EPJ Web of Conferences **146**, 10016 (2017) DOI: 10.1051/epjconf/201714610016
ND2016

The study of structure in 224–234 thorium nuclei within the framework IBM

Su Youn Lee,[a], Young Jun Lee, and J.H. Lee

Department of Physics, Dong-Eui University, Busan 614-714, Korea

Abstract. An investigation has been made of the behaviour of nuclear structure as a function of an increase in neutron number from ^{224}Th to ^{234}Th. Thorium of mass number 234 is a typical rotor nucleus that can be explained by the SU(3) limit of the interacting boson model(IBM) in the algebraic nuclear model. Furthermore, $^{224-232}$Th lie on the path of the symmetry-breaking phase transition. Moreover, the nuclear structure of ^{224}Th can be explained using X(5) symmetry. However, as $^{226-230}$Th nuclei are not fully symmetrical nuclei, they can be represented by adding a perturbed term to express symmetry breaking. Through the following three calculation steps, we identified the tendency of change in nuclear structure. Firstly, the structure of ^{232}Th is described using the matrix elements of the Hamiltonian and the electric quadrupole operator between basis states of the SU(3) limit in IBM. Secondly, the low-lying energy levels and E2 transition ratios corresponding to the observable physical values are calculated by adding a perturbed term with the first-order Casimir operator of the U(5) limit to the SU(3) Hamiltonian in IBM. We compared the results with experimental data of $^{224-234}$Th. Lastly, the potential of the Bohr Hamiltonian is represented by a harmonic oscillator, as a result of which the structure of $^{224-234}$Th could be expressed in closed form by an approximate separation of variables. The results of these theoretical predictions clarify nuclear structure changes in Thorium nuclei over mass numbers of practical significance.

1. Introduction

The interacting boson model (IBM) [1] is one of the representative models for the study of nuclear structure. This model describes various types of nuclear collective states in even-even nuclei on the basis of the group theoretical approach. The even-even nucleus is considered as an inner core combined with bosons which represent pairs of identical valence nucleons. There are three dynamical symmetries [U(5), SU(3) and O(6) limit] connected with subgroups in the process of group reduction starting with the top group U(6). These three limits correspond to the geometric model [2], i.e., U(5) [3] to an harmonic vibrator, SU(3) [4] to an axially symmetric deformed rotor, and O(6) [5] to a γ-unstable rotor. Most nuclei with collective structures have mixed properties instead of only one symmetry among them. The nuclear structure with symmetry breakings have been described using more than one symmetry or explained by phase transitions between the dynamical symmetries. The critical point symmetries, E(5) [6] and X(5) [7], have been recently proposed to describe nuclei at the points of phase transitions between different dynamical symmetries. The critical point symmetries are based on the special solutions of the Bohr Hamiltonian [8] with potentials of the special form instead of the algebraic descriptions of three limits of the IBM. The E(5) critical point symmetry corresponds to the phase transition between U(5) and O(6) while the X(5) critical point symmetry describes the transition between U(5) and SU(3) nuclei.

We have studied the collective properties of even Th nuclei based on the SU(3) limit of the IBM and the X(5) critical point symmetry. We explain the collective properties of low-lying states in ^{234}Th by using the SU(3) limit of the IBM, since this nucleus is known to have the typical rotational structure. The structure of $^{226-232}$Th exhibits a slight breaking of the SU(3) symmetry in the direction of U(5), so we have added the d-boson number operator n_d, which is the main term of the U(5) symmetric Hamiltonian, to the SU(3) Hamiltonian of the IBM[9]. Such small breaking of the low-lying energy levels and the $E2$ transition rates in $^{226-232}$Th can be explained by applying perturbation theory rather than direct diagonalization. Since there is evidence for coexisting phases in ^{224}Th, i.e., vibrational and rotational nuclei at low energy, we have analysed this nucleus by using the X(5) symmetry located at the critical point between U(5) and SU(3) symmetries.

The predicted theoretical results for energy spectra and $B(E2)$ values of low-lying states of the even Thorium nuclei are compared with experimental data, and the change of nuclear structures is analysed on the basis the neutron number in the even Thorium nuclei.

2. Dynamical symmetry and symmetry breaking

The SU(3) limit of the IBM is suitable to describe collective properties of axially symmetric deformed rotating nuclei. The IBM Hamiltonian with the SU(3)

[a] e-mail: syyi@deu.ac.kr

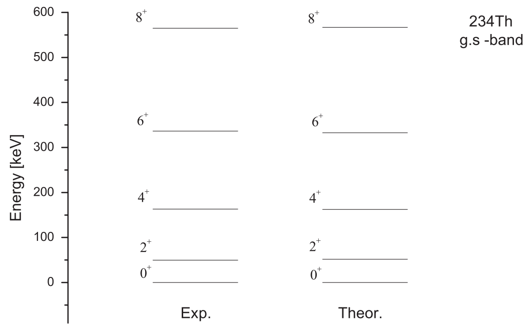

Figure 1. Energy level of the SU(3) and the corresponding experimental data for ^{234}Th [11].

dynamical symmetry can be written as

$$H_{SU(3)} = \kappa_1' L \cdot L + \kappa_2' Q \cdot Q. \quad (1)$$

L and Q are the angular momentum operator and the quadrupole operator, respectively, defined as

$$L = \sqrt{10}(d^\dagger \times \tilde{d})^{(1)},$$
$$Q = (s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s})^{(2)} + \chi(d^\dagger \times \tilde{d})^{(2)}, \quad (2)$$

where $s^\dagger$ and $d^\dagger$ are boson creation operators with angular momenta $L = 0$ and 2, respectively. $\tilde{d}_\mu = (-1)^\mu d_{-\mu}$ is the d-boson annihilation operator. χ in Eq. (2) is the structure parameter of the quadrupole operator of the IBM and corresponds to the value of $-\sqrt{7}/2$ in the SU(3) limit.

The energy eigenvalues of the SU(3) Hamiltonian have the closed form

$$E_{SU(3)} = E_0 + \kappa_1 L(L+1) + \kappa_2(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu) \quad (3)$$

in the SU(3) basis denoted as $|[N](\lambda,\mu)KLM\rangle$. E_0 is the binding energy, and the coefficients in Eq. (3) are related to those in Eq. (1) by $\kappa_1 = \kappa_1' - \frac{3}{8}\kappa_2'$ and $\kappa_2 = \frac{\kappa_2'}{2}$ respectively. The g.s-band energy level of ^{234}Th is reproduced as shown in the Fig. 1 by applying pure SU(3) limit.

The important features of deformed nuclei cannot be described by the pure SU(3) symmetry of the IBM. Therefore, a realisitic calculation will require considerable breaking of SU(3) symmetry. In this paper, we have added the d-boson number operator n_d to the SU(3) Hamiltonian as a perturbation to describe the breaking of the SU(3) symmetry in the direction of U(5) [10]:

$$H = H_{SU(3)} + \epsilon n_d. \quad (4)$$

Energies and wave functions obtained using the perturbation theory were constructed analytically in previous works [9] and are adopted in this paper. In this paper, we use their results. The parameters fitted using the results of the previous work are listed in Table 1. The $B(E2)$ values for $L_i \to L_f$ $E2$ transitions are given by

$$B(E2; L_i \to L_f) = \frac{|\langle L_i||T(E2)||L_f\rangle|^2}{2L_i + 1}. \quad (5)$$

The electric quadrupole transition operator $T(E2) = eQ$ consists of a generator Q in the SU(3) limit and e is the effective charge. Transition probabilities for ^{230}Th and ^{232}Th calculated using SU(3) $\to$ U(5) symmetry breaking and SU(3) symmetry are compared with the available experimental data in Table 2.

Table 1. Values of parameters for the energy spectra of $^{226-232}$Th (keV) [12–15].

	κ_1	κ_2	ϵ
^{226}Th	7.05	−7.69	392.08
^{228}Th	7.76	−7.33	161.99
^{230}Th	7.23	−5.37	148.45
^{232}Th	7.72	−5.22	20.69

Table 2. Theoretical and experimental $B(E2)$ transitions for ^{230,232}Th [16, 17].

		^{230}Th		^{232}Th	
L_i	L_f	Exp.	Theor.	Exp.	Theor.
2_1	0_1	100	100	100	100
4_1	2_1	136	140	144	141
6_1	4_1	167	149	165	151
8_1	6_1		149	173	151

3. Critical point symmetry and Phase transition

The critical point symmetries describe systems undergoing phase transitions between the dynamical symmetries of the algebraic structure. These symmetries are not based on group theoretical descriptions that are solved algebraically but on particular potentials amenable to analytic descriptions in terms of the zeros of special functions. X(5) symmetry based on zeros for the special functions can also be used to discuss the U(5)-SU(3) shape phase transition in nuclei. We have studied nuclear structure at the critical point of the U(5)-SU(3)[vibrator to deformed symmetric rotors] shape phase transition.

The Bohr Hamiltonian can be written in the following form [8]:

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2}\left(\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4}\sum_k \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)}\right)\right] + V(\beta,\gamma) \quad (6)$$

The space of this Hamiltonian is five-dimensional with two intrinsic variables β and γ, and three Euler angles θ_i ($i = 1, 2, 3$). X(5) symmetry deals mainly with potentials of the type $V(\beta,\gamma) = u(\beta) + v(\gamma)/\beta^2$, where the terms with β and γ are taken as an infinite-well potential and a harmonic oscillator $v(\gamma) = A\gamma^2$ respectively. The β-part solution of the Schrödinger equation obtained by inserting these potentials in Eq. (6) is then known to be given in terms of the Bessel function and the γ-part solution is a confluent hypergeometric function. The expression of the spectrum is

$$E_\gamma = 2(n_\gamma + 1)\sqrt{A + \frac{2}{3}[L(L+1) - K^2] - \frac{27}{20}} - 3. \quad (7)$$

The relative positions of all bandheads and the internal structures are determined using by theory with only one parameter A. The X(5) symmetry energy corresponding to Eq. (7) has been applied to ^{224}Th, and theoretical results are compared with the available experimental data taken from the X(5) symmetry, as in Fig. 2.

Harmonic oscillators in both β and γ variables are constructed as a potential of the form $V(\beta,\gamma) = a^2\beta^2 + c^2\gamma^2$

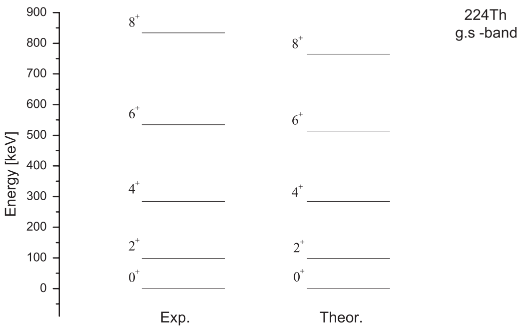

Figure 2. Energy spectra of the X(5) and the corresponding experimental data for ^{224}Th [18].

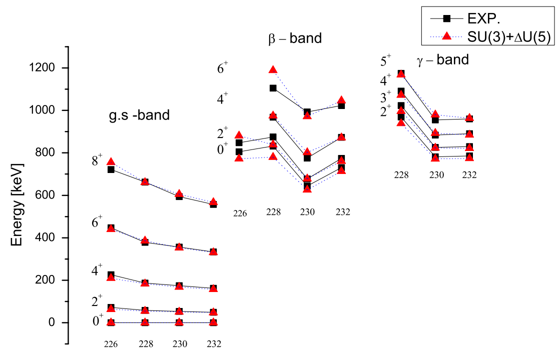

Figure 3. Energy spectra of the SU(3) → U(5) and the corresponding experimental data for $^{226-232}$Th [12–15].

in Eq. (6) to study the transition between the U(5) and the X(5) symmetries for axially-symmetric prolate nuclei. We consider that a potential of the oscillator-type differential equation in Eq. (6) may be approximately separated into β and γ parts. Therefore, the solution of this equation is expressed in terms of the Bessel function and associated Laguerre polynomials. Closed expressions are derived for the energy spectra as follows [19]:

$$E = E_0 + a\left(2n_\beta + 1 + \sqrt{\frac{L(L+1)-K^2}{3} + \frac{9}{4}}\right) + bn_\gamma. \tag{8}$$

The results of Eq. (8) when applied in the present calculation to ^{224}Th are not in agreement with experimental data.

Electromagnetic transition rates can be calculated by adopting matrix elements of the transition operators. Particularly interesting are the matrix elements of the quadrupole operator

$$T_\mu(E2) = t\beta\left[\mathcal{D}^{(2)}_{\mu,0}(\theta_i)\cos\gamma + \frac{1}{\sqrt{2}}(\mathcal{D}^{(2)}_{\mu,2}(\theta_i) + \mathcal{D}^{(2)}_{\mu,-2}(\theta_i))\sin\gamma\right], \tag{9}$$

where t is a scale factor. The γ, θ_i part of the calculation can be performed in the standard way. $B(E2)$ transition probabilities can be calculated using the quadrupole operator in Eq. (9), with the only difference being that the integrals over β have the form

$$B(E2; L_i \to L_f) = (L_i 2 L_f | K_i 0 K_f)^2 \times \left(\int_0^\infty \xi_{n_{\beta_i},\tau_i}(\beta)\xi_{n_{\beta_f},\tau_f}(\beta)\beta^5 d\beta\right)^2. \tag{10}$$

Difficult to determine the merit and validity of our calculations for ^{224}Th because of the lack of comparable experimental data.

4. Summary and conclusions

In this section, we discuss the results of the calculation with the symmetry and symmetry breaking for $^{224-234}$Th. Our predictions for $^{224-234}$Th isotopes on the basis of symmetry and symmetry breaking in the IBM are similar. Lowering of the ground-state band energy with increasing neutron number was clarified, as shown in Fig. 3. Increases of mass number from 228 to 232 exhibit a sharp decrease in level energy for every band. Energy values of the ground-state band are all reasonably close to constant between 226 and 232, while the β and γ bands tend to decrease sharply at mass number 230.

The ratios $R_{4/2} = E(4_1/2_1)$ and $R_{\beta/2} = E(0_\beta/2_1)$ are listed in Table 3. Theoretical and experimental $R_{4/2}$ values for ^{224}Th reflect the transitional nuclei with X(5) symmetry, while the nuclear structure of $^{226-232}$Th can be explained by introducing symmetry breaking in the direction of U(5) from SU(3). Our calculations also predict rotational character for ^{234}Th.

Table 3. Theoretical and experimental $R_{4/2}$ and $R_{\beta/2}$ for $^{226-234}$Th [11–15].

	$R_{4/2}$		$R_{\beta/2}$	
	Exp.	Theor.	Exp.	Theor.
226	3.14	3.32		
228	3.23	3.33	14.40	14.15
230	3.27	3.33	12.10	12.38
232	3.28	3.33	14.80	15.11
234	3.29	3.33		

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2015R1C1A2A 01052083).

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