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Response to a localized stress perturbation during a biaxial test

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Abstract. Numerical simulations performed to complement experimental studies of a biaxial test show that stress and strain are both structured at the mesoscopic scale. Certain features can be explained by a calculation from linear elasticity.

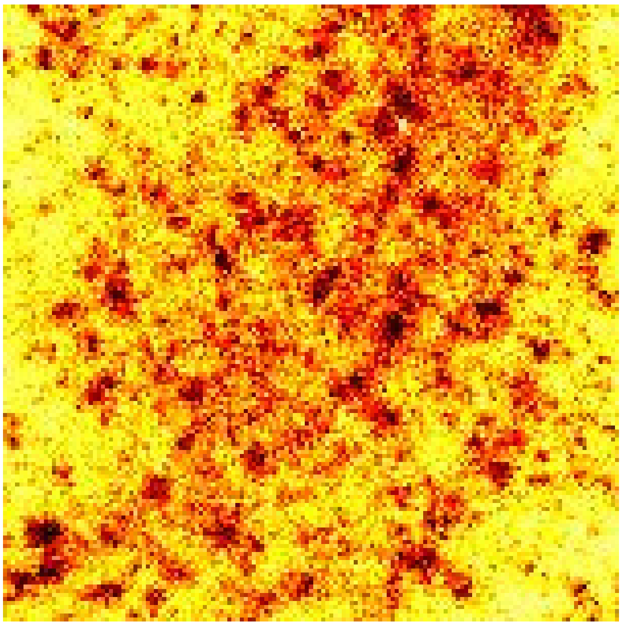

Figure 1. Experimental image showing strain inhomogeneities. The material is being compressed along the vertical axis, while a constant stress is applied at the horizontal boundaries.

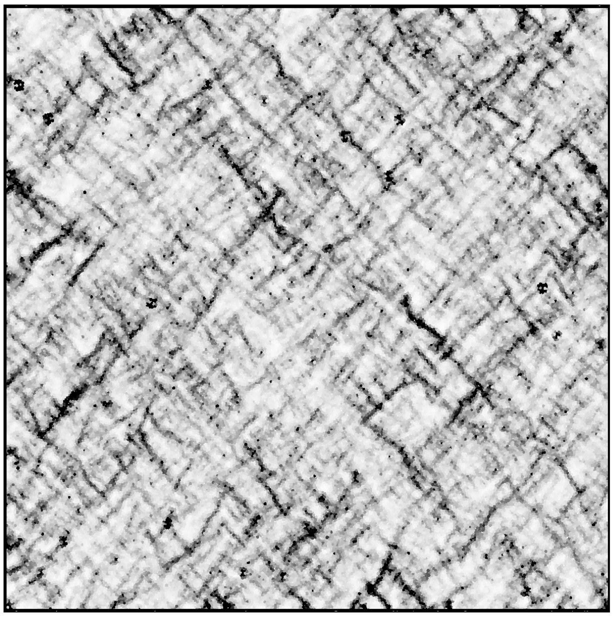

Figure 2. Simulation of a two-dimensional biaxial test with $N = 65536 = 256^2$ disks with friction coefficient $\mu = 0.2$. Imposed strain is 0.1%.

1 Introduction

The biaxial and triaxial tests are often presented as ways of imposing uniform stress and strain on material samples. Optical techniques based on diffusive-wave spectroscopy applied to a biaxial test [1], have been used to show that strain in granular materials is never uniform, even at the beginning of the test. In Fig. 1, we show a typical experimental image. The color code indicates a local measure of strain. with strain being concentrated in dark patches. These patches are themselves embedded in a diffuse irregular network of diagonal bands. It is important to note that these features appear long before failure, and seem to be independent of the formation of the shear band.

When numerical simulations with the similar boundary conditions (Fig. 2) are visualized in an analogous way, the same features appear, with the bands being more prominent [2].

One (yet to be verified) explanation of the diagonal bands is stress redistribution following localized plastic events. As shown by Eshelby [6], such events increase stress in four diagonally oriented lobes, while relaxing stress between these lobes. The increased stress could provoke new events along these directions. After many

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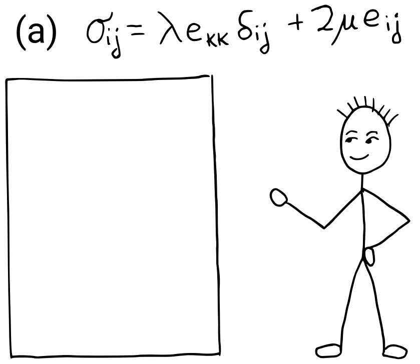
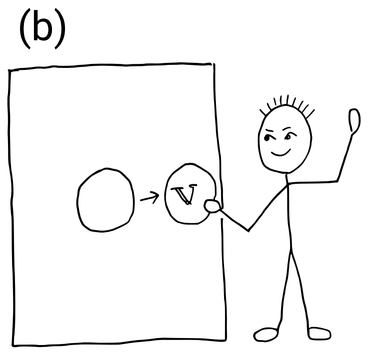
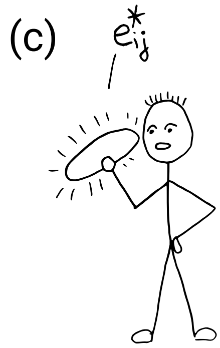


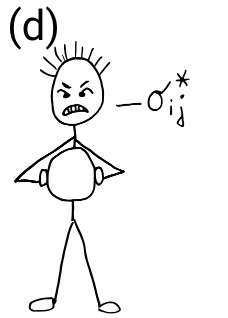
(a) An isotropic elastic medium!



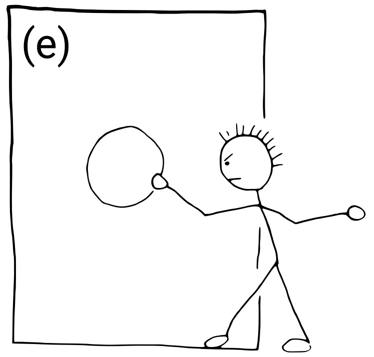
(b) Remove a small region $\mathbb{V}$.



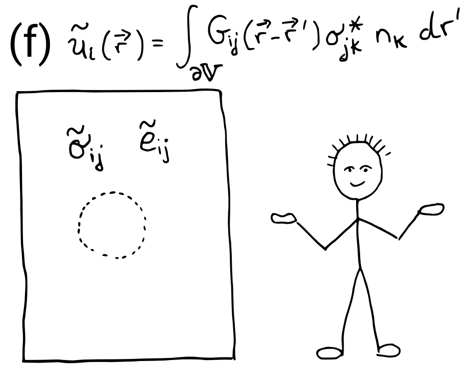
(c) $\mathbb{V}$ changes shape!



(d) The stress $\boldsymbol{\sigma}^*$ returns $\mathbb{V}$ to its original form.



(e) Put $\mathbb{V}$ back into the medium.



(f) The material adjusts to the new state of the inclusion.

Figure 3. The Eshelby calculation. A plastic event is modeled as a change of equilibrium configuration of a region $\mathbb{V}$. The consequences are propagated by a Green function. See text for more details.

events, stress will be concentrated in a network of diagonal bands, like those seen in Figs. 1 and 2.

In this paper, we present evidence supporting this explanation by showing that a local perturbation does indeed redistribute stress as predicted by Eshelby. This is nontrivial, because Eshelby's calculation assumes a linear isotropic elastic medium, whereas a loaded granular material is nonlinear, anisotropic, and contains many sliding (plastic) contacts.

2 Boundary conditions

The experiments of Fig 1 and the simulations of Fig. 2 have similar boundary conditions. A granular sample of initial height H and initial length L is contained by four straight walls. The bottom wall is fixed, while the upper wall descends at a small, constant velocity: thus the change in height ΔH decreases from zero at an imposed constant rate. On the contrary, ΔL is not imposed, but adjusted so as to maintain a constant confining pressure p_0.

Of course, there are also many differences: The simulations are strictly two-dimensional whereas the experiments are done in a three-dimensional plane strain configuration. In the simulations, the walls are perfectly straight and smooth; in the experiments they are composed of a rubber membrane.

Frictionless walls minimize boundary effects, leading to mesoscopic strains that resemble those seen in the center of the experimental sample. Other features of the experiment, such as the formation and persistence of shear bands crossing the entire sample, are not well reproduced by the simulations, suggesting that boundary conditions play an important role in the formation of these features.

3 Macro-, meso- and microscales

To think clearly about these questions, we distinguish between three different length scales:

1. The *macroscopic* length scale is the size of the sample. The macroscopic stress and strain tensors are

$$\boldsymbol{\sigma} = \begin{pmatrix} -p_0 & 0 \\ 0 & -\sigma_y \end{pmatrix} \quad \boldsymbol{\varepsilon} = \begin{pmatrix} \Delta L/L & 0 \\ 0 & \Delta H/H \end{pmatrix}, \qquad (1)$$

   where σ_y is the average normal stress on the upper wall. These tensors are functions of time, but not of space.

2. The *microscopic* scale is the grain diameter or smaller: microscopic quantities describe what happens at individual contacts or grains. Examples of microscopic quantities are contact forces and particle displacements.

3. Finally, between these two scales is the *mesoscopic* scale, where structures arise from the interaction

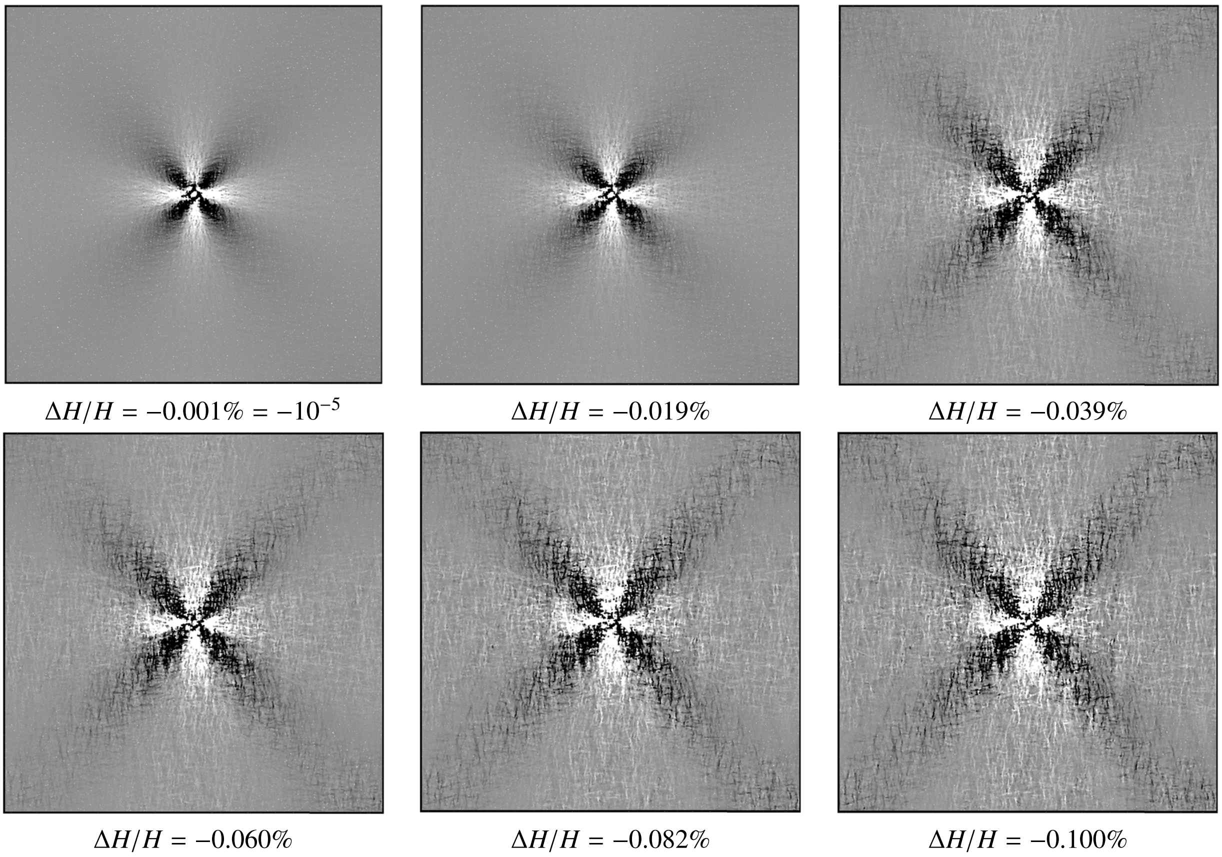


Figure 4. Stress perturbations arising from a perturbing stress applied in the center of the sample. Grayscale indicates the values of $\tilde{\sigma}_{xx} - \tilde{\sigma}_{yy}$ of each grain. Black indicates grains where $\tilde{\sigma}_{xx} - \tilde{\sigma}_{yy}$ increases the load, while where the load is decreased.

of several grains. Examples include force chains and non-affine motions. The strain inhomogeneities shown in Figs. 1 and 2 are mesoscopic. At these length scales, stress and strain can depend on position as well as time. A central question in this article is the interaction between macroscopic and mesoscopic stresses.

4 Elastic theory

In this section, we sketch the calculation of Eshelby [6], who showed how to calculate the effect of local plastic events occurring within an elastic medium. We begin by considering an isotropic elastic medium: Fig 3a. We then imagine isolating the region $\mathbb{V}$ where the event will occur: Fig. 3b.

The plastic event is modeled as a change of the zero stress configuration of $\mathbb{V}$. Instead of zero stress occurring at $\boldsymbol{\varepsilon} = 0$, it occurs at $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon^*}$. If $\mathbb{V}$ were circular and isolated from the rest of the elastic medium, it would adopt an elliptical form given by $\boldsymbol{\varepsilon^*}$, as shown in Fig. 3c.

It remains to calculate the effect of this new zero-stress configuration on the rest of the elastic medium. To do this, we first use the elastic stress-strain relation to calculate the stress $\boldsymbol{\sigma}^*$ needed to return $\mathbb{V}$ to its original form, see Fig. 3d.

Then $\mathbb{V}$ can be put back (mentally) into the elastic medium - see Fig. 3e. It will exert an additional stress $\boldsymbol{\sigma}^*$ on the boundary of $\mathbb{V}$. Using a Green function, the displacements $\tilde{\boldsymbol{u}}$ provoked by this additional stress are calculated everywhere in the medium (Fig. 3f). From these displacements, and additional strain $\tilde{\boldsymbol{\varepsilon}}$ can be found. Finally, the elastic stress-strain relation can be used again to find the corresponding stress $\tilde{\boldsymbol{\sigma}}$.

We will consider $\boldsymbol{\varepsilon}^*$ having the same directions as the macroscopic stress, that is, that height of $\mathbb{V}$ decreases while its length increases:

$$\boldsymbol{\varepsilon}^* = \begin{pmatrix} \varepsilon_1^* & 0 \\ 0 & \varepsilon_2^* \end{pmatrix}, \tag{2}$$

with $\varepsilon_1^* > 0$ and $\varepsilon_2^* < 0$.

One can then ask where $\tilde{\boldsymbol{\sigma}}^*$ will add to the macroscopic stress, possibly provoking new plastic events, and where, on the contrary, it will cancel the macroscopic stress, stabilizing the material. In the case of the biaxial test, the shear stresses are given by $\sigma_y - p_0$. We therefore calculate $\tilde{\sigma}_{yy} - \tilde{\sigma}_{xx}$. This quantity has a positive sign in four diagonal lobes radiating from $\mathbb{V}$. In these regions, $\tilde{\boldsymbol{\sigma}}$ adds to the macroscopic stress, possibly triggering new plastic events that concentrate further stress along other diagonal lobes. Superimposing many such events leads to concentrations of strain in diagonal bands, such as those seen in Fig. 1 and 2.

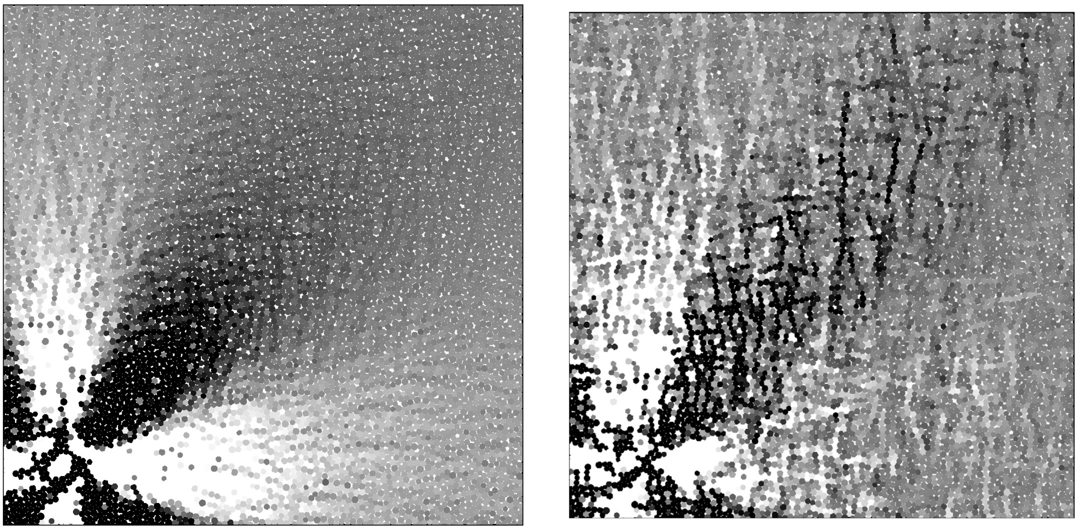


Figure 5. Magnified versions of the first and last panels of Fig. 4. The thresholds for the first panel were reduced to $3 \times 10^{-5} p_0$. One sees that the smooth elastic response co-exists peacefully with the granular microsctructure and the force chain meso-structure.

5 Numerical study of stress redistribution

The explanation advanced above explains the distribution of plastic events using a theory that reposes exclusively on calculations using elasticity. Several objections are possible:

1. We use only elasticity to explain plasticity,
2. An elastic medium has an extremely simple microscopic structure, but granular materials have complicated meso- and microscopic physics,
3. In the theory, the medium is isotropic, but a loaded granular material in a biaxial test is strongly anisotropic.

To investigate these questions, we apply a stress $\boldsymbol{\sigma}^*$ to the boundary of a small circular region $\mathbb{V}$ at the center of a numerically simulated granular material undergoing a biaxial test [3]. This stress can be applied by adjusting the forces of those contacts that cross the boundary of $\mathbb{V}$. We then examine the resulting change in stress $\tilde{\sigma}_{yy} - \tilde{\sigma}_{xx}$ provoked by the perturbing stress $\boldsymbol{\sigma}^*$. The stress on each grain is estimated by

$$\sigma_{ij} = \frac{1}{\pi R^2} \sum_{\alpha} F_i^{(\alpha)} r_j^{(\alpha)}, \tag{3}$$

where α labels the contacts. $\mathbf{F}^{(\alpha)}$ is the force transmitted by contact α, and $\mathbf{r}^{(\alpha)}$ is a vector pointing from the point of contact to the grain center, and R is the particle radius. The sum runs over all contacts of the grain.

In Fig. 4, we use a gray-scale to show the change in stress on each grain. If the stress change exceed $10^{-4} p_0$, the grain is black; it it is less than $-10^{-4} p_0$, the particle is white. If the stress lies between these two values, then a shade of gray is selected to indicate the stress level.

Fig. 4 confirms that the response of the material is indeed as predicted by the prediction of elastic theory: stress is increased in lobes oriented near 45°, and decreased above, below, and on each side of the inclusion.

Another change is also visible: as the test progresses, the texture of the lobes changes. To show this more clearly, in Fig. 5, we show enlargements of one lobe for the first and last images of Fig. 4. These pictures show that the lobes are not uniform. Instead, force chains can be discerned in both cases. But as the simulation progresses, the force chains become more widely separated, with a greater contrast with the rest of the grains. The elastic solution thus coexists with a changing mesoscale force chain structure.

References

[1] Le Bouil, A. *et al.* Gran. Matt. **16**, 1 (2014).

[2] Le Bouil, A., Amon, A., McNamara, S., and Crassous, J., Phys. Rev. Lett. **112**, 246001 (2014).

[3] S. McNamara, J. Crassous, A. Amon, Phys. Rev. E **94** 022907 (2016).

[4] F. Radjai, D.E. Wolf, M. Jean, J.-J. Moreau, *Phys Rev Lett* **80** 61 (1997)

[5] J.R. Williams and N. Rege, *Mech of Cohesive-Frictional mat* **2** 223 (1997)

[6] J. D. Eshelby, Proc. R. Soc. Lond. A **241**, 376-396 (1957).