EPJ Web of Conferences **143**, 02037 (2017) DOI: 10.1051/epjconf/201714302037
EFM 2016

Radial basis functions in mathematical modelling of flow boiling in minichannels

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Abstract. The paper addresses heat transfer processes in flow boiling in a vertical minichannel of 1.7 mm depth with a smooth heated surface contacting fluid. The heated element for FC-72 flowing in a minichannel was a 0.45 mm thick plate made of Haynes-230 alloy. An infrared camera positioned opposite the central, axially symmetric part of the channel measured the plate temperature. K-type thermocouples and pressure converters were installed at the inlet and outlet of the minichannel. In the study radial basis functions were used to solve a problem concerning heat transfer in a heated plate supplied with the controlled direct current. According to the model assumptions, the problem is treated as two-dimensional and governed by the Poisson equation. The aim of the study lies in determining the temperature field and the heat transfer coefficient. The results were verified by comparing them with those obtained by the Trefftz method.

1 Introduction

Recently, research on heat transfer has focused on phenomena occurring in minichannels so that the solutions can be used in cooling systems for electronic devices. A review of literature relating to flow boiling heat transfer in minichannels was presented in [1-4].

The recent years have shown a dynamic development of meshless numerical methods which do not require troublesome and time-consuming mesh generation like e.g. Finite Element Method (FEM). Two typical examples could be the Trefftz method [5-10] and the Kansa method [11,12]. Analytical-numerical Trefftz method consists in approximating the solution of a partial differential equation with a linear combination of certain basis functions (named T-complete functions) satisfying the given equation. Similarly, in the Kansa method the solution is expressed by a linear combination of the radial basis functions, dependent on the distance of the argument from the chosen points called the centres. In order to specify the unknown coefficients of linear combinations we have to use (in both methods) the prescribed boundary conditions and also (in the Kansa method) the governing equation. The present study employed both these methods for determining two-dimensional distributions of the heating surface temperature. Thus the calculations (by both methods) based on empirical input data gave two alternative, but only slightly different, temperature distributions. Having determined the heating surface temperature distribution, one could estimate the local heat transfer coefficients at the interface between the heated plate and the working fluid.

2 Experimental background

Experimental data was collected in the measurement stand whose essential part is a tested module with a vertical minichannel 1.7 mm deep, 16 mm wide and 180 mm long, asymmetrically heated. The heated element for a boiling liquid flowing in a minichannel was a 0.45 mm thick plate made of Haynes-230 alloy.

The most important elements of the measurement stand are presented in figure 1a, the cross-section of the test module with a minichannel is shown in figure 1b. The experimental setup was discussed in [1-4, 13].

The working fluid circulating in a flow loop is FC-72 Fluorinert. The temperature of the outer side of the heated plate was measured by an infrared camera. The camera is positioned opposite the central, axially symmetric part of the minichannel. The plate surface was coated with a black paint with known emissivity [3]. Thermal accuracy of the infrared FLIR E60 camera is ± 1°C or ± 1% within the temperature range of 0÷120°C. K-type thermocouples and pressure converters were installed at the inlet and outlet of the minichannel. The two-phase flow patterns on the plate surface in contact with the fluid in the minichannel were observed through a glass pane, using digital SLR camera and illuminated by high power LEDs.

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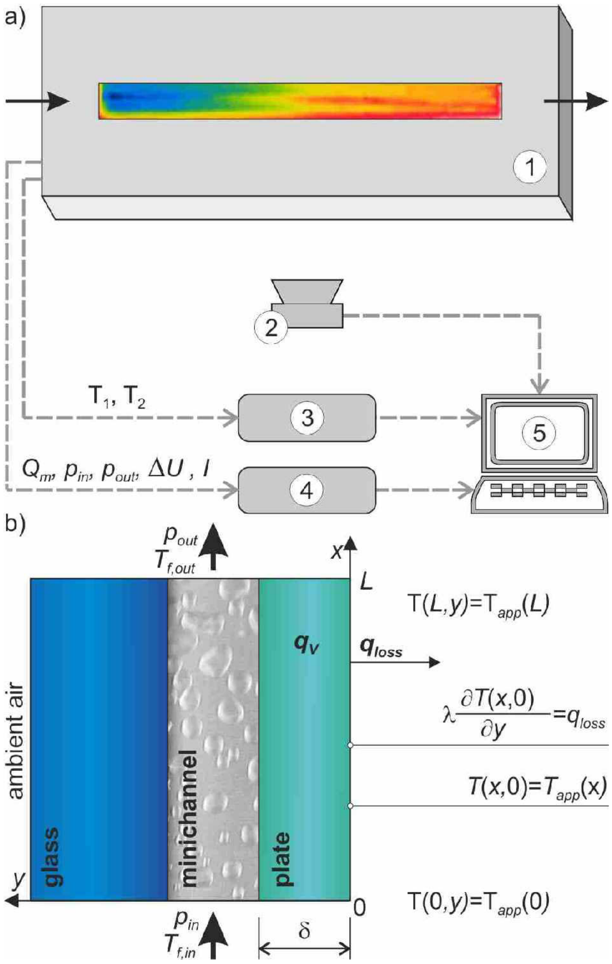

Fig. 1. The schematic diagrams of: a) the measurement stand, b) cross-section of the tested module with a minichannel;
1 - test module with a minichannel, 2 - infrared camera,
3, 4-data acquisition station, 5- computer with special software,
L - minichannel length, λ- heated plate conductivity,
q_V- volumetric heat flux, δ - heated plate thickness,
experimental parameters: Q_m - mass flow rate,
p - pressure, I - current, ΔU- voltage drop.

3 Mathematical model and method

3.1. Model

The mathematical model refers the heat transfer problem in flow boiling of FC- 72 fluorinert flowing through the minichannel. The assumptions include steady state in the test module and slight temperature changes of the plate and fluid along the minichannel width. The proposed approach considered two dimensions: dimension x in the direction of the flow and dimension y perpendicular to the flow, related to the thickness of the heated plate (δ), and to the depth of the minichannel. Further considerations refer only to the central section of the test module. The plate temperature was assumed to satisfy the Poisson equation with an adequate set of boundary conditions

$$\Delta T = -\frac{q_V}{\lambda}\ ,\ (x,y)\in\Omega \tag{1}$$

where $\Omega = \{(x,y)\in R^2 : 0<x<L,\ \ 0<y<\delta\}$, figure 1b. The adequate boundary conditions can be written as

$$T(x,0) = T_{app}(x) \tag{2}$$

$$\lambda \frac{\partial T(x,0)}{\partial y} = q_{loss} \tag{3}$$

$$\text{a) } T(0,y) = T_{app}(0),\ \text{ b) } T(L,y) = T_{app}(L) \tag{4}$$

where $T_{app}(x)$ denotes a polynomial approximating the temperature measurement of the heated plate, q_{loss}- heat flux density transferred from the plate into surrounding and determining as in [3]. The problem described by equation (1) with conditions (2) – (4) is an inverse heat conduction problem.

For the subcooled boiling region, when the void fraction is very low, the fluid temperature T_f changes linearly from the minichannel inlet temperature $T_{f,in}$ to the outlet temperature $T_{f,out}$. For the saturated boiling region, when the bubbly or bubbly-slug boiling occurred in the considerable part of the minichannel domain, we assumed that the fluid temperature is equal the saturation temperature T_{sat}, [4]. Saturation temperature is determined from the assumption of linear distribution of the fluid pressure along the minichannel. The known temperature field of the fluid is used to determine the heat transfer coefficient $\alpha(x)$ at the plate - fluid interface from the following equations:

- for the subcooled boiling region

$$\alpha(x) = \frac{q_V\,\delta - q_{loss}}{T(x,\delta) - T_f(x)} \tag{5a}$$

- for the saturated boiling region

$$\alpha(x) = \frac{q_V\,\delta - q_{loss}}{T(x,\delta) - T_{sat}(x)} \tag{5b}$$

3.2 Numerical method

When considering some problems of mechanics, it often happens that the measurement of a physical quantity inside a region of interest is (for some reason) impossible. In such situations, one could measure the desired parameters on the boundary of a region and then try to search for their distributions (or histories) inside the region. A task like that is a typical inverse problem in which the reasons are to be inferred from the results. The present study addresses two kinds of the inverse problems considering identification of (i) the boundary temperature and (ii) the heat transfer coefficient. Generally, inverse problems are ill-posed [14] and they put great demands (concerning accuracy and stability) on the solution method.

3.2.1 Radial basis function (RBF) method

The concept of RBFs dates from the 1970s when R.L.Hardy [12] suggested using multiquadric functions in cartography for interpolating multidimensional scattered data. Since then, RBFs have found various useful applications including, among others, numerical solution of differential equations.

Consider a linear differential equation (1) subject to the boundary conditions (2) – (4). For the problem (1) – (4) we can use Kansa's collocation method [11], approximating the solution by a sum

$$T_1(x,y) \approx \sum_{j=1}^{M} a_j \phi\left(r_j(x,y)\right) \tag{6}$$

which we refer to a continuous function $\phi:[0,\infty) \to R$ as a radial basis function and where $r_j(x, y)$ denotes the Euclidean distance between (x, y) and the j^{th} node point (x_j, y_j), while a_j – the unknown coefficients.
Some of the nodes are selected to lie in the interior of Ω (for j = 1, 2, ... , K) while others on the boundary (for j = K+1, K+2, ... , M). It is worth noting that an extra term (constant or polynomial) can be added to expansion (6), this however will not be done in this study.

As to RBFs, some of the most commonly used in practical applications are the multiquadrics

$$\phi(r) = \left(r^2 + c^2\right)^{1/2} \tag{7}$$

where $c > 0$ is a shape parameter. These functions will be chosen for further processing. For determining the unknown coefficients a_j we claim that the approximation (6) satisfies the governing equation (1) at interior nodes (x_j, y_j) and the boundary conditions (2) – (4) at boundary nodes (x_j, y_j). Thus we obtain the collocation system which we solve numerically.

3.2.2 Trefftz Method

Two-dimensional temperature distribution T_2 of the heated surface was found by the Trefftz method with use of harmonic polynomials which are T-complete functions for Laplace's equation. More specifically, the solution of the problem (1) – (4) was being sought in the form of a linear combination of harmonic polynomials $h_i(x,y)$ with an added particular solution $\widetilde{h}$ of equation (1), [7-9,15]:

$$T_2(x,y) \approx \widetilde{h}(x,y) + \sum_{i=0}^{N} b_i h_i(x,y) \tag{8}$$

The unknown coefficients b_i were determined on the basis of the known boundary conditions, as described in [8,15].

4 Results

Numerical calculations were performed for the data coming from the experiments where the volumetric heat flux q_V varies from $4.51 \cdot 10^4$ kWm^{-3} to $1.83 \cdot 10^5$ kWm^{-3}, average Reynolds number Re = 2100, average inlet pressure p_{in} = 110 kPa, average inlet liquid subcooling 40 K, flow velocity 0.24 m·s^{-1}. An infrared camera measured the temperature of the heated plate in contact with the ambient air. The temperature measurement was approximated with a 5th degree polynomial $T_{app}(x)$ for which the coefficient of determination R^2 had values no less than 0.9. Applying the Kansa method, we took M = 67 and K = 4. The profile of the function (7) depends on the shape parameter c. With the growth of c, the function becomes more and more flat and thus less sensitive to the changes of the distance between the point (x, y) and the j^{th} center (x_j, y_j). Basically, the proper choice of c is a problem itself, which has not yet been solved and is still being studied. The computation was performed for different values of c ranging from 0.0001 to 0.1 and the results to a large extent agreed with one another. Below we present the numerical results for c = 0.001. Applying the Trefftz method, there were used 10 harmonic polynomials to compute approximate temperature of the heated plate. Figure 2 shows an illustrative two-dimensional distribution of the heated plate temperature calculated with the Kansa method (figure 2a) and the Trefftz method (figure 2b).

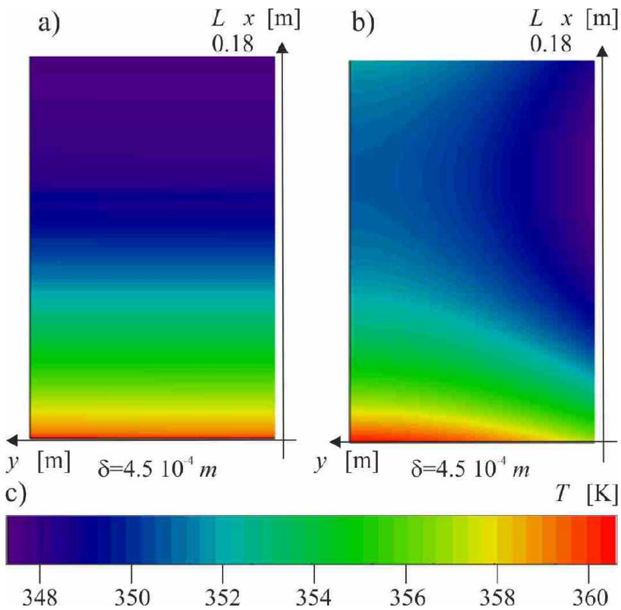

Fig. 2. Temperature of the heated plate obtained by:
a) Kansa method, b) Trefftz method, c) temperature scale, main experimental parameters: q_V=$1.56 \cdot 10^5$ kWm^{-3} , p_{in} = 153 kPa.

In the calculation we considered two boiling regions: subcooled and saturated. It was assumed that in the subcooled boiling region the fluid temperature varied linearly from the measured value at the inlet to the value at the outlet of the minichannel, while in the saturated boiling region it was equal to local saturation temperature. Figures 3 and 4 show the heat transfer coefficient as a function of the distance from the minichannel inlet, calculated from (5), where the plate temperature was determined from equation (6) (figure 3a, figure 4a) or equation (8) (figure 3b, figure 4b).

For the subcooled boiling region, the values of the heat transfer coefficient obtained from both employed methods do not significantly differ from each other, however higher values were obtained using the Trefftz method, see figure 3. The differences between the heat transfer coefficient calculated from the Trefftz and

Kansa methods become more significant in the saturated boiling region, figure 4. The maximum relative error (MRE) of estimating the plate temperature, calculated from the formula

$$MRE = max\left\{\begin{array}{l}\left[\dfrac{\iint\limits_{\Omega}\left(T_1(x,y)-T_2(x,y)\right)^2 dxdy}{\iint\limits_{\Omega}\left(T_1(x,y)\right)^2 dxdy}\right]^{0.5} \\ \left[\dfrac{\iint\limits_{\Omega}\left(T_1(x,y)-T_2(x,y)\right)^2 dxdy}{\iint\limits_{\Omega}\left(T_2(x,y)\right)^2 dxdy}\right]^{0.5}\end{array}\right\} \qquad (9)$$

ranges from 0.61% to 1.21% in the subcooled boiling region and from 0.91% to 1.24% in the saturated boiling region, for each setting of the volumetric heat flux.

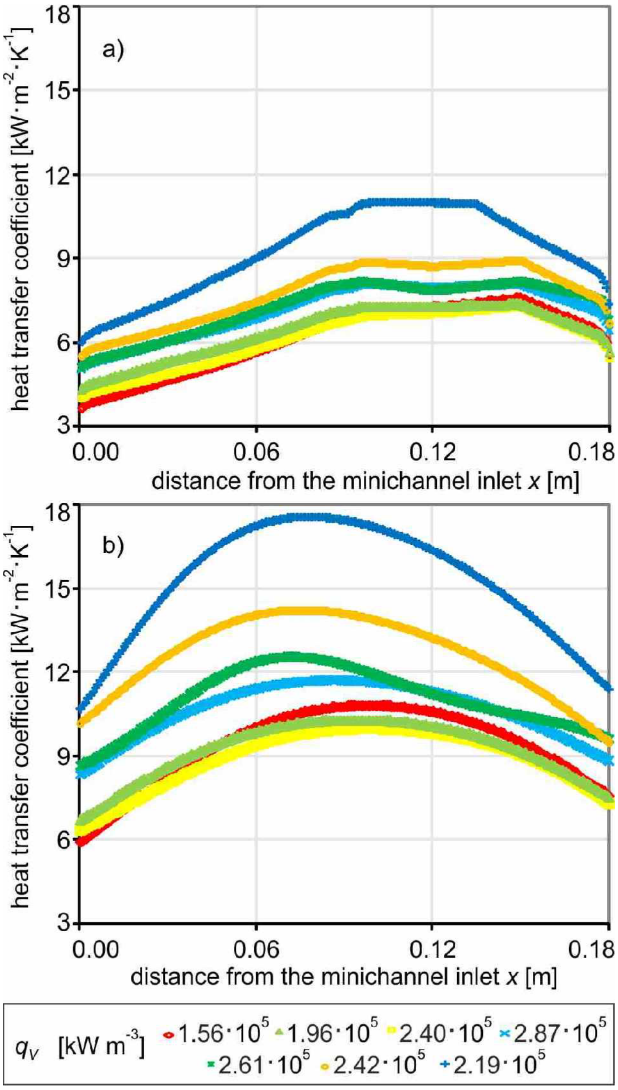

Fig. 3. Heat transfer coefficient vs. the minichannel inlet, the subcooled boiling region. The plate temperature was obtained by: a) Kansa method, b) Trefftz method.

The error of estimating the heat transfer coefficient turned to be less than 16.5%, no matter the plate temperature was obtained by the Kansa method or the Trefftz method. Similar estimates for the heat fluxes q_{loss} and q_v (in both boiling regions) have errors not exceeding 16.3% and 38%, respectively.

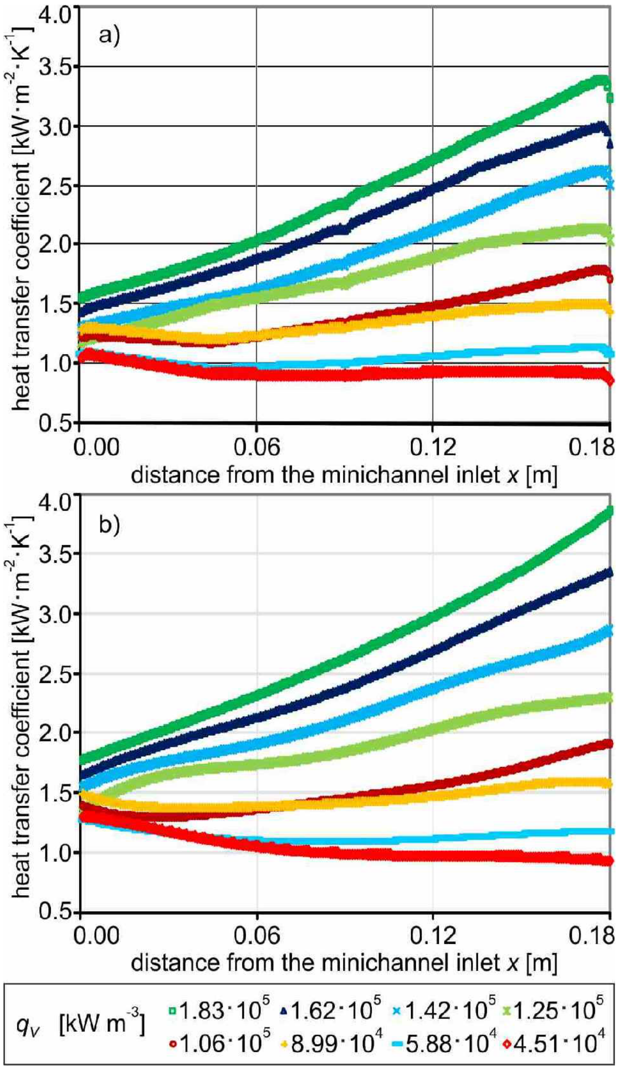

Fig. 4. Heat transfer coefficient vs. the minichannel inlet, the saturated boiling region. The plate temperature was obtained by: a) Kansa method, b) Trefftz method.

5 Conclusions

The paper focused on FC-72 flow boiling heat transfer in a vertical asymmetrically heated minichannel of 1.7 mm depth. Two numerical methods were applied to solve the considered inverse problem: the Kansa method and the Trefftz method. Both gave two-dimensional temperature distributions which agreed with each other, with MRE of the plate temperature no greater than 1.24%**.** Moreover, there was no significant disagreement between the heat transfer coefficients for the subcooled boiling region obtained by these methods. However greater differences between them could be observed in the saturated boiling region. Besides, in both considered boiling regions the heat transfer coefficient had higher values when its estimation was based on the Trefftz method.

Acknowledgements

The research reported herein was partly supported by a grant from the Polish National Science Centre (No. DEC-2013/09/B/ST8/02825).

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