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A DEM study of oedometric compression of model granular materials

Initial state influence, stress ratio, elasticity, irreversibility.

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Abstract. A DEM simulation study of spherical beads with elastic-frictional contacts in *oedometric* compression is carried out for a wide variety of initial states, differing in solid fraction Φ, coordination number z (independent of Φ in dense systems) and inherent anisotropy. Stress ratio $K_0 = \sigma_2/\sigma_1$, along with z, Φ and force and fabric anisotropies are monitored in compressions in which axial stress σ_1 varies by more than 3 orders of magnitude. K_0 tends to remain constant if the material was already one-dimensionally compressed in the assembling stage. Otherwise, it decreases steadily over the investigated stress range. K_0 relates to force and fabric anisotropy parameters by a simple formula. Elastic moduli may express the response to very small stress increments about the transversely isotropic equilibrated states, although oedometric compression proves an essentially anelastic process, mainly due to friction mobilization. Despite apparent nearly reversible increases of axial strain ϵ_1 (or density Φ), especially in dense samples, internal state evolutions are strongly irreversible, as evidenced by changes in z and K_0. Fabric changes are reflected by anisotropic elastic moduli.

1 Introduction

Oedometric compression, an axially symmetric process in which one principal strain component (ε_1) is increased, the others being maintained at zero ($\varepsilon_2 = \varepsilon_3 = 0$), is one of the simplest anisotropic loading processes, representative of natural materials under gravity (e.g. sediments consolidating under their weight). Oedometric compression leads to *transversely isotropic* structures, with the symmetry of revolution about axis 1. It is classically characterized by the ratio K_0 (termed coefficient of earth pressure at rest) of lateral stress $\sigma_2 = \sigma_3$ to axial stress σ_1[1, 2], which is often regarded as a constant. Oedometric compression is usually described as irreversible [2, 3], with a net plastic increase of solid fraction Φ (or a decrease of void ratio $e = -1 + 1/\Phi$) as σ_1 grows, while density changes occurring below the maximum past value of σ_1 (the preconsolidation pressure) are reversible.

The present contribution reports on a systematic DEM study of oedometric compression in a simple model material for a wide variety of initial states, with the aim of exploring, for such a simple case of an anisotropic material state under varying load intensity, stress ratio K_0, elasticity and irreversibility, with reference to microstructural and micromechanical aspects. After a brief description of the model and initial states (Sec. 2), Sec. 3 discusses stress anisotropy and its connections to force networks, and Sec 4 gives a brief account of elastic properties. Sec. 5 discusses irreversiblity, before a few words of conclusion (Sec. 6).

2 Material properties and sample preparation

Slightly polydisperse spherical beads (with diameters between D_1 and $D_2 = 1.2 \times D_1$, uniformly distributed by volume) with elastic-frictional contacts, using the simplified Hertz-Mindlin model of [4] and the elastic properties of glass (plane strain modulus $\tilde{E} = E/(1-\nu^2) = 77$ GPa), and friction coefficient $\mu = 0.3$, are enclosed in a fully periodic cuboidal box. They are first assembled into a solid pack under low stress, $\sigma_0 = 10$ kPa (or, in dimensionless form, $\kappa = (\tilde{E}/\sigma_1)^{2/3} = 39000$), by subjecting a disordered granular gas to either an isotropic ($\sigma_1 = \sigma_2 = \sigma_3 = \sigma_0$) or an oedometric ($\sigma_1 = \sigma_0$, $\varepsilon_2 = \varepsilon_3 = 0$) compression until contact forces accurately balance the applied load. One may regard the beads as frictionless in this assembling procedure, and possibly implement a “mixing" or “vibration" stage, as described in Ref. [4]. As a result, one obtains the 6 different initial states of Tab. 1, characterized by solid fraction Φ, fraction of *rattlers* (grains that carry no load) x_0, and coordination number z (z si the average contact number per grain, and $z^* = z/(1-x_0)$ denotes the coordination number of the force-carrying network). Those states are labelled with a three-letter code: the first letter, D or L, corresponds to dense or loose states, the second one is L for low or H for high coordination number, while the third one is i if the initial confinement

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is isotropic, o if it is oedometric. Note the peculiarity of DLi and DLo states which, as previously reported [4], are nearly as dense as DHo-DHi ones (with Φ values close to random close packing), but as poorly coordinated as the looser ones (LLo-LLi). Initial stresses, isotropic ($K_0 = 1$)

Table 1. Solid fraction Φ, coordination numbers z and z^*, rattler fraction x_0, and stress ratio K_0 for the different initial states. Values averaged over 3 configurations of $N = 4000$ grains.

	LLo	LLi	DHo	DHi	DLo	DLi
Φ	0.584	0.589	0.639	0.638	0.634	0.637
z	4.22	4.14	5.98	5.99	4.06	4.17
$z^\star$	4.63	4.63	6.07	6.07	4.54	4.65
$x_0(\%)$	8.8	10.3	1.5	1.3	10.4	10.4
K_0	0.72	1	0.94	1	0.51	1

by construction in "XYi" states, might exhibit significant levels of anisotropy in oedometrically prepared ones.

3 Compression and stress ratio

All six initial states are subjected to an oedometric compression with σ_1 growing to 31.2 MPa ($\kappa \simeq 180$). In this process, while Φ increases by 0.01 to 0.02, coordination number z varies as in Fig. 1. In general, a gradual increase

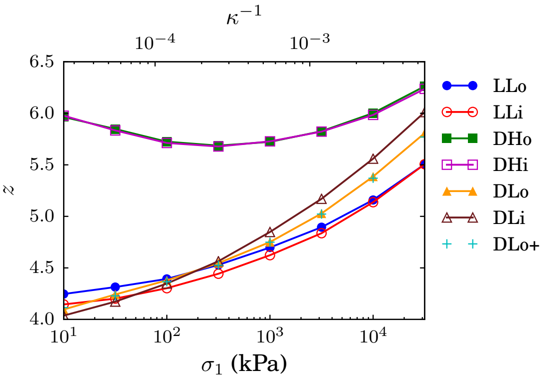

Figure 1. Coordination number z versus axial stress σ_1 for different initial states (DLo+ = DLo state with larger sample size).

of z under compression is observed, except for an initial stage in systems prepared with large coordination (DHo, DHi), in which z first decreases to a minimum. This shows that oedometric compression is a complex process that differs from a homogeneous uniaxial shrinking of the contact structure. The contact network, beyond its density, should also be characterized by fabric anisotropy, which is encoded, to first order, in parameter $\tilde{c}_2$ ($\tilde{c}_2 = 0$ in isotropic states):

$$\tilde{c}_2 = \langle n_1^2 \rangle - \frac{1}{3}. \tag{1}$$

$\tilde{c}_2$, initially zero in "XYi" systems by construction, gradually grows with σ_1, reaching typical values of 0.02–0.04 under high presure. It stays constant, near 0.04, in LLo systems, and achieves its highest values in the DLo states under low stress, slightly decreasing above 1 MPa (down to $\simeq 0.06$ for the highest σ_1 value).

Fig. 2 plots stress ratio K_0 versus σ_1 in the oedometric compression of all six initial states up to a large axial stress value. Initially equal to 1 for "XYi" systems, and

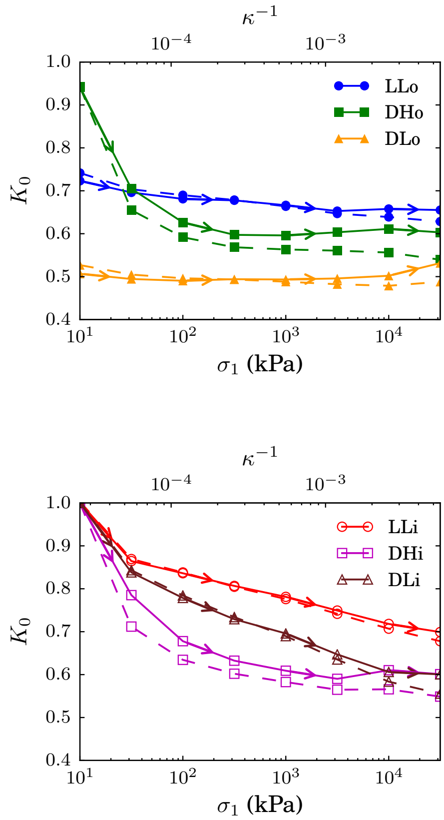

Figure 2. K_0 versus axial stress σ_1 in oedometric compression (dots and solid lines), in oedometrically (top) and isotropically (bottom) assembled systems. Dots joined by dashed lines: predictions of Eq. 3.

close to 1 in DHo ones, K_0 decreases, fast in DHo, much more gradually in isotropically assembled samples. It only stays roughly constant (near 0.5 in DLo, 0.7 in LLo, and somewhat above 0.6 in DHo for $\sigma_1 \geq 200$ kPa) in oedometrically assembled systems. K_0, using the classical formula relating stresses to contact forces in static granular packs [4], readily related to anisotropy parameters. A force anisotropy parameter, $\tilde{f}_2$ is defined from the angular distribution of average normal force, $\mathcal{F}(|n_1|)$, normalized by the average normal force in all contacts. By axial symmetry $\mathcal{F}$ is a function of the sole coordinate n_1 of the unit contact normal vector $\mathbf{n}$. One writes

$$\tilde{f}_2 = \frac{1}{4\pi} \int_\Sigma \mathcal{F}(|n_1|) n_1^2 d^2\mathbf{n} - \frac{1}{3}. \tag{2}$$

Using the standard formula for stresses [4], noting that the contributions of tangential forces to stresses $sigma_1$ $sigma_2$ are negligible, and dealing with anisotropies to leading order, one readily obtains an approximation for principal stress ratio $K_0 = \sigma_2/\sigma_1$:

$$K_0 = \frac{\sigma_{22}}{\sigma_{11}} \simeq \frac{2 - 3(\tilde{c}_2 + \tilde{f}_2)}{2 + 3(\tilde{c}_2 + \tilde{f}_2)}. \tag{3}$$

As apparent in Fig. 2, relation (3) provides a good approximation of K_0, which is thus simply related to anisotropy parameters $\tilde{c}_2$ and $\tilde{f}_2$ (as remarked in [5, 6]). $\tilde{f}_2$ steadily increases with σ_1, from zero in initially isotropic assemblies, the faster in highly coordinate systems, up to about 0.11 in the MPa range. $\tilde{f}_2$ and $\tilde{c}_2$ are of the same order in the most anisotropic systems, DLo and LLo.

4 Elastic properties

An elastic response is probed on subjecting well-equilibrated states along the compression curve to small stress increments that do not cause rearrangements and for which the effects of friction are negligible [7]. Elastic moduli are evaluated on building the stiffness matrix of the contact network, involving normal and tangential stiffnesses associated with small displacement and rotation increments [7]. Given the transversely isotropic symmetry in oedometric compression, there are 5 independent moduli [8, 9]. Defining a quasielastic range for incre-

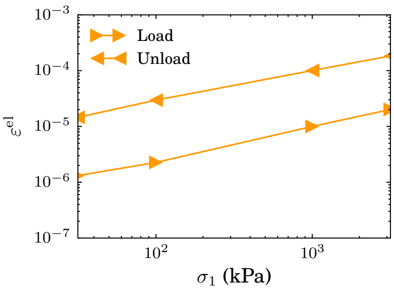

Figure 3. Quasielastic range versus σ_1 in one DLo system, for positive (loading) and negative (unloading) axial strain increments.

ments of axial strains, $\Delta\varepsilon_1$, by the requirement that axial strain increment $\Delta\sigma_1$ differs from the elastic response $C_{11}\Delta\varepsilon_1$ by less than 5%, its typical variation with σ_1, for both signs of $\Delta\varepsilon_1$, is shown in Fig. 3. A fully elastic response for the whole compression curve would imply stress ratios $\Delta\sigma_2/\Delta\sigma_1$ equal to the corresponding ratio of elastic constants, C_{12}/C_{11}. However, those two quantities strongly differ (typically one has $C_{12}/C_{11} < 0.4$, while $\Delta\sigma_2/\Delta\sigma_1$ values are similar to K_0). The oedometric compression entails friction mobilization and sliding (most frequent, somewhat surprisingly, in contacts with normal direction parallel to the transverse plane) and the quasielastic range is only associated to arrested, stabilized configurations (obtained after tiny creep intervals) in which full friction mobilization is suppressed. A full study of elastic moduli [10] reveals, on the one hand, similar trends as in isotropic systems [7], as to their dependence

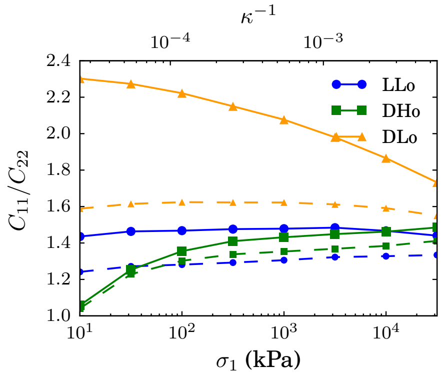

Figure 4. C_{11}/C_{22} versus σ_1 or κ^{-1} in initially anisotropic sample series. Dashed lines: prediction of the Voigt approximation.

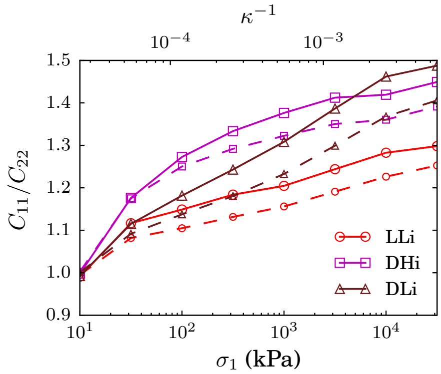

Figure 5. Analog of Fig. 4 for initially isotropic systems.

on stresses and coordination number (which is far mode influential than solid fraction), and, on the other hand, that moduli reflect the geometric and mechanical anisotropy of oedometrically compressed systems. Thus, Figs. 4-5 shows how the ratio of moduli C_{11} and C_{22} (associated to longitudinal waves propagating in the axial and transverse directions) achieves large values in the most anisotropic systems (DLo, and LLo), while lower ratios characterize the gradual gains of fabric (a moderate effect) and force anisotropy (a faster phenomenon) as initially isotropic packs are oedometrically loaded. Those figures also show the predictions of the Voigt ("effective medium theory") prediction, based on an assumption of homogeneous strains doan to the grain scale, for such ratios of moduli. While the approximation itself fails to correctly predict the values of the moduli [7, 10] its estimate of relative difference $C_{11}/C_{22} - 1$ is correct to within 20% for small levels of anisotropy (as $C_{11}/C_{22} < 1.2$). The variety of numerical results on elastic moduli for the different assembling procedures compares well [10] to that of published laboratory results on spherical bead assemblies [8, 9].

5 Unloading and irreversibilities

A compression cycle in which σ_1, after its maximum, is reduced back to its initial value, results in the variations of void index e and coordination number z shown in Fig. 6 The amount of irreversible compression (decrease pf e) after one cycle is remarkably small, compared to typical experimental results on sands [11], a difference we attribute to the absence of plasticity or damage at the contact leve in the numerical model. The irreversibility is quite conspicuous, however, as regards z, which, as observed with isotropic bead packs [12], decreases to low values after one cycle in initially highly coordinated systems (this effect exists, to a smaller extent, in smaller stress cycles).

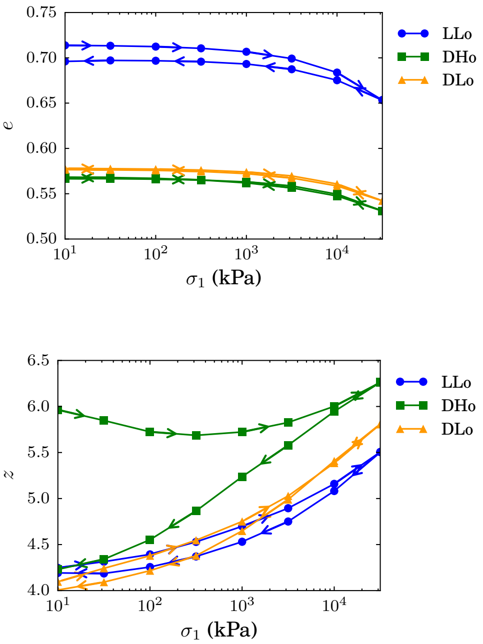

Figure 6. Void ratio e (top) and coordination number z (bottom) versus σ_1 in oedometric compression cycle.

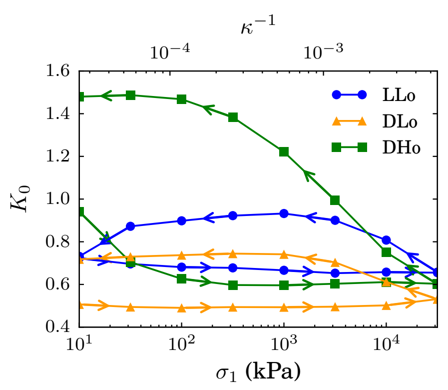

Figure 7. K_0 versus σ_1 in oedometric compression cycle.

Stress ratio K_0 also exhibits strongly irreversible behavior, with values larger than 1 obtained after a cycle (Fig. 7), as transverse stress σ_2 decreases slower than controlled, axial stress σ_1 along the unloading path.

6 Conclusions

This communication reports on some salient aspects of a systematic numerical study of oedometric compression of spherical grain assemblies [10], which investigates a wide variety of different initial states, subjected to a large increase of axial stress, and substantially extends the domains explored in the previous DEM literature [5, 6].

The coefficient of earth pressure, K_0, can be regarded as constant in systems prepared under similar initial oedometric load as the studied test. Otherwise, it appears to vary, reflecting a growing load-induced anisotropy. K_0 relates to both fabric and force anisotropy in a simple way. Despite the smallness of irreversible density changes, oedometric compression is a complex process involving friction-induced irreversible internal evolutions in the contact network, with possible decreases of initially large coordination numbers and hysteretic stress states. Elastic properties reflect material anisotropies and, given encouraging comparisons with experimental results, could provide access to fabric variables.

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