

Photons from the Color Glass Condensate in p+A collisions

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Abstract. We report on a first NLO computation of photon production in p+A collisions at collider energies within the Color Glass Condensate framework, significantly extending previous LO results. At central rapidities, our result is the dominant contribution and probes multi-gluon correlators in nuclei. At high photon momenta, the result is directly sensitive to the nuclear gluon distribution. The NLO result contains two classes of processes, the annihilation process and the process with $q\bar{q}$ pair together with a photon in the final state. Using the McLerran-Venugopalan model, we show full numerical results for the photon spectrum coming from the annihilation process.

1 Introduction

Nuclei at high energies are governed by the dynamics of saturated gluons described within the Color Glass Condensate (CGC) effective theory [1–3]. The CGC introduces a new dynamical scale, called the saturation scale Q_s , characterizing gluon correlations in nuclei with $k_\perp \lesssim Q_s$. The CGC description should well describe the physics already at the energy reach of the RHIC and the LHC, where typically $x \sim 10^{-2}$ and $x \sim 10^{-3}$ at central rapidity is expected, respectively. Anticipating new experimental results on photon spectrum in p+A collisions from RHIC and LHC we aim at a detailed theoretical description that would help constrain fundamental multi-gluon correlators as predicted by the CGC.

The LO contribution for photon production is valence quark bremsstrahlung (see leftmost diagram in Fig. 1). The CGC result for the $O(\alpha_e)$ cross section was calculated by Gelis and Jalilian-Marian in [4] through which the photon spectrum becomes sensitive to the two-point gluon correlator of the nucleus. This is a good description for central rapidities at low energies or, alternatively, in forward kinematics, where the valence quarks dominate the proton wave function. When the center of mass energy is increased, the gluon component of the proton contributes significantly to photon production in the central rapidity region.

Working within the dilute-dense approximation, appropriate for the p+A kinematics, we investigate NLO processes with a gluon in place of the valence quark being emitted from the proton. In Fig. 1 (middle and right) the gluon emits a quark-antiquark pair. The pair either annihilates back to a photon (class II process) or goes to the final state (class III process). The class II process was calculated in [5], while the class III process was calculated in [6]. The inclusive rate in both cases is of the order $O(\alpha\alpha_s)$. To compare with valence quark bremsstrahlung we must multiply with the respective

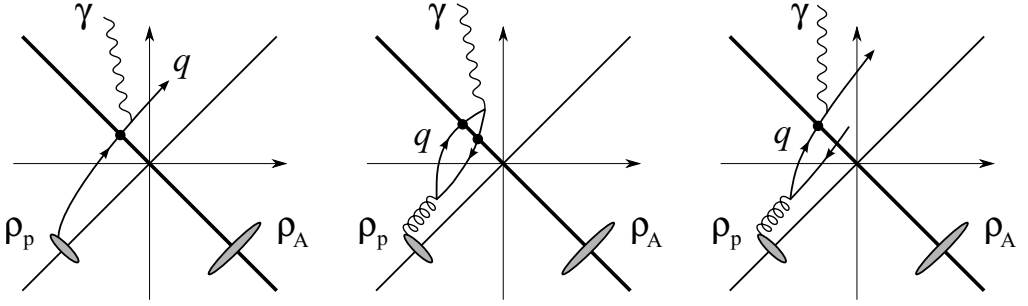


Figure 1. Leftmost diagram belongs to the leading order contribution giving the rate of $O(\alpha)$. The middle (class II) and the rightmost (class III) processes are typical examples of next-to-leading order contributions resulting in rates of $O(\alpha\alpha_s)$.

distribution functions for quarks f_q and for gluons f_g inside the proton. We consider the high energy regime where the new processes become dominant as $f_q \ll \alpha_s f_g$.

2 Calculation of the diagrams

We solve the classical Yang-Mills equations with the dilute-dense approximation for the color sources of the proton ρ_p and of the nucleus ρ_A : $\rho_p \ll \rho_A$. This allows for a systematic computation of the gluon field in powers of ρ_p [7, 8]. We denote the resulting $O(\rho_p^{0,1})$ gluon field as $\mathcal{A}_{(0,1)}^\mu(x)$. In the $\mathcal{A}^+ = 0$ gauge the field of the nucleus alone, $\mathcal{A}_{(0)}^\mu$, is

$$\mathcal{A}_{(0)}^\mu(x) = -g\delta^{\mu-}\delta(x^+)\frac{1}{\partial_\perp^2}\rho_A(\mathbf{x}_\perp). \quad (1)$$

The field $\mathcal{A}_{(1)}^\mu$ is given in Fourier space as [8, 9]

$$\mathcal{A}_{(1)}^\mu(q) = \frac{ig\delta^{\mu i}q_\perp^i}{(q^+ + i\epsilon)(q^- - i\epsilon)}\frac{\rho_p(\mathbf{q}_\perp)}{q_\perp^2} + \frac{ig}{q^2 + iq^+\epsilon} \int_{\mathbf{k}_{1\perp}} \int_{\mathbf{x}_\perp} e^{i(\mathbf{q}_\perp - \mathbf{k}_{1\perp})\cdot\mathbf{x}_\perp} C^\mu(q, \mathbf{k}_{1\perp}) U(\mathbf{x}_\perp) \frac{\rho_p(\mathbf{k}_{1\perp})}{k_{1\perp}^2}, \quad (2)$$

where

$$C^+(q, \mathbf{k}_{1\perp}) = 0, \quad C^-(q, \mathbf{k}_{1\perp}) = \frac{-2\mathbf{k}_{1\perp} \cdot (\mathbf{q}_\perp - \mathbf{k}_{1\perp})}{q^+ + i\epsilon}, \quad C_\perp(q; \mathbf{k}_{1\perp}) = \frac{\mathbf{q}_\perp \cdot \mathbf{k}_{1\perp}^2}{(q^+ + i\epsilon)(q^- + i\epsilon)} - 2\mathbf{k}_{1\perp}, \quad (3)$$

and $U(\mathbf{x}_\perp)$ is the Wilson line with $\mathcal{A}_{(0)}^\mu$ in the adjoint representation

$$U(\mathbf{x}_\perp) = \mathcal{P}_+ \exp \left[ig \int_{-\infty}^{\infty} dz^+ \mathcal{A}_{(0)}^-(z^+, \mathbf{x}_\perp) \cdot T \right]. \quad (4)$$

The quark propagator $S_{(0)}(x, y)$ is given by the Dirac equation in the $\mathcal{A}_{(0)}^\mu$ background [10] as

$$S_{(0)}(x, y) \equiv S_F(x - y) + i\theta(x^+)\theta(-y^+) \int_z \delta(z^+) [\tilde{U}(z_\perp) - 1] S_F(x - z) \gamma^+ S_F(z - y) - i\theta(-x^+)\theta(y^+) \int_z \delta(z^+) [\tilde{U}^\dagger(z_\perp) - 1] S_F(x - z) \gamma^+ S_F(z - y), \quad (5)$$

where $\tilde{U}(\mathbf{x}_\perp)$ is the Wilson line in the fundamental representation, that is, with the fundamental generators t^a , replacing the adjoint ones T^a in Eq. (4).

The LSZ formula for the class II process with a single flavor of charge e in the quark loop is

$$\mathcal{M}_\lambda(\mathbf{k}_\gamma) = eg \int_{xy} e^{-ik_\gamma \cdot x} \text{Tr}[\not{\epsilon}_\lambda(\mathbf{k}_\gamma) S_{(0)}(x, y) \mathcal{A}_{(1)}(y) S_{(0)}(y, x)], \quad (6)$$

where \mathbf{k}_γ is the photon momentum. We have checked that (6) satisfies the photon Ward identity. To calculate the rate we square the amplitude and perform a color average over the sources. The result can be summarized as [5]

$$\begin{aligned} \frac{dN}{d^2\mathbf{k}_{\gamma\perp} d\eta_{k_\gamma}} &= \frac{\alpha_e \alpha_s}{16\pi^8} \frac{N_c}{N_c^2 - 1} \int_0^1 dx dx' \int_{\mathbf{y}_\perp \mathbf{y}'_\perp \mathbf{z}_\perp \mathbf{z}'_\perp} e^{i\mathbf{k}_{\gamma\perp} \cdot \mathbf{r}_\perp} S(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \\ &\times \int_{\mathbf{k}_{1\perp}} e^{i\mathbf{k}_{1\perp} \cdot \mathbf{r}_\perp} \varphi_p(\mathbf{k}_{1\perp}) [(\hat{\mathbf{z}}_\perp - \hat{\mathbf{y}}_\perp) \cdot (\hat{\mathbf{z}}'_\perp - \hat{\mathbf{y}}'_\perp) \Psi_1 \Psi_1^* + \Psi_2 \Psi_2^* + 2(\hat{\mathbf{z}}_\perp - \hat{\mathbf{y}}_\perp) \cdot \hat{\mathbf{k}}_{1\perp} \Psi_1 \Psi_2^*]. \end{aligned} \quad (7)$$

where

$$S(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv \frac{1}{N_c} \langle \text{Tr}_c [\tilde{U}(\mathbf{y}_\perp) T_F^a \tilde{U}^\dagger(\mathbf{z}_\perp)] \text{Tr}_c [\tilde{U}(\mathbf{z}'_\perp) T_F^a \tilde{U}^\dagger(\mathbf{y}'_\perp)] \rangle, \quad (8)$$

$\varphi_p(\mathbf{k}_{1\perp})$ is the unintegrated gluon distribution in the proton, $\mathbf{r}_\perp = (1-x)\mathbf{y}_\perp + x\mathbf{z}_\perp - (1-x')\mathbf{y}'_\perp - x'\mathbf{z}'_\perp$. The quantities $\Psi_{1,2}$ and $\Psi'_{1,2}$ are functions of all the integration variables, and detailed expression are to be found in [5].

We have expanded the Wilson lines to recover the perturbative calculation. We have found that the $O(\rho_p \rho_A)$ contribution vanishes as it should by charge conjugation. The lowest order non-vanishing contribution is $O(\rho_p \rho_A^2)$. We have explicitly checked that this is UV finite as it should by gauge invariance. Taking the zero quark mass limit is completely safe in this amplitude as there are no collinear singularities. Finally, we have checked that the term in the last line of Eq. (7) has a well defined $\mathbf{k}_{1\perp} \rightarrow 0$ limit so that our result can be brought in a collinear factorized form on the proton side.

Turning to the class III process, in addition to the photon, the final state contains a quark with momentum \mathbf{q} and an antiquark with momentum \mathbf{p} . The LSZ formula for the amplitude is

$$\begin{aligned} \mathcal{M}_\lambda(\mathbf{k}_\gamma, \mathbf{q}, \mathbf{p}) &= ieg \int_{xyz} e^{-ik_\gamma \cdot x - iq \cdot y - ip \cdot z} \bar{u}(\mathbf{q})(i\vec{\not{\partial}}_y - m) [S_{(0)}(y, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, x) \not{\epsilon}_\lambda(\mathbf{k}_\gamma) S_{(0)}(x, z) \\ &+ S_{(0)}(y, x) \not{\epsilon}_\lambda(\mathbf{k}_\gamma) S_{(0)}(x, w) \mathcal{A}_{(1)}(w) S_{(0)}(w, z)] (i\vec{\not{\partial}}_z + m) v(\mathbf{p}). \end{aligned} \quad (9)$$

Inserting the quark propagator $S_{(0)}(x, y)$ and the gluon field $\mathcal{A}_{(1)}^\mu(x)$ we find [5]

$$\begin{aligned} \mathcal{M}_\lambda(\mathbf{k}_\gamma, \mathbf{q}, \mathbf{p}) &= -eg^2 \int_{\mathbf{k}_\perp \mathbf{k}_{1\perp}} \int_{\mathbf{x}_\perp \mathbf{y}_\perp} \frac{\rho_p^a(\mathbf{k}_{1\perp})}{\mathbf{k}_{1\perp}^2} e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp + i(\mathbf{k}_{\gamma\perp} + \mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k}_\perp - \mathbf{k}_{1\perp}) \cdot \mathbf{y}_\perp} \\ &\times \bar{u}(\mathbf{q}) \epsilon_{\lambda\mu}(\mathbf{k}_\gamma) \left\{ T_g^\mu(\mathbf{k}_{1\perp}) U(\mathbf{x}_\perp)^{ba} t^b + T_{q\bar{q}}^\mu(\mathbf{k}_\perp, \mathbf{k}_{1\perp}) \tilde{U}(\mathbf{x}_\perp) t^a \tilde{U}^\dagger(\mathbf{y}_\perp) \right\} v(\mathbf{p}). \end{aligned} \quad (10)$$

where the explicit expressions for the functions $T_g^\mu(\mathbf{k}_{1\perp})$ and $T_{q\bar{q}}^\mu(\mathbf{k}_\perp, \mathbf{k}_{1\perp})$ can be found in [5]. We have checked that (10) satisfies the photon Ward identity and also that the leading twist result satisfies the gluon Ward identities. In general, the result for the amplitude is gauge dependent. We have made the calculation in the $\mathcal{A}^+ = 0$ and also in the $\partial_\mu \mathcal{A}^\mu = 0$ gauge [6, 11]. Interestingly, after some algebraic manipulation, the two results for the amplitude, within this particular gauge choices, become identical.

The total cross section for $q\bar{q}\gamma\gamma$ production is found as [6]

$$\begin{aligned} \frac{d\sigma}{d^2\mathbf{k}_{\gamma\perp}d\eta_{k_\gamma}d^2\mathbf{q}_\perp d\eta_q d^2\mathbf{p}_\perp d\eta_p} &= \frac{\alpha_e\alpha_s^2}{256\pi^8 C_F} \int_{\mathbf{k}_{1\perp}\mathbf{k}_{2\perp}} (2\pi)^2 \delta^{(2)}(\mathbf{k}_{\gamma\perp} + \mathbf{q}_\perp + \mathbf{p}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) \frac{\varphi_p(\mathbf{k}_{1\perp})}{k_{1\perp}^2 k_{2\perp}^2} \\ &\times \left\{ \int_{\mathbf{k}_\perp\mathbf{k}'_\perp} \text{Tr}[(\not{q} + m)T_{q\bar{q}}^\mu(\mathbf{k}_{1\perp}, \mathbf{k}_\perp)(m - \not{p})\gamma^0 T_{q\bar{q}\mu}^\dagger(\mathbf{k}_{1\perp}, \mathbf{k}'_\perp)\gamma^0] \phi_A^{q\bar{q},q\bar{q}}(\mathbf{k}_\perp, \mathbf{k}_{2\perp} - \mathbf{k}_\perp; \mathbf{k}'_\perp, \mathbf{k}_{2\perp} - \mathbf{k}'_\perp) \right. \\ &+ \int_{\mathbf{k}_\perp} \text{Tr}[(\not{q} + m)T_{q\bar{q}}^\mu(\mathbf{k}_{1\perp}, \mathbf{k}_\perp)(m - \not{p})\gamma^0 T_{g\mu}^\dagger(\mathbf{k}_{1\perp})\gamma^0] \phi_A^{q\bar{q},g}(\mathbf{k}_\perp, \mathbf{k}_{2\perp} - \mathbf{k}_\perp; \mathbf{k}_{2\perp}) + \text{h. c.} \\ &\left. + \text{Tr}[(\not{q} + m)T_g^\mu(\mathbf{k}_{1\perp})(m - \not{p})\gamma^0 T_{g\mu}^\dagger(\mathbf{k}_{1\perp})\gamma^0] \phi_A^{g,g}(\mathbf{k}_{1\perp}) \right\}. \end{aligned} \quad (11)$$

The cross section depends on three different generalized unintegrated gluon distribution functions of the nucleus $\phi_A^{q\bar{q},q\bar{q}}$, $\phi_A^{q\bar{q},g}$ and $\phi_A^{g,g}$. They correspond to different Wilson line correlators in the fundamental and in the adjoint representation identical to the ones found for $q\bar{q}$ production [12]. The inclusive photon production is found by integrating over the quark and antiquark phase space.

Expanding the Wilson lines we find a contribution at $O(\rho_p\rho_A)$ (leading twist). This is the k_\perp -factorized photon production as it would be obtained by a pQCD computation. Taking the collinear limit on the proton ($\mathbf{k}_{1\perp} \rightarrow 0$) and on the nucleus ($\mathbf{k}_{2\perp} \rightarrow 0$) side, the result can be expressed in terms of gluon distribution functions of the proton and the nucleus and the inclusive cross section is given as

$$\frac{d\sigma}{d^2\mathbf{k}_{\gamma\perp}d\eta_{k_\gamma}} = \frac{1}{16} \int_0^\infty \frac{dq^+}{q^+} \frac{dp^+}{p^+} \int_{\mathbf{q}_\perp\mathbf{p}_\perp} (2\pi)^2 \delta^{(2)}(\mathbf{k}_{\gamma\perp} + \mathbf{q}_\perp + \mathbf{p}_\perp) x_p f_{g,p}(x_p, Q^2) x_A f_{g,A}(x_A, Q^2) |\mathcal{M}_{gg \rightarrow q\bar{q}\gamma}|^2. \quad (12)$$

From this result we get a clear relation between the photon spectrum and the gluon distribution in the nuclei.

In the limit of $\mathbf{k}_{\gamma\perp} \rightarrow 0$ the amplitude $\mathcal{M}_{gg \rightarrow q\bar{q}\gamma}$ factorizes in accordance to the soft photon theorem to a radiative part and the remaining, non-radiative part, responsible for $q\bar{q}$ production. The radiative part gives an infrared enhancement to the inclusive cross section as $1/k_{\gamma\perp}^2$.

3 Numerical results

Here we discuss the numerical computation of the class II process. The general analytical expression (7) is valid for a single massive quark flavor. For simplicity we will take the limit of zero quark mass.

The respective Wilson line correlator occurring in the class II process is referred to in the literature as the inelastic quadrupole [13]. Interestingly, within the large N_c approximation, the inelastic quadrupole does not factorize. This means that in principle one must handle a simultaneous integration over all the coordinates of the quadrupole.

For the numerical calculations we assume the case of a large nucleus, that is $A \gg 1$, in which case approximate translational invariance in the transverse plane holds. This turns one transverse plane integration into a volume factor πR_A^2 , where R_A is the nuclear radii. For the calculation of the inelastic quadrupole we use the McLerran Venugopalan model $\langle \rho_A^a(\mathbf{x}_\perp) \rho_A^b(\mathbf{y}_\perp) \rangle \equiv g^2 \delta^{ab} \mu_A^2 \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$ [14–16]. The saturation scale in the MV model is defined as

$$Q_s^2 \equiv \frac{N_c^2 - 1}{4N_c} g^4 \mu_A^2. \quad (13)$$

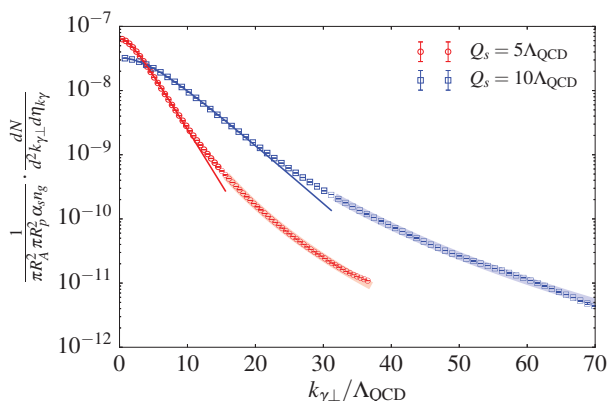


Figure 2. Photon spectrum from the class II diagram. The results are shown for a single quark flavor and in the chiral limit.

Finally, we determine $\varphi_p(\mathbf{k}_{1\perp})$ from the MV model for the proton as $k_{1\perp}^2 \varphi_p(\mathbf{k}_{1\perp}) = \pi(N_c^2 - 1)g^4 \mu_p^2 \pi R_p^2$.

The numerical integration is optimized through a combination of the MISER Monte Carlo algorithm and a Quasi Discrete Hankel Transform algorithm, for details see [6]. The computation is performed for two different values of the saturation scale $Q_s = 5\Lambda_{\text{QCD}}$ and $Q_s = 10\Lambda_{\text{QCD}}$. The numerical results for the transverse momentum spectrum of the photon is shown on Fig. 2, where we factorize the transverse proton density fluctuation parameter as $n_g \equiv (N_c^2 - 1)g^4 \mu_p^2 / (4N_c)$.

The parametrization of the soft part of the spectrum (up to $k_{\gamma\perp} \sim 2Q_s$) is possible by an exponential as $\exp\left(-\sqrt{k_{\gamma\perp}^2 + (0.5Q_s)^2}/0.5Q_s\right)$, while semi-hard part with $k_{\gamma\perp} \gtrsim 2Q_s$ is well fitted by a power law tail $(\log(k_{\gamma\perp}/Q_s))^{1.5}/k_{\gamma\perp}^{5.6}$. The thin lines on Fig. 2 are the exponential fit, while the thick lines are the power law fit.

4 Conclusions

We have performed a detailed analytical calculation of NLO photon rates in p+A collisions. The NLO photon rates probe multi-gluon correlations in the nuclei and as such substantiate and complement related investigations using hadron production [12, 17–21]. Our results will allow a direct study of the nuclear gluon distribution function, as well as the higher-twist corrections thereof, in a systematic way. Furthermore, they may help resolve an experimental puzzle [22] concerning the excess of soft photons in comparison to the predictions by the Low-Burnett-Kroll soft photon theorem. Relative to hadrons, photons are free from hadronization uncertainties and dedicated photon measurements would go a long way towards exploring the physics of gluon saturation.

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References

- [1] E. Iancu, R. Venugopalan, *The Color glass condensate and high-energy scattering in QCD*, in *In *Hwa, R.C. (ed.) et al.: Quark gluon plasma** 249-3363 (2003), hep-ph/0303204
- [2] J. Jalilian-Marian, Y.V. Kovchegov, Prog. Part. Nucl. Phys. **56**, 104 (2006), hep-ph/0505052
- [3] F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, Ann. Rev. Nucl. Part. Sci. **60**, 463 (2010), 1002.0333
- [4] F. Gelis, J. Jalilian-Marian, Phys. Rev. **D66**, 014021 (2002), hep-ph/0205037
- [5] S. Benic, K. Fukushima (2016), 1602.01989
- [6] S. Benic, K. Fukushima, O. Garcia-Montero, R. Venugopalan (2016), 1609.09424
- [7] J.P. Blaizot, F. Gelis, R. Venugopalan, Nucl. Phys. **A743**, 13 (2004), hep-ph/0402256
- [8] K. Fukushima, Y. Hidaka, Nucl. Phys. **A813**, 171 (2008), 0806.2143
- [9] F. Gelis, Y. Mehtar-Tani, Phys. Rev. **D73**, 034019 (2006), hep-ph/0512079
- [10] L.D. McLerran, R. Venugopalan, Phys. Rev. **D59**, 094002 (1999), hep-ph/9809427
- [11] O. Garcia-Montero, *Prompt photon production in pA collisions as a probe of many-body gluodynamics at high energies*, Master's Thesis, Ruprecht-Karls-Universität Heidelberg (2016)
- [12] J.P. Blaizot, F. Gelis, R. Venugopalan, Nucl. Phys. **A743**, 57 (2004), hep-ph/0402257
- [13] F. Dominguez, C. Marquet, B.W. Xiao, F. Yuan, Phys. Rev. **D83**, 105005 (2011), 1101.0715
- [14] L.D. McLerran, R. Venugopalan, Phys. Rev. **D49**, 2233 (1994), hep-ph/9309289
- [15] L.D. McLerran, R. Venugopalan, Phys. Rev. **D49**, 3352 (1994), hep-ph/9311205
- [16] L.D. McLerran, R. Venugopalan, Phys. Rev. **D50**, 2225 (1994), hep-ph/9402335
- [17] H. Fujii, F. Gelis, R. Venugopalan, Nucl. Phys. **A780**, 146 (2006), hep-ph/0603099
- [18] H. Fujii, K. Watanabe, Nucl. Phys. **A915**, 1 (2013), 1304.2221
- [19] Z.B. Kang, Y.Q. Ma, R. Venugopalan, JHEP **01**, 056 (2014), 1309.7337
- [20] Y.Q. Ma, R. Venugopalan, H.F. Zhang, Phys. Rev. **D92**, 071901 (2015), 1503.07772
- [21] Y.Q. Ma, R. Venugopalan, Phys. Rev. Lett. **113**, 192301 (2014), 1408.4075
- [22] A. Beligianni et al., Phys. Lett. **B548**, 129 (2002)