Effects of diffraction in pp and pA collisions

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Abstract. Diffractive excitation is a large fraction of the pp cross section, also at high energies. Diffraction has been described by multi-Regge diagrams, or in the Good-Walker formalism as a result of fluctuations. The two formalisms are, however, just different sides of the same phenomenon. The dipole cascade formalism in impact parameter space is well suited to describe diffractive excitation including effects of saturation. Diffractive excitation is also an important effect in pA scattering, where the Glauber formalism has been used to estimate the number of NN subcollisions and of "wounded" nucleons. Diffractive excitation has here been either neglected or included in a simplified way, not including excitation of target nucleons. In this talk we discuss how these effects can be included with the help of the dipole cascade model DIPSY.

1 Introduction. The role of pertubation theory and unitarity constraints

Hera has shown that the parton density at small x grows rapidly $\sim 1/x^{1.3}$, as predicted by the perturbative BFKL pomeron. For pp collisions this implies a very large probability for gluon-gluon subcollisions, which implies that unitarity constraints are very important. These constraints lead to saturation of the gluon density, and suppression of partons with $k_\perp < Q_s^2$. This may explain why models based on multiple perturbative partonic subcollisions (like Pythia [1, 2]) are very successful at high energies. One can then ask: if perturbative physics dominates, is it then possible to calculate the result from basic principles, without input pdf's?

Unitarity constraints and saturation are most easily described in impact parameter space. Rescattering is represented by a convolution in $\mathbf{k}_\perp$-space, which simplifies to a product in $\mathbf{b}$-space. The small size of $\mathrm{Re}\,A_{el}^{pp}$ indicates that the pp interaction is driven by absorption. If the absorption probability in Born approximation equals $2F(b)$, the optical theorem and the eikonal approximation gives the result: $d\sigma_{el}/d^2b = T^2 = (1-e^{-F})^2$, $d\sigma_{tot}/d^2b = 2T = 2(1-e^{-F})$.

2 Dipole cascade evolution

Mueller's dipole cascade model [3, 4] is a formulation of LL BFKL evolution in impact parameter space. A colour charge is always screened by an accompanying anticharge, forming a colour dipole. The dipole emits bremsstrahlung gluons coherently, which splits the dipole in two dipoles. The new

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dipoles split repeatedly, developing into a cascade. When two cascades collide, s-channel unitarity is restored in the eikonal approximation.

The *Lund dipole cascade model, DIPSY* [5–7], is a generalization of Mueller's model. Thus it is based on BFKL evolution, but includes a set of important corrections:

1) Important non-leading effects in BFKL evolution. (The most essential are related to energy conservation and running α_s.)

2) Saturation from pomeron loops in the evolution. (Not included by Mueller or in the BK equation.)

3) Confinement, which also implies t-channel unitarity.

4) It is implemented in the DIPSY MC it gives also fluctuations and correlations.

5) It can be applied to collisions between electrons, protons, and nuclei.

3 Fluctuations and diffractive excitation

3.1 Good–Walker formalism

A projectile with a substructure may be diffractively excited to a different mass eigenstate. This can be described in the Good–Walker formalism as the result of fluctuations [8]. Assume that the projectile is a linear combination of diffractive eigenstates, Φ_n, with definite eigenvalues T_n. The elastic amplitude is then given by $\langle\Psi_{in}|T(b)|\Psi_{in}\rangle$, where the average is taken over both projectile and target states. The differential total and elastic cross sections are then given by

$$d\sigma_{tot}/d^2b = 2\langle T\rangle, \quad d\sigma_{el}/d^2b = \langle T\rangle^2. \tag{1}$$

The total diffractive cross section, including elastic scattering, is given by $\langle T^2\rangle$. Diffractive excitation is obtained subtracting the elastic, and thus given by the fluctuations $\langle T^2\rangle - \langle T\rangle^2$. For a fluctuating projectile scattering against a fluctuating target, the single and dubble excitations are obtained by separate averages over projectile and target states, below denoted by subscripts p and t respectively. Thus the cross sections for single excitation of the projectile and of the target, and for double excitation are given by:

$$\begin{aligned} d\sigma_{SD,p}/d^2b &= \langle\langle T\rangle_t^2\rangle_p - \langle T\rangle_{p,t}^2; & (2)\\ d\sigma_{SD,t}/d^2b &= \langle\langle T\rangle_p^2\rangle_t - \langle T\rangle_{p,t}^2; & (3)\\ d\sigma_{DD}/d^2b &= \langle T^2\rangle_{p,t} - \langle\langle T\rangle_t^2\rangle_p - \langle\langle T\rangle_p^2\rangle_t + \langle T\rangle_{p,t}^2. & (4)\end{aligned}$$

3.2 Diffractive eigenstates

The parton cascades discussed in sec. 2 can come on shell through interaction with a target. The BFKL evolution has a stochastic nature with large fluctuations, and following Hatta *et.al* [9] we assume that these cascades represent the diffractive eigenstates in high energy collisions. (A similar approach was presented by Miettinen and Pumplin [10].) In ref. [11] it is also shown that the fluctuations in the BFKL dynamics imply that this Good–Walker formalism actually agrees with the more commonly used triple-pomeron analysis. The Good–Walker formalism has, however, the advantage that it is more easy to account for saturation effects.

Fig. 1 shows a comparison between DIPSY and CDF results for single diffraction *vs* max M_X^2 [12]. It is also possible to calculate diffractive final states, and fig. 1 shows the result for $dn_{ch}/d\eta$ in $p\overline{p}$ collisions [13]. Note in particular that the MC is here only tuned to $\sigma_{\rm tot}$ and $\sigma_{\rm el}$, with no new parameter. (The MC result is too large for high η due to the lack of valence quarks in the model.)

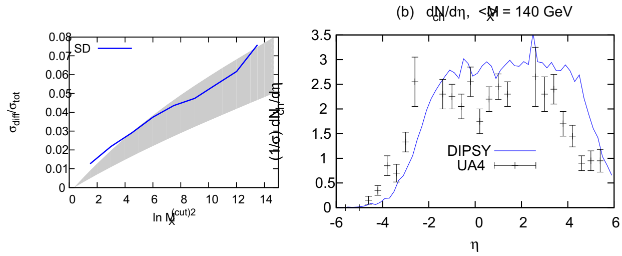

Figure 1. *Left*: $\int dM_X^2\, d\sigma_{SD}/dM_X^2$ for $M_X < M_X^{(cut)}$. The shaded area is an estimate of CDF results. *Right*: $dn_{ch}/d\eta$ in $\mathrm{p}\bar{\mathrm{p}}$ collisions at 546 GeV and $\langle M_X \rangle$= 140 GeV. Data from UA4 [14].

4 Collisions with nuclei

In collisions with nuclei, understanding the initial state is essential for the interpretation of collective final state effects. Studies of pA collisions can then be valuable to test models for the initial state in AA. DIPSY here gives a full partonic picture for the initial state in collisions with nuclei, in form of a dense gluon soup, and the model accounts for saturation within the cascades, finite size effects, and for correlations and fluctuations in the partonic state.

4.1 The Glauber model and Gribov corrections

The Glauber model is frequently used in analyses of experimental data, *e.g.* for estimating centrality or the number of "wounded" nucleons and binary NN collisions. In **b**-space rescattering is given by a product of amplitudes. For a projectile proton at impact parameter **b**, hitting a nucleus with nucleon positions $\mathbf{b}_\nu$ ($\nu = 1, \ldots, A$), the amplitude $T = 1 - S$ will then be determined by the relation

$$S^{(pA)}(\mathbf{b}) = \prod_{\nu=1}^{A} S^{(pp,\nu)}(\mathbf{b} - \mathbf{b}_\nu). \tag{5}$$

Gribov pointed out that diffractive excitation gives significant corretions to Glauber's formulae [15]. In pA collisions the projectile is frozen in the same state, k, during the passage through the nucleus, while the target nucleons are in different, uncorrelated states l_ν. In the Good–Walker formalism the elastic pA scattering amplitude is then given by the average

$$\langle\, T^{(pA)}(\mathbf{b}) \,\rangle = 1 - \langle\, \langle\, \prod_{\nu} \langle\, \{1 - T_{k,l_\nu}^{(pp,\nu)}(\mathbf{b} - \mathbf{b}_\nu)\}\, \rangle_{l_\nu} \rangle_{\mathbf{b}_\nu} \rangle_k, \tag{6}$$

with $d\sigma_{tot}^{pA}/d^2b = 2\,\langle T^{(pA)}(\mathbf{b})\rangle$ and $d\sigma_{el}^{pA}/d^2b = \langle T^{(pA)}(\mathbf{b})\rangle^2$. Note that the target average is over the pp amplitude to first power, but the average over projectile states contains also higher powers of $T^{(pp)}$.

4.2 Specification of wounded nucleons

Białas and coworkers noted that the central particle density in pA collisions is approximately determined by the number of participating, or wounded, nucleons: $\frac{dN^{pA}}{d\eta} \approx \frac{1+N_w^t}{2}\frac{dN^{pp}}{d\eta}$ [16, 17].

In most applications all inelastically interacting nucleons are treated as wounded nucleons, also those who are diffractively excited. This may be OK for forward observables, where these nucleons contribute to particle production. If the excited mass has a distribution $\sigma_{SD}/dM_X^2 \sim dM_X^2/(M_X^2)^{1+\epsilon}$ excited nucleons contribute significantly to central particle production at LHC only if ϵ is very small. For the (weakly) favoured value $\epsilon \approx 0.1$ [18] their contribution is suppressed by a factor $\sim 1/2$, and for larger ϵ-values even more. We thus conclude that depending on the observable under consideration, wounded nucleons ought to correspond either to non-diffractive, "absorbed", nucleons or to inelastic nucleons including diffraction ("inclusively wounded" nucleons).

i) Wounded nucleons = absorbed nucleons (inel. non diffr.)

The absorption probability is given by $d\sigma_{abs}/d^2b = 1 - S^2$. As the S-matrix factorizes, this is also the case for S^2, and the absorptive cross section is given by

$$d\sigma^{pA}_{abs}/d^2b = \langle 1 - \prod_\nu (S^{(pp,\nu)})^2 \rangle = 1 - \langle \prod_\nu \langle \{1 - T^{(pp,\nu)}(\mathbf{b} - \mathbf{b}_\nu)\}^2 \rangle_{l_\nu} \rangle_k. \tag{7}$$

Note that this expression includes the target average of $T^{(pp)}$ squared, taken to different powers n before averaging over projectile states.

ii) Inclusively wounded nucleons

Here we get

$$d\sigma^{pA}_{winc}/d^2b = 1 - \langle \prod_\nu \{1 - \langle T^{(pp,\nu)}(\mathbf{b} - \mathbf{b}_\nu)\rangle^2_{l_\nu}\} \rangle_k. \tag{8}$$

Just like the total and elastic cross sections, this expression includes only the target average of $T(\mathbf{b})$ to first power.

4.3 Simple approximations

Many analyses of experimental data use simple approximations for the pp amplitude:

i) Black disc model: $T^{pp}(b) = \theta(R - b)$

Here the amplitude depends on a single parameter R, and it gives the result $\sigma^{pp}_{abs} = \sigma^{pp}_{el} = \sigma^{pp}_{tot}/2$. Thus diffraction is totally neglected. If R is chosen to reproduce σ^{pp}_{inel}, then σ^{pA}_{tot} will be overestimated, while if it is choosed to reproduce σ^{pp}_{tot}, σ^{pA}_{inel} will be correspondingly underestimated [19].

ii) Gray disc model: Projectile is absorbed with probability a, for $b < R$

This model has two parameters, which makes it possible to reproduce *e.g.* σ^{pp}_{tot} and σ^{pp}_{el}. It is, however, not possible to distinguish SD_{target}, SD_{proj}, and DD, and therefore it is not possible to take separate averages over projectile and target states, as in eqs. (6-8) (see also ref. [19]).

4.4 The model by Strikman and coworkers

This model, sometime called the Glauber–Gribov (GG) model [20, 21], accounts for a fluctuating projectile, but not for fluctuations in the target. If the fluctuating pp total cross section, averaged over target states, is denoted $\hat{\sigma}_{tot}$, *i.e.* $\hat{\sigma}_{tot} = 2\int d^2b \,\langle\, T^{(pp)}(b)\,\rangle_{targ}$, then the pp total cross section is given by the average over projectile states: $\sigma^{(pp)}_{tot} = \langle\hat{\sigma}_{tot}\rangle_{proj}$. The distribution in values for $\hat{\sigma}_{tot}$ is assumed to have the following form:

$$\frac{dP}{d\,\hat{\sigma}_{tot}} = \rho\,\frac{\hat{\sigma}_{tot}}{\hat{\sigma}_{tot} + \sigma^{tot}_0} \exp\left\{-\frac{(\hat{\sigma}_{tot}/\sigma^{tot}_0 - 1)^2}{\Omega^2}\right\}. \tag{9}$$

Here Ω is a parameter determining the fluctuations, related to $\sigma^{(pp)}_{SD}$, σ^{tot}_0 is fixed from $\sigma^{(pp)}_{tot}$, and ρ is a normalization constant.

i) Wounded nucleons = absorbed nucleons

Define the fluctuating absorptive pp cross section:

$$\hat{\sigma}_{abs} = \int d^2b\, \langle\, \{2T^{(pp)}(b) - T^{(pp)2}(b)\}\,\rangle_{targ}; \tag{10}$$

$$\sigma^{(pp)}_{abs} = \langle\hat{\sigma}_{abs}\rangle_{proj}. \tag{11}$$

The same functional form is used as for σ_{tot} in eq. (9), but the width parameter Ω need not be the same. Note that σ_0^{abs} ought to be adjusted to reproduce $\sigma^{pp}_{inel\,ND}$, but is often tuned to $\sigma^{pp}_{inel\,tot}$.

ii) Wounded nucleons include target excitations

Here $\hat{\sigma}_{abs}$ should be exchanged for $\hat{\sigma}_{wincl}$ obtained by replacing $\langle T^{(pp)2}(b)\rangle_{targ}$ by $\langle T^{(pp)}(b)\rangle^2_{targ}$ in eq. (10).

In fig. 2 we compare the result from DIPSY with the analytic form in eq. (9), with averages and widths tuned to the DIPSY cross sections. We note that the DIPSY results have a more asymmetric form with a longer tail to large cross sections, which can be well fitted by a log-normal distribution.

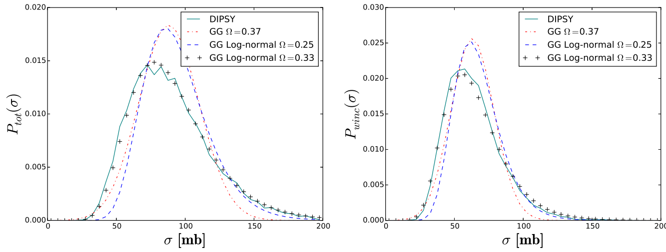

Figure 2. Prob. distrib. for the total (left) and the wounded incl. (right) cross sections. The solid line shows the DIPSY result, and the dotted line the GG form with the same average and width. Plus signs and dashed line are log-normal fits.

4.5 Final states

The old Fritiof model [22] was inspired by the wounded nucleon idea, and it worked quite well in the fixed target energy range. We have now constructed an updated version, called FritiofP8, in form of a toy model incorporating diffractive excitation using Pythia8. If different branches of a proton cascade interact via gluon exchange with two target nucleons, the result will be one dipole chain stretching in rapidity from the proton up to a point where it branches off to two chains, continuing to the two target nucleons. The result will be the same as one chain stretched over the full rapidity range, plus one chain stretched between the second target nucleon and the branching point, which can be placed randomly in rapidity. The result of the secondary chain is then similar to the contribution from a diffractively excited nucleon, as generated by the Pythia8 MC. This picture can be directly generalized to a case, where the proton interacts absorptively with several target nucleons. If diffractive target nucleons are produced with a mass distribution $d\sigma/dM_X^2 \sim 1/M_X^2$, which is the default in Pythia8, then the contribution from secondary absorbed and diffractively excited target nucleons will be very similar.

The FritiofP8 model is implemented in a MC [23], simulating final states obtained from the initial states given by DIPSY. Results for $\sum E_\perp$ in the forward (nucleus) direction is shown in fig. 3 *left*, which do agree very well with results from Atlas [24]. For illustration we also show result from a model called "Absorptive", which assumes that all wounded target nucleons contribute like a pp collision at full energy. This model is similar to the "G-Pythia" model used by Alice. We note that for large $\sum E_\perp$ this model is almost a factor 10 above FritiofP8 and the data. Fig. 3 *right* shows FritiofP8 results for the η-distribution in central pPb collisions, compared with the absorptive model and Atlas data [24]. We see that also here the FritiofP8 model agrees quite well with the data.

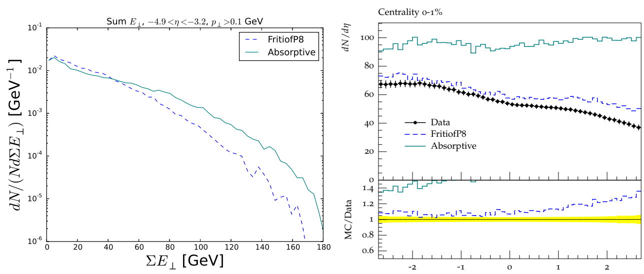

Figure 3. *Left*: $\sum E_\perp$ in forward direction from FritiofP8 (dashed) compared with the "Absorptive" model (solid). *Right*: η-distribution in central collisions from FritiofP8 (dashed), compared with the "Absorptive" model (solid) and with data from Atlas [24].

5 Conclusions

Saturation suppresses low-$p_{\perp}$ gluons, which implies that high energy hadronic collisions are dominantly perturbative.

The DIPSY dipole cascade model is based on BFKL dynamics with non-leading corrections and saturation. Implemented in a MC it includes correlations and fluctuations. It gives a fair description of DIS and pp data, with no input pdf's, and can give the initial condition in p*A* and *AA* collisions.

The Glauber model is frequently used in experimental analyses. Gribov pointed out the importance of diffractive scattering, but this is frequently not taken into account, or treated in an improper way.

A significant fraction of the interacting target nucleons are diffractively excited, and their contribution depends on the observable under study. For observables which obtain small contributions from diffractively excited target nucleons, the distribution in the GG model should be normalized to $\sigma_{inel.ND} \approx 2/3\,\sigma_{inel}$.

A simple model based on Fritiof, called FritiofP8, which combines DIPSY and PYTHIA8, works rather well for min bias final states in p*A* collisions.

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