The first unbound states of mirror ^{9}Be and ^{9}B nuclei

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Abstract. The structures of the first excited states of mirror ^{9}Be and ^{9}B nuclei are studied by using the $\alpha + \alpha + N$ three-body model and the complex scaling method. The resonance energy with a decay width of the $1/2^+$ state of ^{9}B is calculated by taking into account the consistency with photodisintegration cross sections of ^{9}Be into the $1/2^+$ state. We also compare the results with the measured data and other theories.

1. Introduction

A study of mirror nuclei plays an important role in understanding of the nuclear structure. Considerable experimental and theoretical efforts have been done so far to determine the location of the low-lying states of ^{9}Be and ^{9}B, specially the first unbound $1/2^+$ state. The existence of the $1/2^+$ states is a long-standing problem and has been a topic of interest in relation to the breakup mechanism of ^{9}Be. Recently, we have investigated the photodisintegration cross section of ^{9}Be and also discussed the structure of the excited $1/2^+$ state of ^{9}Be by applying the complex scaling method (CSM) [1,2] to the $\alpha + \alpha + n$ three-body model. In the results, we found that the $1/2^+$ state has a virtual-state character of an s-wave neutron in the ^{8}Be(g.s.)+n system [3].

It is particularly interest to compare the structure of the ^{9}Be$(1/2^+)$ state with the same spin-parity state of the mirror nucleus ^{9}B [4]. The situation of its analog ^{9}B$(1/2^+)$ state is also unclear experimentally and theoretically. This state has been studied experimentally but there is considerable ambiguity of measured energies and decay widths [5, 6]. In Ref. [7] this state was not observed. However, recent updated information [8] and measurement [9] have reported the energy and width of the ^{9}B$(1/2^+)$ state. Therefore, a special attention must be paid on the determination of the ^{9}B$(1/2^+)$ state by applying the appropriate theoretical model. Our purpose in this study is to investigate the energy with the decay width of the first excited $1/2^+$ state of ^{9}B together with the mirror state in ^{9}Be.

2. Method

2.1. $\alpha + \alpha + N$ three-body model

In order to study the first unbound $1/2^+$ state of mirror ^{9}Be and ^{9}B nuclei, we use the $\alpha + \alpha + n$ and $\alpha + \alpha + p$ three-cluster models for ^{9}Be and ^{9}B, respectively. We solve the Schrödinger equation for the $\alpha + \alpha + N$ system ($N = n$ for ^{9}Be and p for ^{9}B) using the complex-scaled orthogonality condition model [10]. The Schrödinger equation is given as

$$\hat{H}\Psi_J^\nu = E_\nu \Psi_J^\nu, \tag{1}$$

where J is the total spin of the $\alpha + \alpha + N$ system and ν is the state index.

The Hamiltonian for the relative motion of the $\alpha + \alpha + N$ three-body system is given as

$$\hat{H} = \sum_{i=1}^{3} t_i - T_{\rm c.m.} + \sum_{i=1}^{2} V_{\alpha N}(\boldsymbol{\xi}_i) + V_{\alpha\alpha} + V_{\rm PF} + V_3, \tag{2}$$

where t_i and $T_{\rm c.m.}$ are kinetic operators for each particle and the center-of-mass of the system, respectively. The interactions between the nucleon and the i-th α particle are given as $V_{\alpha N}(\boldsymbol{\xi}_i)$, where $\boldsymbol{\xi}_i$ is the relative coordinate between them. We here employ the KKNN potential [11] for $V_{\alpha N}$. For the α-α interaction $V_{\alpha\alpha}$ we employ a folding potential [12] of the effective NN interaction and the Coulomb interaction:

$$V_{\alpha\alpha}(r) = v_0 \exp\left(-ar^2\right) + \frac{4e^2}{r}{\rm erf}\left(\beta r\right), \tag{3}$$

where $v_0 = -106.09$ MeV, $a = 0.2009$ fm^{-2}, and $\beta = 0.5972$ fm^{-1}. The pseudo potential $V_{\rm PF} = \lambda|\Phi_{\rm PF}\rangle\langle\Phi_{\rm PF}|$ is the projection operator to remove the Pauli forbidden states from the relative motions of α-α and α-N [13]. The Pauli forbidden states are defined as the harmonic oscillator wave functions by assuming the $(0s)^4$ configuration whose oscillator length is fixed to reproduce the observed charge radius of the α particle. In the present calculation, λ is taken as 10^6 MeV.

In the present calculation, we introduce an $\alpha + \alpha + N$ three-body potential V_3. The explicit form of V_3 is given as

$$V_3 = v_3 \exp\left(-\mu\rho^2\right), \tag{4}$$

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where ρ is the hyperradius of the $\alpha + \alpha + N$ system. The hyperradius is defined as

$$\rho^2 = 2r^2 + \frac{8}{9}R^2, \tag{5}$$

where r is the distance between two alpha particles and R is that between the nucleon and the center-of-mass of the $\alpha + \alpha$ subsystem. The strength $v_3 = 1.10$ Mev and range $\mu = 0.02\,\text{fm}^{-2}$ parameters are chosen so as to reproduce the binding energy and nuclear radius of the ^{9}Be ground state.

We solve the Schrödinger equation with the coupled-rearrangement-channel Gaussian expansion method [14]. In the present calculation, the wave function $\Psi^{\nu}_{J^\pi}$ is given by the summation of basis functions described in different Jacobi systems:

$$\Psi^{\nu}_{J^\pi} = \sum_{cij} C^{\nu}_{cij}(J^\pi)\left[\left[\phi^i_l(\mathbf{r_c}), \phi^j_\lambda(\mathbf{R_c})\right]_L, \chi_{\frac{1}{2}}\right]_{J^\pi}, \tag{6}$$

where $C^{\nu}_{ij}(J^\pi)$ is expansion coefficient and $\chi_{\frac{1}{2}}$ is the spin wave function of the valence nucleon N. The relative coordinates r_c and R_c are those in three kinds of Jacobi coordinate systems indexed by $c(= 1, 2, 3)$, and the indices for the basis functions are represented as i and j. The radial part of the wave function is expanded with the Gaussian basis functions.

2.2. Complex scaling method

In the CSM for the $\alpha + \alpha + N$ model, we transform the relative radial coordinates ξ (r_c and R_c) as

$$U(\theta)\xi U^{-1}(\theta) = \xi e^{i\theta}, \tag{7}$$

where $U(\theta)$ is a complex-scaling operator and θ is a scaling angle being a real number. Applying this transformation to Eq. (1), we obtain the complex-scaled Schrödinger equation as

$$\hat{H}^\theta \Psi^\nu_J(\theta) = E^\theta_\nu \Psi^\nu_J(\theta). \tag{8}$$

The complex-scaled Hamiltonian and wave function are given as

$$\hat{H}^\theta = U(\theta)\hat{H}U^{-1}(\theta) \quad \text{and} \quad \Psi^\nu_J(\theta) = U(\theta)\Psi^\nu_J, \tag{9}$$

respectively. By solving the complex scaled Schrödinger equation with appropriate Gaussian basis functions as an L^2 basis, we obtain the energy eigenvalues E^θ_ν and eigenstates $\Psi^\nu_{J^\pi}(\theta)$ [2]. The energy eigenvalues E^θ_ν are obtained in the complex energy plane. The bound states are obtained on the negative energy axis independently from θ as well as the ordinary bound states. Because of a finite number of basis states, the continuum states are discretized with complex energies distributed on the rotated branch cut (2θ-line). The complex energies of resonant states are obtained as $E^\theta_\nu = E^{res}_\nu - i\Gamma_\nu/2$, when $\tan^{-1}(\Gamma_\nu/2E^{res}_\nu) < 2\theta$. The resonance energies E^{res}_ν and the decay widths Γ_ν are independent of the scaling angle θ.

3. Results

3.1. The first unbound state of ^{9}Be

In our previous work [3], we reported that no resonance solutions of the $1/2^+$ state are found without and with the three-body potential of $v_3 = 1.10$ MeV in the CSM of the scaling angle $\theta = 15°$. However, the CSM cannot reproduce broad resonances of $\Gamma/(2E_r) > \tan(2\theta)$ and virtual states as isolated solutions [2]. So our results of the CSM means nothing but that there is no $1/2^+$ state as a sharp resonance. We also discussed the photodisintegration cross section from the $3/2^-$ ground state to $1/2^+$ states. In Fig. 1, we show the calculated cross sections using Eq. (11) in Ref. [3] in comparison with the two sets of the observed data which commonly have a peak just above the ^{8}Be + n threshold. The results show that the calculated cross sections without and with three-body potential of $v_3 = 1.10$ MeV are underestimated and cannot reproduce the observed low-lying peak.

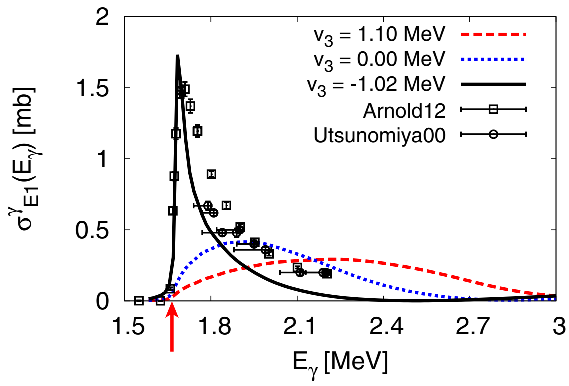

Figure 1. Calculated photodisintegration cross sections of ^{9}Be in comparison with the experimental data. The experimental data are taken from Refs. [15] and [16]. The arrow indicates the threshold energy of the ^{8}Be(0^+) + n channel.

To discuss the observed sharp peak just above the ^{8}Be + n threshold in the photodisintegration cross section, we change the strength, v_3, for the $1/2^+$ state, but fixing its range μ in Eq. (4) as the same as used in the ground state (TABLE I of [3]). We here take the strength as $v_3 = -1.02$ MeV for the $1/2^+$ state and obtain the cross section as shown as the black (solid) line in Fig. 1. Our result reproduces the observed peak by using the attractive three-body potential. We confirm that the calculated cross section rapidly increases just above the ^{8}Be(0^+) + n threshold and there is negligibly small strength below this threshold. We also find that the calculated cross sections show the strong dependence on the strengths of the three-body potentials as shown in Fig. 1. This result is interesting and suggests the existence of the three-body unbound state of ^{9}Be($1/2^+$), as a resonance or virtual state which is closely connected with the ^{8}Be(0^+) + n configuration. In relation to the cross section, we discuss the character of the $1/2^+$ state.

We investigate the origin of the low-lying peak of the cross section above the ^{8}Be(0^+) + n threshold in more detail. For this purpose, we show the distribution of the energy eigenvalues of the $1/2^+$ states in the CSM. In Fig. 2, we show the distribution of the energy eigenvalues for the $1/2^+$ states measured from the $\alpha + \alpha + N$ threshold energy, calculated with $v_3 = -1.02$ MeV, which reproduces the observed peak as shown in Fig. 1. In the present calculation, we find no resonances in the energy eigenvalue distribution. All energy eigenvalues are located on the 2θ-lines, corresponding to the branch cuts for the $\alpha + \alpha + n$, ^{8}Be(0^+) + n, and ^{5}He($3/2^-$) + α

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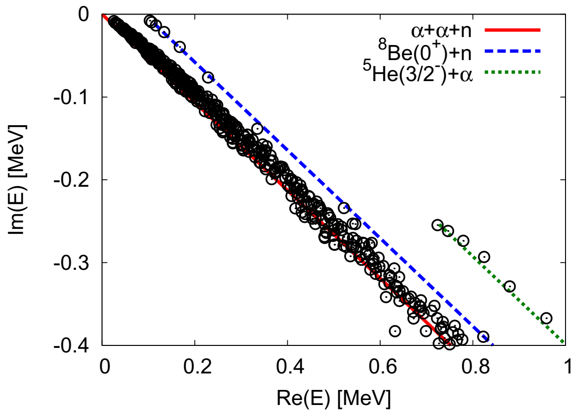

Figure 2. Distribution of energy eigenvalues of $1/2^+$ states solved with the strength $v_3 = -1.02$ MeV and scaling angle $\theta = 15$ degrees.

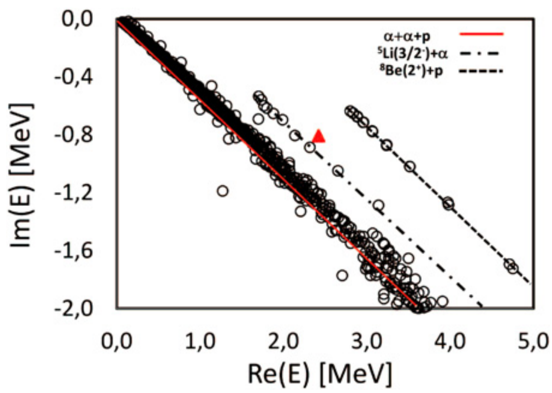

Figure 3. Distribution of energy eigenvalues of the ^{9}B$(1/2^+)$ state and the same parameters are used with Fig. 2.

continuum states. In the CSM, continuum states include the virtual states and broad resonance states, which are located on the second Riemann sheet covered by the rotated first Riemann sheet. The contributions from these states to the cross section are scattered into the continuum solutions obtained on the 2θ-lines. Decomposing the strength of the photodisintegration into each continuum solution on the different branch cuts, we find that the contribution from the ^{8}Be$(0^+) + n$ continuum solutions is dominant.

In the previous work [3], we further investigated the peak position of the cross section by changing the strength v_3 and obtained resonance poles below the ^{8}Be(0^+) + n threshold for $v_3 < -1.3$ MeV. When we take a more attractive potential of $v_3 < -1.8$, bound solutions of $1/2^+$ appear. From these analysis, we conclude that the ^{9}Be$(1/2^+)$ state is a virtual state of the ^{8}Be$(0^+) + n$ dominant configuration [3].

3.2. The first unbound state of ^{9}B

The ^{9}B$(1/2^+)$ state was observed in the ^{9}Be$({}^6$Li,^{6}He$){}^9$B charge exchange reaction [17] but another study [7] of the same reaction reported not finding it. A recent measurement using the ^{6}Li$({}^6$Li,t$){}^9$B reaction has reported a $1/2^+$ state at the excitation energy $E_x = 0.8 - 1.2$ MeV with the decay width $\Gamma > 1$ MeV [9]. Previous theoretical works [18–20] have predicted the energy with a decay width of the $1/2^+$ state by applying various types of approaches. The new calculation [20] suggests two close lying $1/2^+$ states with similar decay widths in ^{9}B. They reported that these two states have the different structures ^{8}Be(0^+) + p and ^{5}Li$(3/2^-)$ + α, respectively. This unsettled situation of the ^{9}B$(1/2^+)$ state motivates us to study the state in detail. In this calculation, we try to obtain this state based on our well-defined methodology considering its analog state in ^{9}Be [3] as well.

In Fig. 3, we show the distribution of the energy eigenvalues of the $1/2^+$ states calculated with $v_3 = -1.02$ MeV in the $\alpha + \alpha + p$ model. We find a resonance solution at the energy $E_{res} = 2.42$ MeV with a decay width $\Gamma = 1.61$ MeV, which is presented by a solid triangle in Fig. 3. In addition to the resonance solution, the $\alpha + \alpha + p$ three-body and ^{8}Be$(0^+) + p$ continuum solutions are obtained degenerately. Other two kinds of continuum solutions correspond to ^{5}Li$(3/2^-) + \alpha$ and ^{8}Be$(2^+) + p$ ones.

We investigate the consistency with the results of the ^{9}Be$(1/2^+)$ and the experiments of ^{9}B. First we calculate the ^{9}B$(1/2^+)$ state without three-body potential ($v_3 = 0$), and obtain $E_{res} = 2.59$ MeV with a decay width $\Gamma = 1.51$ MeV. This implies that the ^{9}B$(1/2^+)$ solution has a weak dependence of the three-body potential. The reason is considered to come from the additional Coulomb interaction due to the valence proton in the $\alpha + \alpha + p$ system. In order to confirm the different results in ^{9}Be and ^{9}B, we performed the calculation of ^{9}B by reducing the charge of the valence proton [21]. The resonance solutions for the $1/2^+$ state are obtained for the ranges of charge with $(1 - 0.5)e$. Unfortunately, the analytical continuation to zero charge was not succeeded to evaluate the solutions.

Garrido et al. [20] used the similar three-body model and obtained two resonance states in the ^{9}B$(1/2^+)$ state but there is no other theoretical work which supports the second state so far. In our case, we searched for the second state but we couldn't confirmed this state in the present calculation. The energy of the second resonance state in Ref. [20] is calculated 2.05 MeV with a decay width 1.6 MeV. The calculated energy is lower than the value we computed, but the decay width is the same in our estimation.

4. Summary

The structures of the first excited states of mirror ^{9}Be and ^{9}B nuclei are studied by using the $\alpha + \alpha + N$ three-body model and the complex scaling method. The first excited state of ^{9}Be is observed in photodisintegration cross sections showing a sharp peak just above the ^{8}Be(0^+) + n threshold. Reproducing this observed cross section, we discussed that the ^{9}Be$(1/2^+)$ has a virtual-state character of the s-wave neutron around the ^{8}Be(0^+) cluster. Its analog ^{9}B$(1/2^+)$ state is also calculated at the resonance energy $E_{res} = 2.42$ MeV with the decay width $\Gamma = 1.61$ MeV. Although there are difference in the obtained values of energy and decay width, the calculated result suggests the resonance energy higher than the recent measured values [9] and only one $1/2^+$ state differently from Ref. [20].

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