The connection between period spectra and constraints in white dwarf asteroseismology

Agnès Bischoff-Kim

Penn State Worthington Scranton, Dunmore, PA 18512, USA

Abstract. White dwarfs are the end product of evolution for around 98% of the stars in our Galaxy. Buried in their interiors are the records of physical processes that take place during earlier stages in the life of the star. In recent years, a well-established theory of non-radial oscillations, improved white dwarf models, year of expertise built up in the field of white dwarf asteroseismic fitting, and computing power have culminated in the asteroseismology finally delivering what it promised: a detailed map of the interior structure of white dwarfs. As always in science, new results raise new questions. We perform a number of numerical experiments to better understand the connection between a given set of periods varying in the number of periods and in the set of radial overtones and the quality of the constraints on interior structure one obtains from fitting these periods.

1 Introduction

A few rules of thumb are used in white dwarf asteroseismology: 1) The number of parameters should be of the same order and no greater than the number of periods fitted, and 2) Lower radial overtones (low $k$ modes) carry information about the core while higher $k$ modes do not. The fact that these are only rules of thumb was most recently made clear when two stars each with 6 observed lower $k$ modes (R458 and GD165) were analyzed side by side ([1]). The amount of constraint on core structure and other parameters varied greatly between the two stars, even though their pulsation spectra were similar. This made evident the fact that not all low $k$ modes are created equal. In this work, we investigate the connection between period spectra and constraints.

2 Numerical experimental setup

We picked a fiducial model similar to the best fit model of Giannichele et al. (2016, [1]); that is, a hydrogen layer mass white dwarf of average stellar mass, inside the ZZ Ceti instability strip. The properties of the model are listed in Table 1. For that fiducial model we computed a set of ($\ell = 1$) modes, then went on to sample that list as if we were observing actual stars, which often only show a few of the possible excited modes. We selected 5 modes, using weight functions to guide our choices. Weight functions determine where in the model a mode is strongly trapped ([2]). For our first fit, we specifically selected modes that appeared strongly trapped in the core, while for the second fit, we selected lower $k$ modes that were not strongly trapped in the core.

We fit the chosen periods of the fiducial model to models in an extensive grid of DAV models ([3]). For each fit, we calculate the fitness function
Table 1. Properties of the fiducial model. $X_0$ is the central oxygen abundance and $X_{fm}$ is the location of the edge of the homogeneous C/O core. For more details on how parameters are defined, see [3].

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<table>
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<tbody>
<tr>
<td>$T_{\text{eff}}$</td>
<td>12,000 K</td>
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<tr>
<td>$M_{\text{H}}$</td>
<td>$-4.80$</td>
</tr>
<tr>
<td>Mass</td>
<td>0.595 $M_\odot$</td>
</tr>
<tr>
<td>$X_0$</td>
<td>0.70</td>
</tr>
<tr>
<td>$M_{\text{He}}$</td>
<td>$-2.20$</td>
</tr>
<tr>
<td>$X_{fm}$</td>
<td>0.45 $M_\odot$</td>
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![Figure 1](image1.png)

Figure 1. Dependence of each mode on the central oxygen abundance. Each mode is labeled with its radial overtone number. The vertical dashed line marks the value set for the fiducial model. **left**: Fit strongly sensitive to core structure. **right**: Fit not very sensitive to core structure.

\[ \sigma_{\text{RMS}} = \sqrt{\frac{1}{n_{\text{obs}} \sum_{i=1}^{n_{\text{obs}}}} \left( P_{i}^{\text{calc}} - P_{i}^{\text{obs}} \right)^2}, \]  

where in this case $P_{i}^{\text{obs}}$ refers to the periods of the fiducial model, while $P_{i}^{\text{calc}}$ refers to the periods of each model in the grid. Evidently, when we match the fiducial model to itself in the grid, we get $\sigma_{\text{RMS}} = 0$ s.

3 Results

We find that indeed, when choosing modes that are strongly trapped in the core for our fitting, we obtain tight constraints on core structure, while when we specifically choose modes that show weak trapping in the core, we obtain very little constraints on core structure. While this is a nice demonstration of the usefulness of weight functions, this result also shows strikingly that even very low $k$ modes can be of little help in constraining core structure. This is best illustrated in Figure 1, where we show how sensitive each individual mode is to the central abundance of oxygen in the model.

References