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Statistical Modelling of the Multiplicity Distribution

The Weighted GMD model at LHC energies

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Abstract. The multiplicity distribution is described in terms of models incorporating the stochastic branching of quarks and gluons, along with fluctuations in their production. Results show a reasonable description of charged-particle multiplicity distributions measured in pp collisions by the CMS collaboration at centre-of-mass collision energies of 0.9, 2.36, and 7 TeV for $|\eta| < 0.5$, 1.0, 1.5, 2.0, and 2.4. A prediction for 13 TeV is also given.

1 The Weighted GMD Model

Charged-particle multiplicities are one of the most inclusive variables that can be measured in high energy collisions. Their inclusiveness make them one of the most straightforward variables to define and measure but difficult to understand. However it is clear that the characteristics of various particle production processes drive its evolution across different centre-of-mass energies, and its distribution in phase space. A Poisson distributed (PD) multiplicity for example, indicates the independent emission of single particles, while particle cascades that develop from energetic collisions lead to a broader distribution like the Negative Binomial Distribution (NBD) [1].

Charged-particle multiplicity distributions can be described by the Generalised Multiplicity Distribution (GMD) [2–4] which is a general solution of the stochastic branching equation [5] and is reducible to the the PD, NBD, and Furry-Yule distribution [6]. Given m and k' initial number of quarks and gluons, the stochastic branching equation yields the GMD:

$$P_{GMD}(n; p, k, k') = \frac{\Gamma(n+k)}{\Gamma(n-k'+1)\,\Gamma(k'+k)} (1-p)^{n-k'} (p)^{k'+k}, \tag{1}$$

where n is particle multiplicity, k and p are related the initial quark/gluon numbers and their branching probabilities [2–4]. The average multiplicity of final state hadrons is given by

$$\bar{n} = \frac{k'+k}{p} - k. \tag{2}$$

Within this framework [5], the QCD evolution of quark and gluon numbers during fragmentation are modeled as Markov chains with the following processes: quark bremsstrahlung ($q \to q + g$) and gluon fission ($g \to g + g$). This intuitive picture has been shown to be equivalent to an algorithm

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calculating the multi-parton distributions within a QCD jet in a leading logarithmic approximation [7].

The GMD however is modified when there are fluctuations in the initial gluon numbers, distributed in a way that is determined by the dynamics of the collision. In pp collisions, various dynamical causes like Parton Distribution Functions (PDFs), low momentum transfer collisions, or interactions in the non-perturbative QCD regime in general play a role. These are not well understood from first principles and a statistical approach may provide some insights.

The modification of the GMD due to initial gluon number fluctuations is in general described by the weighted GMD (WGMD) model [8]. Suppose each independent collision produces k' number of gluons distributed according to a distribution $P(k'; x_1, ..., x_r)$ with r parameters, then the WGMD is given by

$$P_{WGMD}(n; x_1, ..., x_r, p, k) = \sum_{k'=0}^{n} P(k'; x_1, ..., x_r) \times P_{GMD}(n; p, k, k'). \tag{3}$$

The observed final state multiplicity distribution is thus a weighted sum of the GMD over the probabilities of the initial states.

The WGMD describes an ensemble of events with varying gluon numbers described by the weight factor. This model opens up possibilities to probe the average gluon number as a function of the collision centre-of-mass energy and pseudorapidity acceptance. Given a different weight distribution (e.g. NBD, Bose-Einstein), correlations in gluon production can also be studied.

In the simplest case, gluons emitted independently by identical sources give rise to a Poisson distribution, i.e. $P(k'; \overline{k'}) = \overline{k'}^{k'} \exp\left(-\overline{k'}\right)/k'!$, where $\overline{k'}$ is the average number of gluons. In this case, the fluctuation can be quantified by the variance which is equal to the mean of the Poisson distribution. This results in a final particle multiplicity with mean $\overline{n} = \frac{\overline{k'}+k}{p} - k$.

2 Results

The Poisson weighted GMD (PGMD) is applied to describe charged-particle multiplicity distributions measured by the CMS collaboration [9]. Figure 1 shows the distributions at centre-of-mass energies of (top left) 0.9, (top right) 2.36, and (bottom left) 7 TeV. Comparison is made with the best fit PGMD and GMD. The interior point algorithm [10, 11] is used to find the parameters that give the minimum χ^2.

In general the evolution of multiplicities across all pseudorapidity windows and centre-of-mass energies is well described by both GMD and PGMD. While the PGMD describes the tail ends of the multiplicity distributions better than the GMD, it does not do as well at low multiplicities. It is however expected that both models would not be able to accommodate both the low multiplicity peak as well as the broadened “shoulder” at higher multiplicities, since diffractive events which have lower multiplicities are not factored in both models.

The advantage that PGMD has over the GMD can be seen in the lower multiplicity bins, especially the $n = 0$ spike, which is is evident in both CMS data and PGMD. This is not possible with the GMD since a Markov branching model with fixed initial number of gluons k' will not be able to describe multiplicities with $n < k'$. This is reflected in the GMD where the best description is reduced to the NBD i.e. $k' = 0$. This constraint limits the applicability of the GMD in the low multiplicity regime.

At 0.9, 2.36, and 7 TeV, the PGMD gives an average number of 1.07, 0.741, and 0.865 gluons respectively in the widest pseudorapidity window of 2.4. For increasing centre-of-mass energies, the parameter p decreases monotonically from 0.135 to 0.0614, reflecting an increasing initial parton

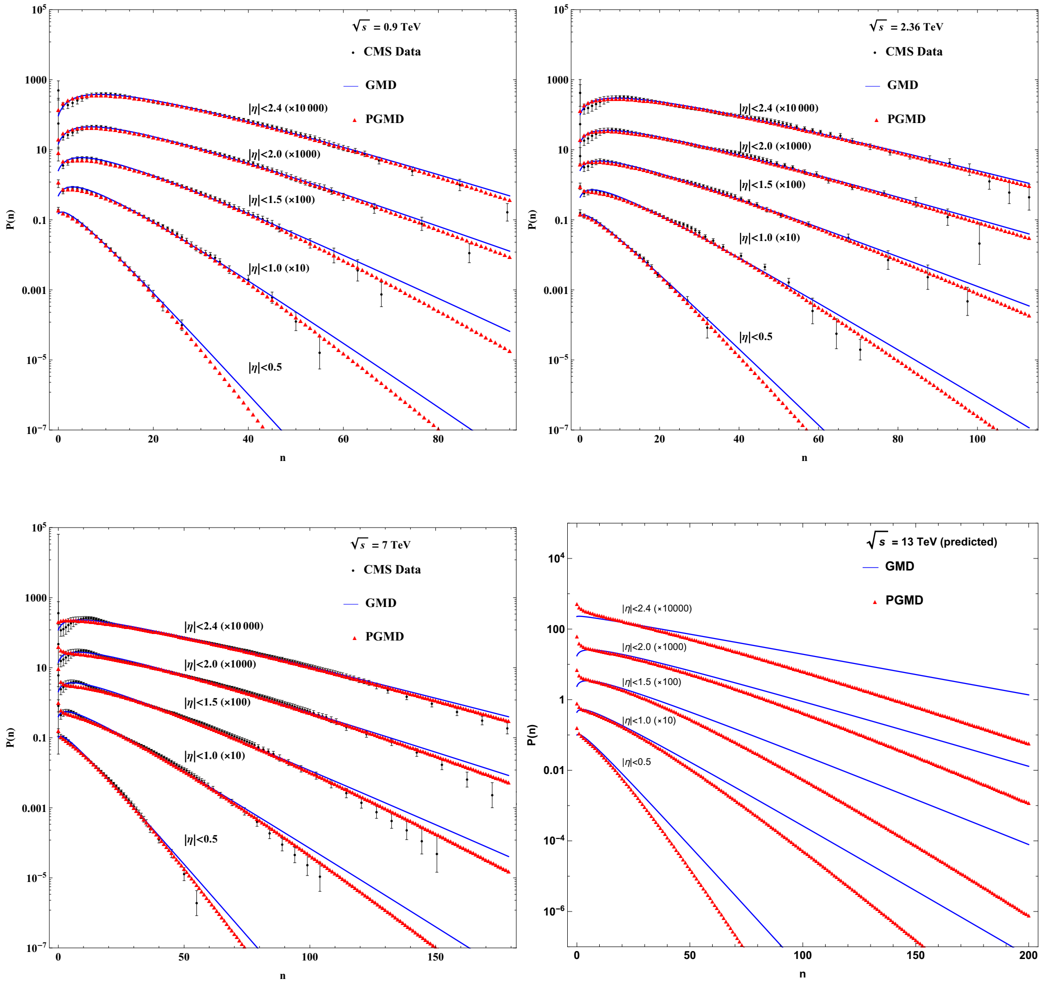

Figure 1. Charged-particle multiplicity distributions at (top left) $\sqrt{s}$ = 0.9, (top right) 2.36, and (bottom left) 7 TeV for $|\eta|$ < 0.5, 1.0, 1.5, 2.0, and 2.4 as measured by the CMS collaboration (circles). Comparison is made between the best fit PGMD (triangle) and GMD (line). The vertical lines in the data points represent the statistical errors and systematic uncertainties added in quadrature. The multiplicity distributions at (bottom right) $\sqrt{s}$ = 13 TeV is predicted by the models.

invariant mass Q as a function of the centre-of-mass energy, given a fixed branching probability [8]. The corresponding mean hadron numbers 18.1, 23.3, and 30.4 are compatible with the experimental values measured by the CMS collaboration [9].

Figure 1 (bottom right) shows predictions of the multiplicity distributions for all pseudorapidity windows at a centre-of-mass energy of 13 TeV. The centre-of-mass energy dependence of the parameter p, which is related to the mean multiplicity $\overline{n}$, follows a power law.

3 Conclusion

The weighted Generalised Multiplicity Distribution model is obtained from a weighted sum of the Generalised Multiplicity Distribution. The model incorporates event-by-event fluctuations in the initial gluon numbers into the Generalised Multiplicity Distribution, giving a more realistic description of *pp* collisions as well as providing a method of obtaining insights into the collision dynamics.

In the case where the initial gluons are produced independently, the gluon numbers are distributed according the the Poisson distribution. The Poisson weighted Generalised Multiplicity Distribution is applied to charged-particle multiplicity distributions measured by the CMS collaboration at centre-of-mass energies of 0.9, 2.36, and 7 TeV. Without the incorporation of statistics describing diffractive events which mainly factor into the low multiplicity bins, both the Generalised Multiplicity Distribution and the Poisson weighted Generalised Multiplicity Distribution fail to describe low multiplicity peak and broad "shoulder" together. However, the reasonable description of the high multiplicity tail and the lowest $n = 0$ spike by the Poisson weighted Multiplicity Distribution suggests that with the addition of diffractive processes, both the peak and the "shoulder" may be described.

Prediction of charged-particle multiplicity distributions for $|\eta| < 0.5$, 1.0, 1.5, 2.0, and 2.4 at the centre-of-mass energy of 13 TeV is given. The main features of the multiplicity distribution are reproduced.

The application of Poisson weights to describe the gluon number production implies that the sources produce independent and uncorrelated single gluons. The use of other distributions as weights, for example the Negative Binomial Distribution or Bose-Einstein distribution may be more physically motivated and would suggest a different picture where the gluons are produced in a correlated manner. Insight on gluon production may be obtained by the comparison of such distributions.

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