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Measuring the leading-order hadronic contribution to the muon g-2 in the space-like region

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Abstract. A new experiment is proposed to measure the running of the electromagnetic coupling constant in the space-like region by scattering high-energy muons on atomic electrons of a low-Z target. The differential cross section of the elastic process $\mu e \to \mu e$ provides direct sensitivity to the leading-order hadronic contribution to the muon anomaly $a_\mu^{\rm HLO}$. It is argued that by using the 150-GeV muon beam available at the CERN North Area, with an average rate of $\sim 1.3 \times 10^7$ muon/s, a statistical uncertainty of $\sim 0.3\%$ can be achieved on $a_\mu^{\rm HLO}$ after two years of data taking. The direct measurement of $a_\mu^{\rm HLO}$ via μe scattering will provide an independent determination and consolidate the theoretical prediction for the muon g-2 in the Standard Model. It will allow therefore a firmer interpretation of the measurements of the future muon g-2 experiments at Fermilab and J-PARC.

1 Introduction

The material presented here is largely based on the recently published paper of Ref. [1].

The long-standing (3–4)σ discrepancy between the experimental value of the muon anomalous magnetic moment $a_\mu = (g-2)/2$ and the Standard Model (SM) prediction, $\Delta a_\mu(\text{Exp} - \text{SM}) \sim (28 \pm 8) \times 10^{-10}$ [2, 3], is considered as one of the most intriguing indications of physics beyond the SM. However, the accuracy of the SM prediction, 5×10^{-10}, is limited by strong interaction effects, which cannot be computed perturbatively at low energies. By using analyticity and unitarity, it was shown [4] that the leading-order (LO) hadronic contribution to the muon g-2, $a_\mu^{\rm HLO}$, can be computed via a dispersion integral of the hadron production cross section in e^+e^- annihilation at low-energy. With this technique, the present error on $a_\mu^{\rm HLO}$, $\sim 4 \times 10^{-10}$ (corresponding to a fractional accuracy of 0.6%), constitutes the main uncertainty of the SM prediction.

From the experimental side, the error achieved by the BNL E821 experiment, $\delta a_\mu^{\rm Exp} = 6.3 \times 10^{-10}$ (corresponding to 0.54 ppm) [6], is dominated by the available statistics. New experiments at Fermilab and J-PARC, aiming at measuring the muon g-2 to a precision of 1.6×10^{-10} (0.14 ppm), are in preparation [7, 8].

Together with a fourfold improved precision on the experimental side, an improvement on the LO hadronic contribution is highly desirable. Differently from the dispersive approach, in Ref. [1] it is proposed to determine $a_\mu^{\rm HLO}$ from a measurement of the effective electromagnetic coupling in the space-like region, where the vacuum polarization (VP) is a smooth function of the squared momentum transfer. The method in Ref. [1] exploits $\mu e \to \mu e$ scattering, while the main ideas were firstly proposed in Ref. [9] by using Bhabha scattering data. [1]

How the hadronic contribution to the running of α can be determined unambiguously through the t-channel μe elastic scattering, from which $a_\mu^{\rm HLO}$ can be obtained, is sketched in the next sections.

2 Theoretical framework

With the help of dispersion relations and the optical theorem, $a_\mu^{\rm HLO}$ is given by the well-known formula [4, 12]

$$a_\mu^{\rm HLO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{\hat{K}(s) R_{\rm had}(s)}{s^2}, \tag{1}$$

where $R_{\rm had}(s)$ is the ratio of the total $e^+e^- \to$ hadrons and the Born $e^+e^- \to \mu^+\mu^-$ cross sections, $\hat{K}(s)$ is a smooth function and m_μ (m_π) is the muon (pion) mass. It should be remarked that $R_{\rm had}(s)$ in the integrand function of Eq. (1) is highly fluctuating at low energy due to hadronic resonances. The dispersive integral in Eq. (1) is usually calculated by using the experimental value of $R_{\rm had}(s)$ up to a certain value of s [5, 13, 14] and by using perturbative QCD (pQCD) [15] in the high-energy tail.

For the calculation of $a_\mu^{\rm HLO}$, an alternative formula can also be exploited [9, 16], namely

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi} \int_0^1 dx\, (1-x)\, \Delta\alpha_{\rm had}[t(x)]\,, \tag{2}$$

[1] It is worth mentioning that a method to determine the running of α by using small-angle Bhabha scattering was described in [10] and applied to LEP data in [11].

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where $\Delta\alpha_{\rm had}(t)$ is the hadronic contribution to the running of the fine-structure constant, evaluated at

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0, \quad (3)$$

the space-like (negative) squared four-momentum transfer. In contrast with the integrand function of Eq. (1), the integrand of Eq. (2) is smooth and free of resonances.

By measuring the running of α,

$$\alpha(t) = \frac{\alpha(0)}{1 - \Delta\alpha(t)}, \quad (4)$$

where $t = q^2 < 0$, the hadronic contribution $\Delta\alpha_{\rm had}(t)$ can be extracted by subtracting from $\Delta\alpha(t)$ the purely leptonic part $\Delta\alpha_{\rm lep}(t)$, which can be calculated in perturbation theory with high accuracy.

Fig. 1 (top) shows $\Delta\alpha_{\rm lep}$ and $\Delta\alpha_{\rm had}$ as a function of the variables x and t. The range $x \in (0, 1)$ corresponds to $t \in (-\infty, 0)$, with $x = 0$ for $t = 0$. The integrand of Eq. (2), calculated with the routine `hadr5n12` [17], which uses time-like hadroproduction data and perturbative QCD, is plotted in Fig. 1 (bottom). The peak of the integrand occurs at $x_{\rm peak} \simeq 0.914$ (corresponding to $t_{\rm peak} \simeq -0.108$ GeV^2) and $\Delta\alpha_{\rm had}(t_{\rm peak}) \simeq 7.86 \times 10^{-4}$.

3 Experimental proposal

Through Eq. (2), $a_\mu^{\rm HLO}$ can be determined by measuring the running of α in the space-like region with a muon beam of $E_\mu = 150$ GeV on a fixed electron target. The proposed measurement is similar to the one used for the determination of the pion form factor, as described in [18], and it is very appealing for the following reasons:

1. at tree-level, $\mu e \to \mu e$ is a t-channel process, making the dependence on t of the differential cross section, when including VP effects, proportional to $|\alpha(t)/\alpha(0)|^2$:

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha(0)}\right|^2. \quad (5)$$

   The VP effect, in the leading photon t-channel exchange, is incorporated in the running of α and gives rise to the factor $|\alpha(t)/\alpha(0)|^2$. It is understood that for a high precision measurement also higher-order radiative corrections must be included;

2. given the incoming muon energy E_μ, in a fixed-target experiment the t variable is related to the energy of the scattered electron E_e^f or its angle θ_e^f. With $E_\mu = 150$ GeV, the angle θ_e^f spans the range (0–31.85) mrad for the electron energy E_e^f in the range (1–139.8) GeV (the low-energy cut at 1 GeV is arbitrary);

3. it turns out that -0.143 GeV$^2 < t < 0$ GeV^2 if $E_\mu = 150$ GeV. This implies that the region of x extends up to 0.93, while the peak of the integrand function of Eq. (2) is at $x_{peak} = 0.914$, corresponding to an electron scattering angle of 1.5 mrad;

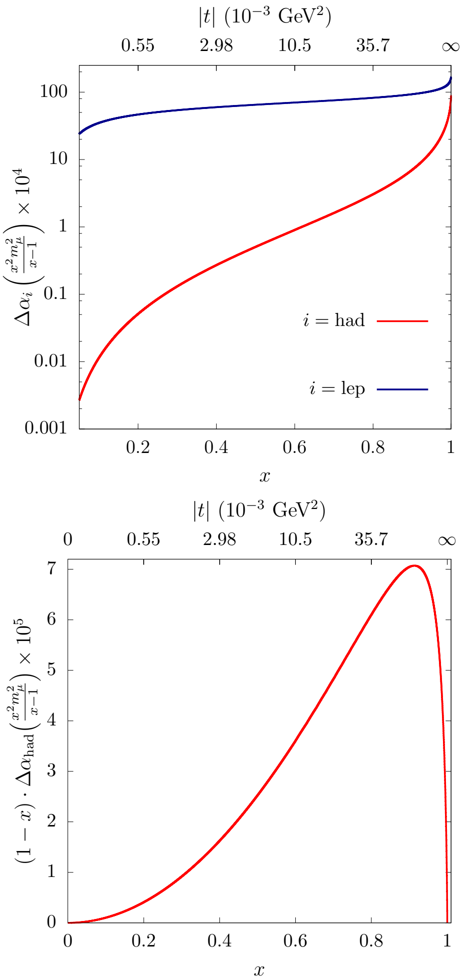

Figure 1. Top: $\Delta\alpha_{\rm had}[t(x)] \times 10^4$ (red) and, for comparison, $\Delta\alpha_{\rm lep}[t(x)] \times 10^4$ (blue), as a function of x and t (upper scale). Bottom: the integrand $(1-x)\Delta\alpha_{\rm had}[t(x)] \times 10^5$ as a function of x and t. The peak value is at $x_{\rm peak} \simeq 0.914$.

4. the angles of the scattered e^- and μ are correlated as shown in Fig. 2 (drawn for $E_\mu = 150$ GeV). This constraint is extremely important to select elastic scattering events, rejecting background events from radiative or inelastic processes and to minimize systematic effects in the determination of t. Note that for scattering angles of (2–3) mrad there is an ambiguity between the outgoing e^- and μ, as their angles and momenta are similar, to be resolved by means of μ/e discrimination;

5. the boosted kinematics allows the same detector to cover the whole acceptance. Many systematic errors, *e.g.* on the efficiency, will cancel out (at least at first order) in the relative ratios of event counts in the high and low q^2 regions (signal and normalization regions).

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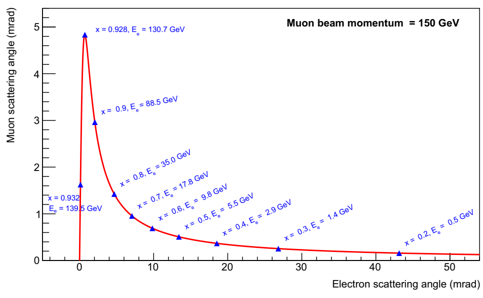

Figure 2. The relation between the μ and e^- scattering angles for 150 GeV incident muon beam. Blue triangles indicate reference values of the x variable and e^- energy.

Assuming a 150 GeV muon beam with an average intensity of $\sim 1.3 \times 10^7$ muons/s, presently available at CERN's North Area, incident on a target consisting of twenty Beryllium layers, each 3 cm thick (see Sect. 4), and two years of data taking with a running time of 2×10^7 s/yr, one can reach an integrated luminosity of about 1.5×10^7 nb^{-1}: taking into account the process cross section and with this luminosity, it is estimated with a simplified simulation that one can reach a statistical sensitivity of roughly 0.3% on the value of $a_\mu^{\rm HLO}$. The integrand in the region $x \in [0.93, 1]$, accounting for 13% of the $a_\mu^{\rm HLO}$ integral, cannot be reached by the proposed experiment, but can be determined using time-like data and perturbative QCD, and/or lattice QCD results [19–21].

4 Considerations on the detector

In order to perform the measurement with the required precision, a dedicated detector is necessary. In this section, a possible and preliminary setup to measure the direction and momentum of the incident muon and the directions of the outgoing electron and muon is sketched.

The CERN 150-GeV muon beam M2 has the characteristics needed for such a measurement. The beam intensity provides the required event yield. Its time structure allows to tag the incident muon while keeping low the background related to incoming particles (*e.g.* electrons). The electron contamination is very small. The beam provides both positive and negative muons, which can be both exploited.

The target consists of atomic electrons. To reach the required statistics, it must contain an adequate amount of material to give a sufficient number of electron scattering centres. The target has to be made of a low-Z material to minimize the impact of multiple scattering and the background due to bremsstrahlung and pair production processes.

A promising idea, presently under study, is to use 20 identical modules, each consisting of a 3 cm thick layer of Be (or C) coupled to 2 Si stations located at a relative distance of one meter from each other and spaced by intermediate air gaps.

The arrangement provides both a distributed target with low-Z and the tracking system. As downstream particle identifiers it is planned to use a calorimeter for the electrons and a muon system for the muons (a filter plus active planes). This particle identifier system is required to solve the muon-electron ambiguity for electron scattering angles around (2–3) mrad.

Preliminary studies of such an apparatus, performed with the help of GEANT4, indicate that a tracking angular resolution for the outgoing particles of ~ 0.02 mrad can be reached using modern silicon strip detectors.

The detector acceptance covers the region of the signal, with the electron emitted at extremely forward angles and high energies, as well as the normalization region, where the electron has much lower energy (around 1 GeV) and an emission angle of some tens of mrad.

The incoming muons have to be tagged and their direction and momentum precisely measured. To this purpose, a detector similar to those used by COMPASS [22] or NA62 [23] can be employed.

5 Systematic uncertainties

Significant contributions of the hadronic VP to the $\mu e \to \mu e$ differential cross section are essentially restricted to electron scattering angles below 10 mrad, corresponding to electron energies above 10 GeV. The net effect of these contributions is to increase the cross section by a few per mille: a precise determination of $a_\mu^{\rm HLO}$ requires not only high statistics, but also a high systematic accuracy, as the final goal of the experiment is equivalent to a determination of the differential cross section with $\sim$10 ppm systematic uncertainty at the peak of the integrand function.

Such an accuracy can be achieved if the efficiency is kept highly uniform over the entire q^2 range, including the normalization region, and over all the detector components. This motivates the choice of a purely angular measurement: an acceptance of tens of mrad can be covered with a single sensor of modern silicon detectors, positioned at a distance of about one meter from the target. It has to be stressed that particle identification is necessary to solve the electron-muon ambiguity in the region below 5 mrad. The wrong assignment probability can be measured with the data by using the rate of muon-muon and electron-electron events.

Another requirement for reaching very high accuracy is to measure all the relevant contributions to systematic uncertainties from the data themselves. An important effect, which distinguishes the normalization from the signal region, is multiple scattering, as the electron energy in the normalization region is as low as 1 GeV. Multiple scattering breaks the muon-electron two-body angular correlation, moving events out of the kinematic line in the 2D plot of Fig. 2. In addition, multiple scattering in general causes acoplanarity, while two-body events are planar, within the resolution. These facts allow effects to be modelled and measured using data. An additional handle on multiple scattering could be the inclusion of a thin layer in the apparatus, made of the same material as the main target modules. This possibility will be studied in detail with simulation.

The challenge of the proposed measurement is the feasibility of achieving a systematic uncertainty at the level of 10 ppm. This is the key point from the experimental side. In order to demonstrate that such a precision can be realistic, a very detailed optimization of the experimental apparatus is necessary. Tests with beams (electrons and muons), and with one or two modules of the detector, will be necessary and a crucial tool to understand if and to what extent the systematic uncertainties can be kept under control. They will provide a proof-of-concept of the proposed method.

From the theoretical side, the control of the systematic uncertainties requires the development of high-precision Monte Carlo tools, including all the relevant QED radiative corrections to reach the needed theoretical precision. To this aim, exact fixed order (NLO, NNLO) calculations, properly matched to leading-logarithmic corrections resummed up to all orders of perturbation theory, are mandatory to achieve the necessary theoretical accuracy on the relevant differential cross sections.

Tools to calculate Bhabha scattering exist, like for instance the `BabaYaga` event generator [24] (see also Ref. [25] and references therein), which implement exact NLO corrections matched with leading-logarithmic resummation, ensuring that the differential cross section is theoretically under control at the $O(10^{-4})$ level. The same algorithmic framework can be extended to $\mu e \to \mu e$ scattering and generalized to include exact diagrammatic NNLO corrections, which are not completely known yet for this process.

Work is in progress to extend the available Monte Carlo tools to $\mu e \to \mu e$ scattering and to quantify the achievable accuracy in the computation of the ratio of signal and normalization cross sections, by means of dedicated and realistic simulations.

6 Conclusions

A novel approach is under study to measure the running of α in the space-like region which can be used to determine a_μ^{HLO}, the leading hadronic contribution to the muon g-2. The proposed experiment exploits the process $\mu e \to \mu e$ by scattering high-energy muons on atomic electrons of a low-Z target. The experiment is primarily based on a precise measurement of the scattering angles of the two outgoing particles, from which the q^2 of the muon-electron interaction can be directly accessed.

An advantage of the muon beam is the possibility of employing a modular apparatus, with the target subdivided in subsequent layers. A low-Z solid target is preferred in order to provide the required event rate, limiting at the same time the effect of multiple scattering as well as of other types of muon interactions (pair production, bremsstrahlung and nuclear interactions).

The normalization of the cross section is provided by the very same $\mu e \to \mu e$ process in the low-q^2 region, where the effect of the hadronic corrections on $\alpha(t)$ is negligible. Such a simple and robust technique has the potential to keep systematic effects under control, aiming to reach a systematic uncertainty of the same order as the statistical one. For this purpose a preliminary detector layout has been described. By considering a beam of 150 GeV muons with an average intensity of $\sim 1.3 \times 10^7$ muon/s, currently available at the CERN North Area, a statistical uncertainty of $\sim 0.3\%$ can be achieved on a_μ^{HLO} in two years of data taking.

A test performed using a single detector module, exploiting the muon beam facility, can provide a validation of the proposed method and it is scheduled for the next future.

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References

[1] G. Abbiendi *et al.*, Eur. Phys. J. C **77** (2017) no.3, 139.

[2] T. Blum, A. Denig, I. Logashenko, E. de Rafael, B. Lee Roberts, T. Teubner and G. Venanzoni, arXiv:1311.2198 [hep-ph].

[3] F. Jegerlehner, EPJ Web Conf. **118** (2016) 01016.

[4] C. Bouchiat, L. Michel, J. Phys. Radium 22 (1961) 121;
L. Durand, Phys. Rev. **128** (1962) 441 [Erratum-ibid. **129** (1963) 2835];
M. Gourdin, E. De Rafael, Nucl. Phys. B **10** (1969) 667.

[5] F. Jegerlehner, A. Nyffeler, Phys. Rept. **477** (2009) 1.

[6] G. W. Bennett et al. (The g-2 Collaboration), Phys. Rev. **D73** (2006) 072003.

[7] J. Grange *et al.* [Muon g-2 Collaboration], arXiv:1501.06858 [physics.ins-det].

[8] N. Saito [J-PARC g-2/EDM Collaboration], AIP Conf. Proc. **1467** (2012) 45.

[9] C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni, Phys. Lett. B **746** (2015) 325.

[10] A.B. Arbuzov, D. Haidt, C. Matteuzzi, M. Paganoni and L. Trentadue, Eur. Phys. J. C **34** (2004) 267.

[11] G. Abbiendi *et al.* [OPAL Collaboration], Eur. Phys. J. C **45** (2006) 1.

[12] F. Jegerlehner, "The anomalous magnetic moment of the muon", Springer Tracts Mod. Phys. **226**, 2008.

[13] M. Davier, A. Hoecker, B. Malaescu, Z. Zhang, Eur. Phys. J. C **71** (2011) 1515 [Erratum-ibid. C **72** (2012) 1874].

DOI: 10.1051/epjconf/201714201007

[14] K. Hagiwara, R. Liao, A.D. Martin, D. Nomura, T. Teubner, J. Phys. G **38** (2011) 085003.

[15] R.V. Harlander, M. Steinhauser, Comput. Phys. Commun. **153** (2003) 244.

[16] B.E. Lautrup, A. Peterman, E. de Rafael, Phys. Rept. **3** (1972) 193.

[17] S. Eidelman and F. Jegerlehner, Z. Phys. C **67** (1995) 585; F. Jegerlehner, Nucl. Phys. Proc. Suppl. **181-182** (2008) 135.

[18] S. R. Amendolia *et al.*, Phys. Lett. B **146** (1984) 116; S. R. Amendolia *et al.* [NA7 Collaboration], Nucl. Phys. B **277** (1986) 168.

[19] F. Jegerlehner, Nucl.Phys.Proc.Suppl. 181-182 (2008) 135-140.

[20] M. Marinkovic, P. Boyle, L. Del Debbio, A. Juettner, K. Maltman, A. Portelli, PoS LATTICE2014 (2015) 160, http://inspirehep.net/record/1345265.

[21] M. Golterman, K. Maltman, S. Peris, Phys. Rev. D90 (2014) 7, 074508.

[22] P. Abbon *et al.* [COMPASS Collaboration], Nucl. Instrum. Meth. A **577** (2007) 455.

[23] Mathieu Perrin-Terrin for the NA62 Collaboration, Proceed. 24th IWVD, Santa Fe, June 2015, pp.016.

[24] G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini and F. Piccinini, Nucl. Phys. B **758** (2006) 227; C. M. Carloni Calame, Phys. Lett. B **520** (2001) 16; C. M. Carloni Calame, C. Lunardini, G. Montagna, O. Nicrosini and F. Piccinini, Nucl. Phys. B **584** (2000) 459.

[25] S. Actis *et al.* [Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies Collaboration], Eur. Phys. J. C **66** (2010) 585.