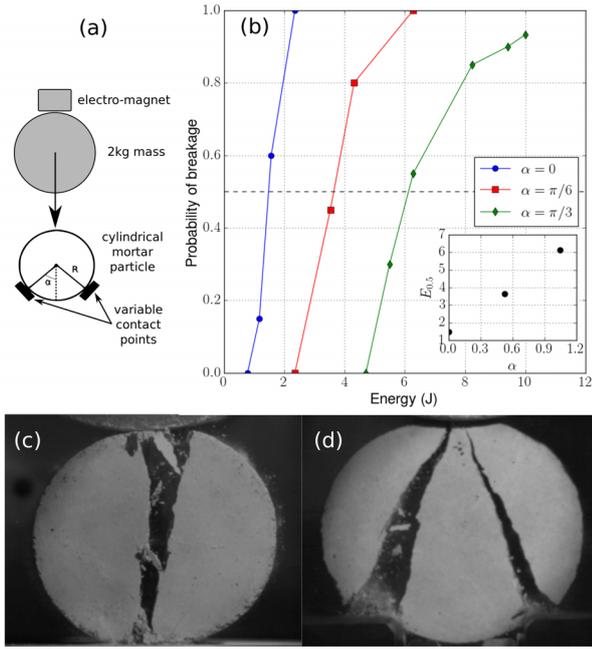


Figure 1. (a) Sketch of the experimental setup. A 2kg mass was dropped from an electro-magnet from a variable height on a cylindrical mortar sample; (b) probability of breakage of the sample as a function of the impact energy and the angle between base contact points. Fracture pattern of the sample from high speed video recordings for (c) $\alpha = 0$ and (d) $\alpha = \pi/6$.

tern is observed, with the development of a vertical crack between the two contact edges. On the other hand, when three contact edges are present, two cracks develop between the impact edge and the two bottom contact edges. Experiments prove that the particle strength and the fracture pattern are strongly influenced by the contacts configuration.

3 Cohesive interaction model

Numerical simulations using discrete Element Methods (DEM) have already been successfully used to describe the elastic behavior and rupture mechanism of a rock piece [4–6]. In these methods, a grain is represented by a collection of particles with contact bonds to model inter-particles cohesion. In a recent work [1], we introduced a cohesive interaction model for Discrete Element Methods which allows to model cohesion between contacting and non-contacting particles, suitable for any kind of particle shape. The idea behind the model is to treat independently contacts and cohesive interactions. Contacts may be modelled in different ways, for example using unilateral contact laws. Cohesion is set inside the modelled material by applying forces which oppose relative motion between particles. This relative displacement is computed between two cohesion points, each located on one of the two interacting particles. Cohesive interactions consist of a spring and a damper for both normal and tangential components. As cohesion is treated separately from contact, this allows

to apply cohesive forces even if particles are not touching each other, whatever their shape. More than one cohesive interaction may be set between interacting particles to resist rotation, bending and torsion. In this work we study the behavior of a solid grain discretized by a space-filling tessellation (Voronoi). For the sake of simplicity, only one cohesive interaction is set between the centers of the Voronoi cells which share a face. The expression of the cohesive force in the reference frame of the cohesive interaction plane writes:

$$\mathbf{F} = F_n \cdot \mathbf{n} + F_t \cdot \mathbf{t} + F_s \cdot \mathbf{s}, \quad (1)$$

with the normal (n) and tangential (t, s) components given by:

$$F_i = -k_i^s \Delta_i - k_i^d U_i, \quad \text{with } i = n, t, s \quad (2)$$

where the Δ_i are the components of the relative displacements in the cohesive interaction plane reference frame, and the k_i^s are the corresponding stiffnesses. Energy dissipation is accounted for through viscous damping: U_i denotes the relative velocity of the interaction points along direction i and k_i^d the corresponding damping coefficients.

In order to determine the stiffnesses, an analogy is made with the continuum elastic modulus of a solid. Thus, the normal spring stiffness k_n^s is derived from the Young's modulus E_μ of an elastic body as :

$$k_n^s = \frac{E_\mu S}{l_0}, \quad (3)$$

where S is the cohesive interaction surface, taken here equal to the area of the face shared by the Voronoi cells in contact. The initial length of a cohesive contact l_0 is the distance between the two cohesion points when the cohesive interactions are set. Since the cohesive interaction surface S and length l_0 differ for each pair of particles, a microscopic stiffness heterogeneity is thus introduced. The tangential spring stiffness is determined from the shear modulus, which corresponds to the shear stress to shear strain ratio. As it was shown in [1], the ratio of the normal to tangential stiffnesses may be expressed as follows:

$$\frac{k_n^s}{k_t^s} = \frac{k_n^s}{k_s^s} = 2(1 + \nu_\mu). \quad (4)$$

The damping coefficients k_i^d are calculated to obtain a critical damping which prevents oscillations at contact. Cohesive interaction may break in tension or shear when the relative displacement reaches one of the following threshold values:

$$\Delta_n^{max} = \frac{l_0}{E_\mu} \cdot \sigma_r^n, \quad (5)$$

$$\Delta_{t,s}^{max} = \frac{2(1 + \nu_\mu) l_0}{E_\mu} \cdot \sigma_r^t, \quad (6)$$

with σ_r^n and σ_r^t the microscopic strength in tension and shear which are input parameters of the model, taken identical for all the interactions. The threshold displacements

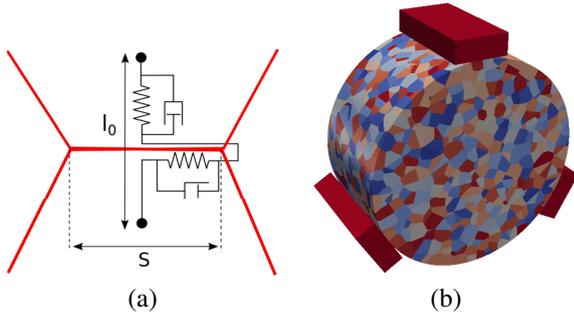


Figure 2. (a) 2D sketch of the cohesive interaction set between the centers of two Voronoi cells, with a spring-dashpot system along the normal and tangential components, (b) face view of the 3d Voronoi tessellation generated with Neper [7] (different colors for visualization purposes only).

have been chosen identical in tension and shear, which implies that:

$$\sigma_r^t = \frac{\sigma_r^n}{2(1 + \nu_\mu)} \quad (7)$$

In the following, the rupture threshold will be referred as σ_r , with $\sigma_r^n = \sigma_r$ and σ_r^t given by Eq. (7).

4 Numerical simulations

The model described in the previous section has been implemented in the Contact Dynamics framework LMG90 [8]. It is applied here to study the crushing of a cylindrical particle subjected to a three-edges loading.

The simulated particle is a cylinder of diameter d and thickness $0.5d$, which was discretized by a 3d Voronoi tessellation built by means of the Neper software [7]. Each cell of the tessellation was imported as a convex polyhedron for the simulation, for a total of 5000 polyhedra. An example of a sample generated from a Voronoi tessellation is shown in Fig. 2b.

Three walls are placed in contact with the sample. Two fixed walls are positioned symmetrically at the bottom, with their normal pointing towards the centerline of the cylinder; at the top, a wall is moved at constant velocity to apply the load on the body. The velocity is chosen of the same order of magnitude as in the impact test experiments. As in the experiments, the effect of angle α value on particle strength is investigated. Angle α (as defined in Fig. 1a) is varied between 0 and $\pi/3$. 6 repetitions were made for each angle. Simulations were performed without gravity. The stress scale is given by the microscopic Young's modulus E_μ , the mass scale by the mass of the cylinder m , the length scale by the cylinder diameter d , and the time scale by $\sqrt{\frac{m}{E_\mu d}}$. Several parameters were varied in the present study; the results discussed here were obtained for $\nu_\mu = 0.25$, $\sigma_r = 10^{-3}$, and a friction coefficient $\mu = 0.3$ between polyhedra.

The force-displacement curves obtained from the simulations are plotted in Fig. 3a. We can see that each curve displays a linear, elastic part, then reaches a maximum and decreases gently, suggesting the occurrence of progressive

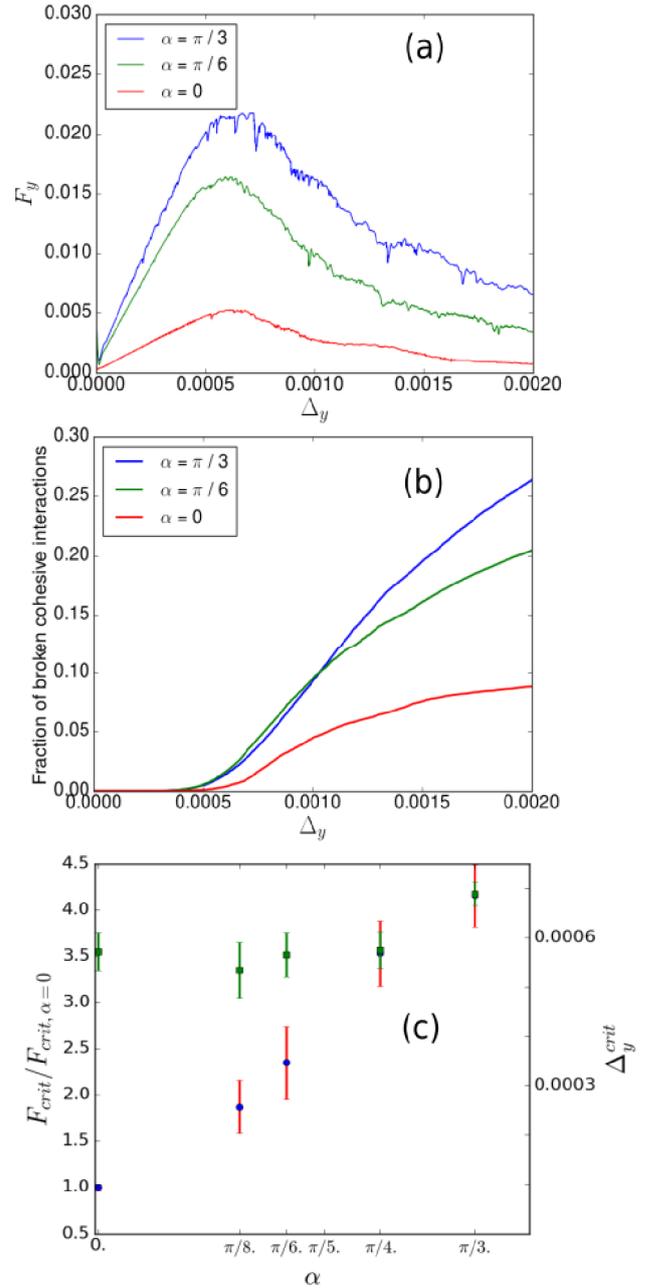


Figure 3. (a) Force-displacement curves from numerical simulations for three values of angle α ; (b) fraction of broken cohesive interactions as a function of top wall displacement; (c) ratio between force maximum for a given value of α and force maximum for $\alpha = 0$ (●), critical displacement at force maximum (■).

damage of the grain. This behavior is confirmed by the evolution of the number of broken cohesive interactions (Fig. 3b): for small displacements no breakage of the cohesive interactions occurs, then the sample is progressively damaged.

The contact distribution strongly influences the slope of the force-displacement curve as well as the maximum of the force. The effect of angle α on the maximum of the force is shown in detail in Figure 3c, where the force is normalized by its value for $\alpha = 0$. The particle strength

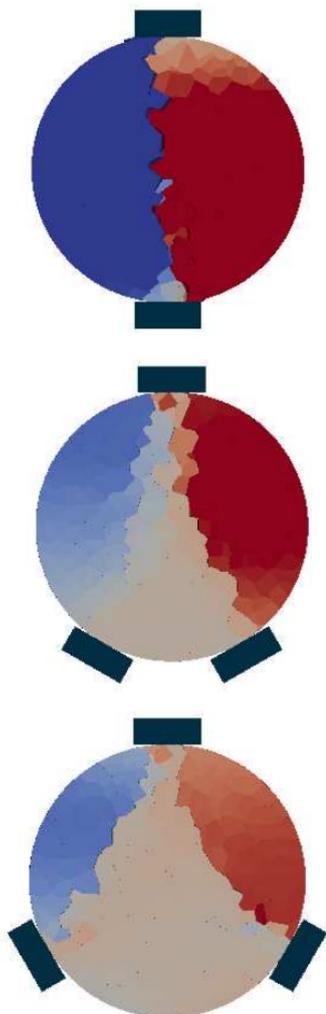


Figure 4. Fracture pattern for the numerical simulations for three values of angle α : from top to bottom, $\alpha = 0, \pi/6, \pi/3$. Colors correspond to the total displacement in the horizontal direction.

seems to increase fairly linearly with α , and the maximum force at $\alpha = \pi/3$ is approximately four times its value at $\alpha = 0$. This is in good agreement with the experimental data on impact crushing (see inset of Fig.2). By contrast, the displacements corresponding to the force maxima are similar, possibly because displacements in our Voronoi tessellation made of non-deformable particles concentrate in the fractures, hence they scale with the maximum displacement permitted in each cohesive bond crossed by the fracture.

Figure 4 displays the fracture patterns obtained for three values of angle α . As in the experiments, for $\alpha = 0$ a vertical crack is formed, and for three contact edges two cracks form between the impact and bottom contact edges.

5 Conclusions

The effect of the contact orientation distribution on the crushing of granular materials was studied by means of experimental and numerical tools. At first, a simple dynamic experiment was presented to highlight the effect of the location of three contact edges on the fracture pattern and the strength of a model cylindrical grain. The sample was placed on two bottom contact edges and received an impact by a falling weight at the top. The energy required to fracture the particle was shown to be an increasing function of the angle between the plane connecting a bottom contact edge to the centerline of the cylinder and a vertical plane. Peculiar crack patterns were also observed, connecting the impact edge to the other contact edges. A discrete model of grain fracture [1] was then employed for a numerical analysis of the effect of contact anisotropy. The cylindrical grain was discretized by means of a space filling Voronoi tessellation, and submitted to a compression test for different values of angle α . In agreement with experiments, simulations predict a strong effect of the contact orientation on the strength of the particle as well as similar fracture patterns. In particular, a linear reversible elastic behavior was found for small displacements of the top boundary, and a progressive damage was experienced for larger displacements. The proposed numerical model appears therefore a promising tool for understanding fracture mechanisms at microscale with the purpose of optimizing the crushing process. Future work will deal with more complex grain shapes and configuration of contacts.

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