Amplification of short-wavelength radiation by relativistic electron beams moving near the impedance surfaces

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Electrodynamical properties of a wide class of surfaces can be described by introduction of an impedance boundary condition on the surface, namely, a linear relation binding the tangential components of the electric and magnetic fields. One of the most well-known is the Leontovich boundary condition valid at the surface of the lossy metal with its skin depth much smaller than the wavelength. Approximate impedance boundary conditions can also be used for description of periodical gratings with a period small in the scale of the wavelength. Impedance surfaces support evanescent waves with a phase velocity that can be smaller than the velocity of light, thus making it possible to provide their interaction with relativistic electron beams. Relativistic TWTs based on such mechanisms operate in centimeter wave bands [1]. In shorter wavelength bands, impedance systems also look viable due to the fact that the excitation of evanescent wave allow solving the transverse mode selection problem inevitable in oversized systems.
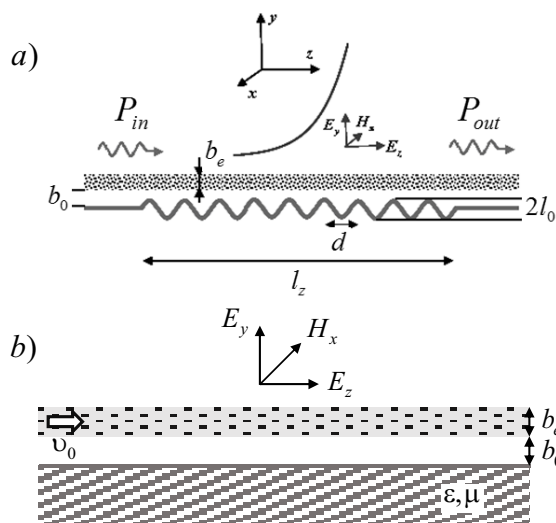

Fig. 1. Scheme of a surface-wave amplifier: an relativistic electron beam interacts synchronously with a decelerated surface wave near the corrugated impedance surface (a); amplification of the field in the beam near a resistive surface (b)

In the paper, we present the results of quasi-optical approach applied to the problem of electron beam excitation of high-frequency surface waves existing near the impedance surfaces. Two types of surfaces are taken into consideration, one of them being the lossy metal surface and the other is the ideal metal corrugation. For the case of shallow sine corrugation we obtain explicitly the expression for impedance based on the coupled wavebeam approach and the surface magnetic current technique [2].

We consider a relativistic electron beam moving in z direction and interacting with a co-moving incident

wavebeam near the impedance surface located at $y = 0$ with an impedance of χ. The radiation field can be presented via its magnetic component,

$$H_x = \text{Re}\left(H_x^\omega (z, y) e^{i(\omega t - kz)} \right),$$

where H_x^ω is the slowly varying amplitude.

The beam-wave interaction can be described within the 2D model by the self-consistent set of equations comprising the parabolic equation for the wavebeam supplemented by the equations of electron motion

$$\frac{\partial C}{\partial Z} + i \frac{\partial^2 C}{\partial Y^2} + i \hat\chi \delta(Y) C = i \frac{1}{B_e} \frac{\partial}{\partial Y}\left(F(Y) J \right), \quad (1)$$

$$\frac{\partial^2 \theta}{\partial Z^2} = \text{Re}\left(i \frac{\partial C}{\partial Y} e^{i\theta} - i \frac{1}{B_e}\left(F(Y) J \right) e^{i\theta} \right), \quad (2)$$

$$\theta\big|_{Z=0} = \theta_0 \in \left[0, 2\pi\right], \quad \frac{\partial\theta}{\partial Z}\bigg|_{Z=0} = \Delta, \quad (3)$$

where $J = \pi^{-1} \int_0^{2\pi} e^{-i\theta} d\theta_0$ is the amplitude of electron current. Wave amplitude at the input of the interaction space satisfies the boundary condition $C_{Z=0} = C_0(Y)$, where C_0 is the transverse profile of the incident field. Equations (1)-(3) are put down using the normalized variables $C = \sqrt{2} e\mu H_x^\omega / mc^2 \gamma_0 k G^{3/2}$, $Z = Gkz$ $Y = \sqrt{2G} ky$, $\hat\chi = \sqrt{2/G}\chi$, θ is the electron phase with respect to the wave, $\Delta = \left(1 - \beta_0\right)/G\beta_0$, $k = \omega/c$, $F(Y)$ is the transverse current distribution function. Normalization parameter (similar to Pierce parameter) has the following form:

$$G = \left(2\sqrt{2}\, \frac{eI_0}{mc^3} \frac{\mu}{\gamma_0} \lambda \right)^{\frac{2}{3}}.$$

In the case of the sine corrugation on the surface of the ideal metal (Fig. 1a), the impedance is determined by a relation [2]

$$\chi = 2\alpha^2 \left(\frac{k}{\sqrt{(k+\bar h)^2 - k^2}} + \frac{k}{\sqrt{(k-\bar h)^2 - k^2}} \right), \quad (4)$$

where $\bar h = 2\pi/\bar d$ is the lattice wavenumber and $\alpha = l_0 \bar h /8$ is the coupling coefficient proportional to the corrugation depth l_0.

If the impedance boundary conditions are set on the surface of a conductor (Fig.1b), than the impedance is determined by Leontovich formulas as

$$\chi = iw/2, \quad w = \sqrt{\mu/\varepsilon}. \quad (5)$$

Here w is the wave resistance of the material. For the case of impedance corrugation, χ is purely real, while for the lossy case, χ is complex, and there is an energy flow directed inside the metal.

In the case when the corrugated material is non-ideal metal with high conductivity, surface impedance would be a sum of (4) and (5).

For the electron beam with the width of b_e moving at the distance of b_0 from the surface, a linear theory can be constructed and a dispersion equation can be obtained. We have shown that in both cases (***a*** and ***b*** in Fig. 1) only one root of the dispersion equation satisfies the requirements on wave evanescence and energy flow direction. Thus, only one regime of amplification can be realized. In this regime, in the case of corrugated surface, the slow evanescent wave can be amplified using the Cherenkov mechanism of interaction while in the case of lossy surface, the dissipative instability takes place leading to amplification of the spatial charge fields of the beam. This conclusion is illustrated by Fig. 2 where the transverse distribution of electric and magnetic field is presented on the linear stage of amplification for the case of the resistive instability (Fig. 2a,b). If the absolute value of impedance tends to zero or the distance between the beam and the surface increases, the fields tend to the intrinsic field of the beam localized near it. At the same time, for purely real impedance (case *a* in Fig.1), the electric field tends to evanescent profile confined at the corrugation (Fig.2 c,d).
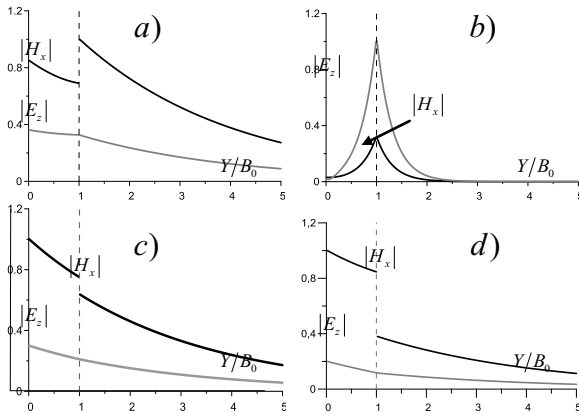
Fig. 2. Transverse distributions of electric and magnetic field amplitudes at the linear stage of instability at $\Delta = 9$, $\sqrt{2Gk}b_0 = 0.1$, $b_e = 0$, $\hat{\chi} = 3 - 3i$ (a), $\hat{\chi} = 0.5(1-i)$ (b), $\hat{\chi} = 3$ (c), $\hat{\chi} = 2$ (d)

Based on the nonlinear theory given above, let us analyze the possibility of realization of the relativistic sub-mm range amplifier of surface waves based on the corrugated metallic surface operating at 0.9 mm wavelength. We take the electrons energy to be of 1 MeV, linear injection current density of 1.2 kA/cm, corrugation period of 0.25 mm, corrugation depth of 0.136 mm, the distance between the corrugation and the beam of 0.1 mm, and beam thickness 0.2 mm. These values corres-

pond to the following dimensionless parameters: $G = 8.8 \times 10^{-3}$, $\sqrt{2Gk}b_0 = 0.1$, $\sqrt{2Gk}b_e = 0.2$, $\hat{\chi} = 3.5$, $\Delta = 7$. For the linear input power density of 50 kW/cm, the output power density of 550 MW/cm (which corresponds to an amplification factor of up to 40 dB and an efficiency of 45%) was reached at the optimal length of about 6 cm. Spatial distribution of the magnetic field amplitude of the amplified field is presented in Fig. 3 together with the dependence of integral radiation power on the z-coordinate.
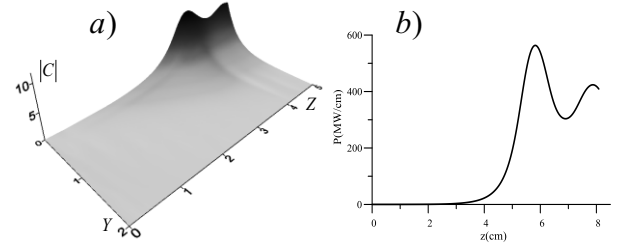
Fig. 3. Spatial distribution of the normalized amplitude of the amplified wave (a) and z dependence of power density (b) in the case of purely real impedance

We also have undertaken simulations of the dissipative amplifier operating in THz frequency range at a wavelength of 0.3 mm. We considered a relativistic electron beam with following parameters: particles energy of 1 MeV, linear current density of 1.3 kA/cm, width of electron beam of 0.1 mm and the distance between the beam and the resistive surface of 0.1 mm. corresponding to the normalized values: $G = 4.3 \times 10^{-3}$, $\sqrt{2Gk}b_0 = 0.2$, $\sqrt{2Gk}b_e = 0.2$, $\hat{\chi} = 3.35 - 0.3i$, $\hat{\Delta} = 14$. The output power density in saturation regime is 10.6 MW/cm with a power gain of up to 28 dB (for a power density of the input signal of about 16 kW/cm). Saturation is reached at a length of about 7 cm. Wave and electron efficiencies are rather small (0.8% and 4%, correspondingly); nevertheless, absolute value of the output power is of interest for THz wavelength range.

References

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