EPJ Web of Conferences **140**, 03082 (2017) DOI: 10.1051/epjconf/201714003082
Powders & Grains 2017

Size Segregation of Binary Granular Mixtures

Towards more appropriate scaling of percolation velocity

Anurag Tripathi[1,⋆] and *Mohit* Nema[1]

[1] *Indian Institute of Technology Kanpur, 208016, India*

Abstract. We study size segregation of binary granular mixtures flowing over an inclined plane using DEM simulations. We critically examine the recently proposed scaling of the species percolation velocity with the local shear rate for different inclination angles and size ratios for an equal volume mixture of large and small spherical particles. Our DEM simulations explore a much wider range of inclination angles and shear rates for three different size ratios of large and small particles. Our results suggest that while the scaling of percolation velocity with the shear rate seems to work well for any given inclination angle, this scaling is unable to capture the influence of inclination angle on the segregation. We overcome this limitation and propose a more appropriate way of scaling the percolation velocity. This scaling, when used along with the convection-diffusion equation, is able to predict the steady state segregation of binary mixtures flowing at different inclination angles for different size ratios.

1 Introduction

Understanding and quantitatively predicting granular segregation due to size difference have been focus of researchers for decades [1, 2]. One highly promising approach toward quantitative prediction of size segregation is proposed recently by [3, 4] where the authors have studied segregation of binary granular mixtures in heap flow and in rotating cylinders. The authors report that percolation velocity of the species in the segregation direction, when scaled by the local shear rate [5, 6], shows a linear relation with the concentration of the other species with a constant of proportionality that depends logarithmically on the size ratio of the grains. In these studies, the expression for the segregation flux is symmetric in the species concentration. Recent studies [7–9], however, indicate that an asymmetric segregation flux function may be more appropriate to predict the size segregation in binary granular mixtures. In this study, we investigate the shear rate based scaling of the percolation velocity for a binary granular mixture consisting of particles of two different sizes flowing over an inclined plane using DEM simulations. In this configuration, we are able to explore a much wider range of shear rates compared to those used in [3, 4]. Our results establish that the shear rate scaling of the percolation velocity is not appropriate for chute flows. Instead, the percolation velocity scaled by the square root of the local shear rate seems to be more appropriate and can be used to predict the size segregation of binary mixtures for a range of inclination angles and size ratios.

⋆e-mail: anuragt@iitk.ac.in

2 Simulation methodology

Gravity driven flow of spherical, slightly inelastic, frictional particles over a rough bumpy surface is studied by means of discrete element method (DEM) simulations. Simulation box of length $L_x = 20d$ and width $L_z = 20d$ is used with periodic boundaries in x (flow) and z (neutral) direction (see fig.1a). The simulation box height in y direction is kept large enough so that the particles can not detect the presence of the top surface. The particles are initially placed in a simple cubic lattice arrangement with a small gap between the surface of the particles. Total number of particles is chosen to ensure that the layer thickness is around $25d$. The rough base is made by taking a $1.2d$ thick slice of particles from a random closed packed bed of particles of diameter d settled on a flat surface. The diameter (d) and mass (m) of small particles is fixed in all the simulations and three different ratios of large to small particle size ($r = d_L/d$), namely 1.5, 2 and 2.5 are considered. The large particles are placed near the base while the small particles are placed near the surface in the beginning of the simulations. The particles are settled on a horizontal surface ($\theta = 0°$). The inclination angle is then increased to desired angle (θ) and all the properties are observed as the flow becomes steady, fully developed. The contact forces between the particles are modelled using a linear spring (stiffness $k_n = 2 \times 10^5 mg/d$) with a viscous dashpot in the radial direction. The damping coefficient of the dashpot is chosen to obtain normal restitution coefficient ($e_n = 0.88$). A linear spring (stiffness $k_t = 2k_n/7$) is used to account for the elastic interaction in the tangential direction. The value of the friction coefficient between the grains is $\mu = 0.5$ and the spring deformation in the tangential direction is truncated so that the ratio of tangential to normal force for any contact is less than or equal to μ [10, 11]. Velocity-Verlet algorithm is used to integrate the equations of motion for all the particles and the time step of integration in the simulations is chosen to be small enough to resolve head-on collision of two small particles in 50 time steps. All the properties reported are non-dimensionalized using m, d and g where g is acceleration due to gravity. Properties of interest (velocity, packing fraction etc.) are averaged in a bin of thickness d_L spanning the entire simulation box in x and z direction. Fractional volume of the particles in a bin is accounted to accurately measure the properties in each bin. The solids volume fraction (ϕ_i) of species i is obtained as the ratio of the total volume of particles of species i in a bin to the volume of the bin. The local shear rate is obtained by numerically differentiating the velocity profile.

3 Results and discussion

We report results for an equal volume mixture of large and small particles with size ratio $r = 2$ in fig. 1b-d for four different inclination angles. As shown in fig. 1b and fig. 1c, the flow becomes faster (i.e., velocity in the flow direction v_x increases) and less dense (the packing fraction ϕ decreases) with increase in the inclination angle θ. The segregation due to the size difference is evident since the volume concentration of large grains (ϕ_L/ϕ) is close to unity (indicating mostly large grains are present) near free surface (fig. 1d). The large particle concentration reduces as the distance from the free surface increases, indicating that small grains concentrate near the base. Complete segregation will be indicated by a horizontal line in fig. 1d. As the inclination angle increases, the concentration profile moves away from the horizontal due to increased diffusive mixing at higher shear rates.

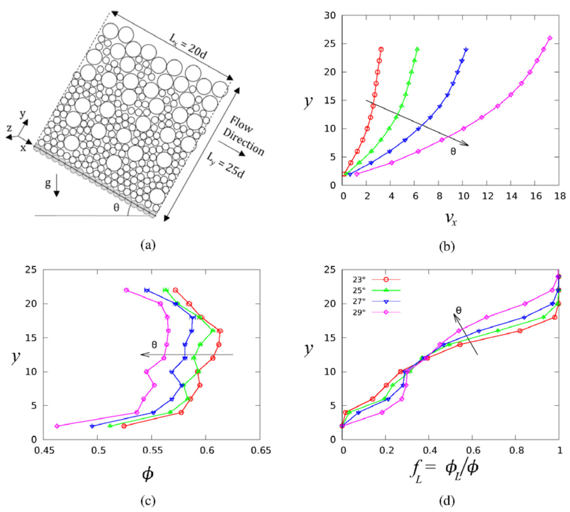

Figure 1: (a) 2D schematic for 50%-50% mixture flowing at inclination angle (θ) for size ratio 2. (b) Velocity (v_x), (c) total solids fraction (ϕ) and (d) concentration (f_L) of large particles.

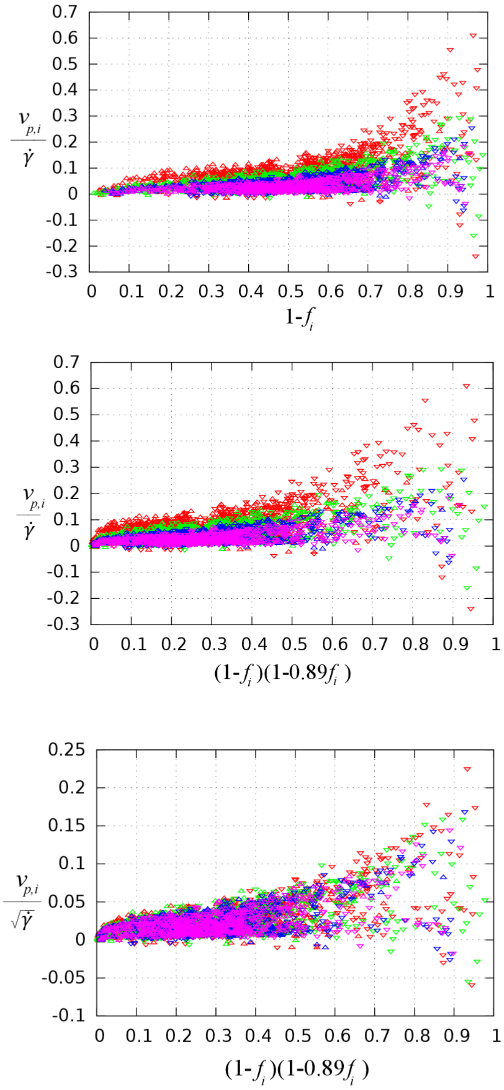

Figure 2: Top panel shows the variation of shear rate scaled percolation velocity with $1 - f_i$ and the middle panel shows the variation of the same with $(1 - f_i)(1 - 0.89 f_i)$ for four different values of θ for size ratio $r = 2$ (colour scheme same as in fig.1). Bottom panel shows the variation of percolation velocity scaled by square root of shear rate with $(1 - f_i)(1 - 0.89 f_i)$.

3.1 Percolation velocity scaling

The percolation velocity of a species in the segregation (y) direction is obtained by subtracting the volume average y-velocity (v_y) of all the grains in a bin from the volume average y-velocity of the species in that bin, i.e., $v_{p,i} = v_i - v_y$. In fig. 2, we plot the magnitude of the percolation velocity $v_{p,i}$ of the large (shown as upward triangles) and small (shown as downward triangles) grains scaled by shear rate or square root of shear rate for size ratio $r = 2$ at four different inclination angles. In fig. 2 (top), we use the symmetric segregation flux expression suggested by [3, 4] and plot the percolation velocity scaled by the local shear rate $\dot{\gamma}$ [5, 6] with the concentration of the other species $(1 - f_i)$. While the linear variation of the shear rate scaled percolation velocity seems reasonably good (for most of the range of $1 - f_i$) for a particular inclination angle, the data for different inclinations do not follow the same linear variation, i.e., data for different inclinations do not fall on top of each other. The slope of the fitted line to this data is referred to as the 'percolation length' in [3] and it is reported that this percolation length varies logarithmically with the size ratio (r). Our results indicate that the slopes of the fitted lines (not shown here) differ substantially for different inclination angles, i.e., the 'percolation length' in [3] depends upon the inclination angle as well.

Next, we explore whether the asymmetric segregation flux function [7, 8] can be used to appropriately describe the variation of the percolation velocity with species concentration. We plot the magnitude of the percolation velocity scaled by the shear rate ($v_{p,i}/\dot{\gamma}$) with $(1 - f_i)(1 - \kappa f_i)$ in fig. 2(middle). While the value of κ might depend upon the size ratio and inclination angle [7, 9], following ref. [8], we have used a constant value of $\kappa = 0.89$ in present study. Again, while the linear variation of the shear rate scaled percolation velocity seems reasonable for a particular inclination angle, data for different inclinations do not follow the same trend and differ significantly from each other. By fitting a straight line through the data, we obtain the slope S_s so that $v_{p,i}/\dot{\gamma} = S_s(1 - f_i)(1 - 0.89 f_i)$. Fig.3 shows the variation of the fitted parameter S_s for three different size ratios $r = 1.5, 2$ and 2.5 at four different inclination angles ($\theta = 23°, 25°, 27°$ and $29°$). It is evident that the value of S_s for a given size varies substantially as the inclination angle is changed. This indicates that the effect of the inclination angle on the percolation velocity is very significant and can not be neglected. We note that in ref. [3, 4], the granular flow over erodible bases were considered and hence a limited range of shear rates were explored since the flow occurred close to the angle of the repose. However, if we scale the percolation velocity ($v_{p,i}$) by the square root of the shear rate ($\sqrt{\dot{\gamma}}$) and plot the variation of $v_{p,i}/\sqrt{\dot{\gamma}}$ with $(1 - f_i)(1 - 0.89 f_i)$, data for different inclinations fall on top of each other as shown in fig. 2(bottom). By fitting a straight line through this data, we obtain the slope S_{ss} so that $v_{p,i}/\sqrt{\dot{\gamma}} = S_{ss}(1 - f_i)(1 - 0.89 f_i)$. Fig.3 shows the variation of the fitted parameter S_{ss} for three different size ratios and four different inclination angles as mentioned above. For a given size, the values of S_{ss} do not differ much from each other suggesting that square root of the shear rate may be a more appropriate quantity to scale the percolation velocity. Further, the linear variation in S_{ss} with size ratio suggest that $S_{ss} = A(r - 1)$ can be used to describe the data for different inclination angles. Thus, the percolation speed for both large and small grains can be related to the other species concentration as

$$v_{p,i} = A\sqrt{\dot{\gamma}}(r - 1)(1 - f_i)(1 - 0.89 f_i) \quad (1)$$

where A is a constant determined from the simulations. We emphasise that the above relation accounts for the dependence of the percolation velocity on the local shear rate ($\dot{\gamma}$) for a wide range of inclination angles (and hence shear rates), species concentration (f_i) for different size ratio of the grains (upto size ratio 2.5) and hence, in principle, might be appropriate for other flow configurations as well.

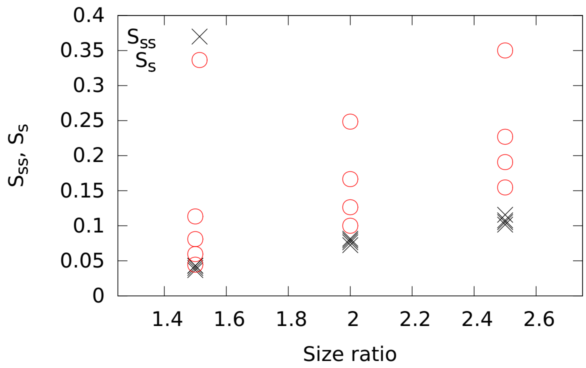

Figure 3: Variation of S_s and S_{ss} with size ratio for four different inclination angles.

3.2 Concentration profile prediction from convection-diffusion equation

The convection-diffusion equation for fully developed, steady chute flow of binary mixture of species reduces to the balance of segregation flux and diffusion flux in the segregation direction (y direction in fig. 1a) and is given as

$$v_{p,i} f_i = D\frac{df_i}{dy} \quad (2)$$

where D is the mass diffusivity of the binary mixture. The diffusivity D is calculated as half the slope of the mean square displacement of the particles in the y direction. Our simulation results indicate that the diffusivity is proportional to the shear rate (as in ref. [3]) and varies quadratically with the volume average diameter d_{mix}, i.e., $D = \beta\dot{\gamma}d_{mix}^2$. Local volume average diameter is given as $d_{mix} = (\phi_S d + \phi_L d_L)/\phi = (1 - f_L + r f_L)d$. The proportionality constant β depends upon the size ratio and we use the value of β obtained from simulations. Using the percolation velocity relation of eq. 1 and the above mentioned relation for the diffusivity in eq. 2, we obtain the following ordinary differential equation for large particle concentation (f_L)

$$\frac{df_L}{dy} = \frac{A(r-1)f_L(1 - f_L)(1 - 0.89 f_L)}{\beta\sqrt{\dot{\gamma}}d^2(1 + (r - 1)f_L)^2} \quad (3)$$

which is solved as an initial value problem using Matlab ODE15s solver to obtain the concentration profile of large particles $f_L(y)$ at steady state.

Since the initial value of $f_L(y = 0)$ is not known, we use the fact that total concentration of the large particles in the layer of height h is known; for a binary mixture having equal volume of large and small grains $\frac{\int_0^h f_L(y)dy}{h} = 0.5$. Starting from an initial guess of $f_L(y = 0)$, we explore the solution $f_L(y)$ of eq. 3 that satisfies the condition that the total concentration of the large particles in the layer is 50%

DOI: 10.1051/epjconf/201714003082

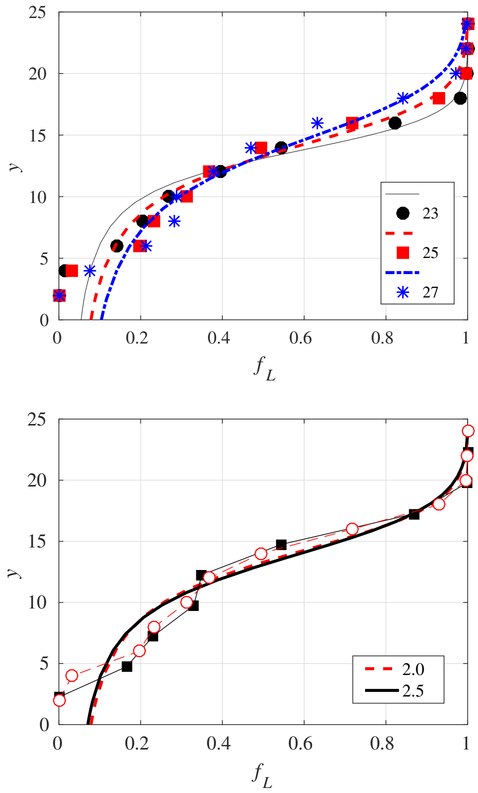

Figure 4: Comparison of the predicted concentration profile (lines) of large particles with simulation results (symbols) for three different values of θ for $r = 2$ (top) and for two different size ratios ($r = 2$ & 2.5) for $\theta = 25°$ (bottom).

within a tolerance limit of 10^{-3}, i.e., $|\frac{\int_0^h f_L(y)dy}{h} - 0.5| < 10^{-3}$. We use bisection method in the interval $0 < f_L < 1$ and iterate for the solutions of eq. 3 so that the tolerance criterion is satisfied. The values of A and β are obtained from the simulations. The shear rate profile used in eq. 3 is obtained by differentiating the function used to fit the steady state velocity profile at any particular inclination angle. Fig. 4(top) shows that the predictions using the percolation velocity scaling in eq. 1 is able to capture the dependence on the inclination angle θ. We emphasize that in the case of shear rate scaling, eq. 3 becomes independent of the shear rate and predicts that the concentration profile remains unaffected by the inclination angle. This is a crucial limitation of the existing segregation flux models which assume the percolation velocity to be proportional to the local shear rate. Fig. 4(bottom) shows that the concentration profile for $r = 2$ and 2.5 from the DEM simulations are nearly identical [10] and the same behaviour is observed in the predicted concentration profiles as well. We note that while the complete segregation in the upper part of the layer is well predicted by eq. 3, it fails to predict the near complete segregation of small particles close to the base observed in simulations.

4 Conclusion

Size segregation of binary granular mixtures, having an equal volume of large and small particles, is studied in chute flow configuration using DEM simulations for different inclination angles and size ratios. Suitability of the shear rate scaling of the percolation velocity, used by various researchers [3–9], has been investigated for both symmetric as well as asymmetric segregation flux functions. While the shear rate based scaling of the percolation velocity seems to be reasonable for a given inclination angle, it fails to describe the data when a wide range of inclination angles (and hence shear rates) are considered, i.e., the ‘percolation length’ used in ref. [3, 4] is found to depend on the inclination angle. This indicates that the shear rate based scaling of the percolation velocity is not appropriate for quantitative description of size segregation for chute flow. Our results establish that square root of the shear rate based scaling of the percolation velocity is more appropriate and is able to predict the large particle concentration for different size ratios and inclination angles in good quantitative agreement with simulation results.

Acknowledgements

AT gratefully acknowledges the financial support from I.I.T. Kanpur through the initiation grant for this study.

References

[1] J. Bridgwater, W. Foo, and D. Stephens, Powder Technology, **41**, 147-158 (1985)

[2] S. Savage and C. Lun, J. Fluid Mech., **189**, 311-335 (1988)

[3] Y. Fan, C. P. Schlick, P. B. Umbanhowar, J. M. Ottino, and R. M. Lueptow, J. Fluid Mech., **741**, 252-279 (2014)

[4] C.P. Schlick, Y. Fan, A.B. Isner, P.B. Umbanhowar, J.M. Ottino, and R.M. Lueptow, AIChE Journal, **61**, 1524-1534 (2015)

[5] B. Marks, P. Rognon, and I. Einav, J. Fluid Mech., **690**, 499-511 (2012)

[6] D.R. Tunuguntla, O. Bokhove, and A.R. Thornton, J. Fluid Mech., **749**, 99-112 (2014)

[7] P. Gajjar and J.M.N.T. Gray, J. Fluid Mech., **757**, 297-329 (2014)

[8] K. van der Vaart, P. Gajjar, G. Epely-Chauvin, N. Andreini, J.M.N.T. Gray, and C. Ancey, Phys. Rev. Lett., **114**, 238001 (2015)

[9] D.R. Tunuguntla, T. Weinhart, and A.R. Thornton, Comp. Part. Mech. (2016) doi:10.1007/s40571-016-0136-1

[10] A. Tripathi and D.V. Khakhar, Phys. Fluids, **23**, 113302 (2011)

[11] A. Tripathi and D.V. Khakhar, J. Fluid Mech., **717**, 643-669 (2013)