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Semileptonic decays of B_c mesons into charmonium states

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Abstract. In this work we study the semileptonic decays of B_c meson. We evaluated $B_c \to D(D^*)$, $B_c \to D_s(D_s^*)$ and $B_c \to \eta_c(J/\psi)$ transitions form factors in the full kinematical region within the covariant quark model. The calculated form factors are used to evaluate the semileptonic decays of B_c meson and it was defined ratios (R_{η_c}, $R_{J/\psi}$, R_D ,R_{D^*}) of the branching ratios, which will be hopefully tested on LHC experiments.We compare the obtained results with the results from other theoretical approaches.

Model

The covariant quark model was developed by G.V.Efimov and M.A.Ivanov [1–3].

The effective Lagrangian describing the transition of a meson $M(q_1\bar{q}_2)$ to its constituent quarks q_1 and $\bar{q}_2$ in model looks like

$$\begin{aligned} \mathcal{L}_{\rm int}(x) &= g_M M(x) \cdot J_M(x) + \text{h.c.}, \\ J_M(x) &= \int dx_1 \int dx_2 F_M(x, x_1, x_2) \bar{q}_2(x_2) \Gamma_M q_1(x_1), \end{aligned} \tag{1}$$

with Γ_M a Dirac matrix which projects onto the spin quantum number of the meson field $M(x)$. The vertex function F_M characterizes the finite size of the meson. Translational invariance requires the function F_M to fulfill the identity $F_M(x + a, x_1 + a, x_2 + a) = F_M(x, x_1, x_2)$ for any four-vector a. A specific form for the vertex function is adopted

$$F_M(x, x_1, x_2) = \delta(x - w_1 x_1 - w_2 x_2) \Phi_M((x_1 - x_2)^2), \tag{2}$$

where Φ_M is the correlation function of the two constituent quarks with masses m_{q_1}, m_{q_2} and the mass ratios $w_i = m_{q_i}/(m_{q_1} + m_{q_2})$.

A simple Gaussian form of the vertex function $\bar{\Phi}_M(-k^2)$ is selected

$$\bar{\Phi}_M(-k^2) = \exp\left(k^2/\Lambda_M^2\right) \tag{3}$$

with the parameter Λ_M linked to the size of the meson. The minus sign in the argument is chosen to indicate that we are working in the Minkowski space. Since k^2 turns into $-k_E^2$ in the Euclidean space, the form (3) has the appropriate fall-off behavior in the Euclidean region. Any choice for Φ_M

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is appropriate as long as it falls off sufficiently fast in the ultraviolet region of the Euclidean space to render the corresponding Feynman diagrams ultraviolet finite. We choose a Gaussian form for calculational convenience.

The fermion propagators for the quarks are given by

$$S_i(k) = \frac{1}{m_{q_i} - \not{k}} \tag{4}$$

with an effective constituent quark mass m_{q_i}.

The so-called *compositeness condition* [4, 5] is used to determine the value of the coupling constants g_M. It means that the renormalization constant Z_M of the elementary meson field $M(x)$ is to be set to zero, i.e.,

$$Z_M \ = \ 1 - \frac{3g_M^2}{4\pi^2}\,\tilde{\Pi}'_M(m_M^2) \ = \ 0, \tag{5}$$

where $\tilde{\Pi}'_M$ is the derivative of the meson mass operator. Its physical meaning in Eq. (5) becomes clear when interpreted as the matrix element between the physical and the corresponding bare state: $Z_M = 0$ implies that the physical state does not contain the bare state and is appropriately described as a bound state. The interaction makes the physical particle dressed, i.e. its mass and wave function have to be renormalized. The condition $Z_M = 0$ also effectively excludes the constituent degrees of freedom from the space of physical states. It thereby guarantees the absence of double counting for the physical observable under consideration, the constituents exist only in virtual states. The tree-level diagram together with the diagrams containing self-energy insertions into the external legs (i.e. the tree-level diagram times $Z_M - 1$) give a common factor Z_M which is equal to zero.

The mass functions for the pseudoscalar meson (spin $S = 0$) and vector meson (spin $S = 1$) are defined as

$$\Pi_P(x-y) \ = \ +i\,\langle T\{J_P(x)J_P(y)\}\rangle_0, \tag{6}$$

$$\Pi_V^{\mu\nu}(x-y) \ = \ -i\,\langle T\{J_V^{\mu}(x)J_V^{\nu}(y)\}\rangle_0. \tag{7}$$

Herein we use the updated values of the model parameters from [6] which are shown in Eq. (8,9).

$m_{u/d}$	m_s	m_c	m_b	λ	
0.241	0.428	1.67	5.05	0.181	GeV

(8)

Λ_{B_c}	Λ_{η_c}	$\Lambda_{J/\psi}$	Λ_D	Λ_{D^*}	Λ_{D_s}	$\Lambda_{D_s^*}$	Λ_B	Λ_{B^*}	Λ_{B_s}	
2.73	3.97	1.74	1.6	1.53	1.75	1.56	1.96	1.8	2.05	GeV

(9)

Semileptonic decays

We give the necessary definitions of the leptonic decay constants, invariant form factors and helicity amplitudes. The leptonic decay constants are defined by

$$M(H_{12} \to \bar{l}\nu) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \mathcal{M}^\mu_H(p)\, \bar{u}_l(k_l)\, O^\mu\, u_\nu(k_\nu),$$

$$\mathcal{M}^\mu_H(p) = -3\, g_{12} \int \frac{d^4k}{(2\pi)^4 i} \widetilde{\Phi}_{12}\left(-k^2\right) \mathrm{tr}\left[\Gamma_H\, \widetilde{S}_2(k - c^2_{12}\, p)\, O^\mu\, \widetilde{S}_1(k + c^1_{12}\, p)\right],$$

$$\Gamma_P = i\gamma^5, \;\; \Gamma_V = \varepsilon_V \cdot \gamma,$$

$$\mathcal{M}^\mu_P(p) = -i f_P p^\mu, \qquad \mathcal{M}^\mu_V(p) = f_V m_V \varepsilon^\mu_V. \tag{10}$$

The semileptonic decays of the B_c-meson may be induced by a b-quark transition.

$$\begin{aligned} M(H_{13} \to H_{23} + \bar{l}\nu) &= \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \mathcal{M}^\mu_{12}(p_1, p_2)\, \bar{u}_l(k_l)\, O^\mu\, u_\nu(k_\nu), \\ \mathcal{M}^\mu_{12} &= -3\, g_{13}\, g_{23} \int \frac{d^4k}{(2\pi)^4 i} \widetilde{\Phi}_{13}\left(-(k + c^3_{13}\, p_1)^2\right) \widetilde{\Phi}_{23}\left(-(k + c^3_{23}\, p_2)^2\right) \\ &\times \mathrm{tr}\left[\, i\gamma^5\, \widetilde{S}_3(k)\, \Gamma_{32}\, \widetilde{S}_2(k + p_2)\, O^\mu\, \widetilde{S}_1(k + p_1)\,\right], \\ &\times \mathrm{tr}\left[\, i\gamma^5\, \widetilde{S}_3(k - p_1)\, O^\mu\, \widetilde{S}_2(k - p_2)\Gamma_{21}\, \widetilde{S}_1(k)\,\right], \end{aligned} \tag{11}$$

where $q_1 \equiv b$ and $q_3 \equiv c$ whereas q_2 denotes either of c, u, d, s.

The invariant form factors for the semileptonic B_c-decay into the hadron with spin $S = 0, 1$ are defined by

$$\mathcal{M}^\mu_{S=0} = P^\mu\, F_+(q^2) + q^\mu\, F_-(q^2), \tag{12}$$

$$\mathcal{M}^\mu_{S=1} = \frac{1}{m_1 + m_2}\, \epsilon^\dagger_\nu \left\{ -\, g^{\mu\nu}\, Pq\, A_0(q^2) + P^\mu\, P^\nu\, A_+(q^2) + q^\mu\, P^\nu\, A_-(q^2) + i\, \varepsilon^{\mu\nu\alpha\beta}\, P_\alpha\, q_\beta\, V(q^2) \right\}, \tag{13}$$

$$P = p_1 + p_2, \qquad q = p_1 - p_2.$$

It is convenient to express all physical observables through the helicity form factors H_m. The helicity form factors H_m can be expressed in terms of the invariant form factors in the following way [7]:

(a) Spin $S = 0$:

$$\begin{aligned} H_t &= \frac{1}{\sqrt{q^2}} \left\{ (m_1^2 - m_2^2)\, F_+ + q^2\, F_- \right\}, \\ H_\pm &= 0\,, \\ H_0 &= \frac{2\, m_1\, |\mathbf{p_2}|}{\sqrt{q^2}}\, F_+\,. \end{aligned} \tag{14}$$

(b) Spin $S = 1$:

$$
\begin{aligned}
H_t &= \frac{1}{m_1+m_2}\frac{m_1\,|\mathbf{p_2}|}{m_2\sqrt{q^2}}\left\{(m_1^2-m_2^2)\,(A_+-A_0)+q^2A_-\right\},\\
H_\pm &= \frac{1}{m_1+m_2}\left\{-(m_1^2-m_2^2)\,A_0\pm 2\,m_1\,|\mathbf{p_2}|\,V\right\},\\
H_0 &= \frac{1}{m_1+m_2}\frac{1}{2\,m_2\sqrt{q^2}}\left\{-(m_1^2-m_2^2)\,(m_1^2-m_2^2-q^2)\,A_0+4\,m_1^2\,|\mathbf{p_2}|^2\,A_+\right\}.
\end{aligned}
\tag{15}
$$

where $|\mathbf{p_2}| = \lambda^{1/2}(m_1^2, m_2^2, q^2)/(2\,m_1)$ is the momentum of the outgoing particles in the B_c rest frame. The semileptonic B_c-decay widths are given by

$$
\begin{aligned}
\Gamma(B_c^-\to M_{\bar{c}c}\,l\bar{\nu}) &= \frac{G_F^2}{(2\,\pi)^3}|V_{cb}|^2\int_{m_l^2}^{q_-^2}dq^2\,\frac{(q^2-m_l^2)^2\,|\mathbf{p}_2|}{12\,m_1^2\,q^2}\\
&\times\left\{\left(1+\frac{m_l^2}{2\,q^2}\right)\sum_{i=\pm,0}\left(H_i^{B_c\to M_{\bar{c}c}}(q^2)\right)^2+\frac{3\,m_l^2}{2\,q^2}\left(H_t^{B_c\to M_{\bar{c}c}}(q^2)\right)^2\right\},\\
\Gamma(B_c^-\to \overline{D}^0\,l\bar{\nu}) &= \frac{G_F^2}{(2\,\pi)^3}|V_{ub}|^2\int_{m_l^2}^{q_-^2}dq^2\,\frac{(q^2-m_l^2)^2\,|\mathbf{p}_2|}{12\,m_1^2\,q^2}\\
&\times\left\{\left(1+\frac{m_l^2}{2\,q^2}\right)\sum_{i=\pm,0}\left(H_i^{B_c\to \overline{D}^0}(q^2)\right)^2+\frac{3\,m_l^2}{2\,q^2}\left(H_t^{B_c\to \overline{D}^0}(q^2)\right)^2\right\},
\end{aligned}
$$

where $q^2 = (m_1 - m_2)^2$, $m_1 \equiv m_{B_c}$, and $m_2 \equiv m_f$. Note that $M_{\bar{c}c}$ and $\overline{D}^0$ denote both the pseudoscalar and vector cases.

Numerical results

We take the following values (16) of the meson masses and the B_c-meson's lifetime from the PDG [8].

m_{B_c}	m_{η_c}	$m_{J/\psi}$	m_D	m_{D^*}	m_{D_s}	$m_{D_s^*}$		τ_{B_c}
6.275	2.984	3.097	1.869	2.010	1.968	2.112	GeV	$0.507\,ps$

(16)

The calculation of the semileptonic decay widths is straightforward. For the CKM-matrix elements we use

$\lvert V_{ud}\rvert$	$\lvert V_{us}\rvert$	$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert$
0.974	0.225	0.220	0.995	0.0405	0.00409

(17)

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The value of the decay constant f_{η_c} was calculated from the branching ratio for the η_c meson decay into two photons using the last data [8]. The quality of the fit may be assessed from the entries in Table 1.

The form factors are calculated in the full kinematical region of momentum transfer squared and are shown in Table 2. The curves are depicted in Fig. 1 and Fig. 2.

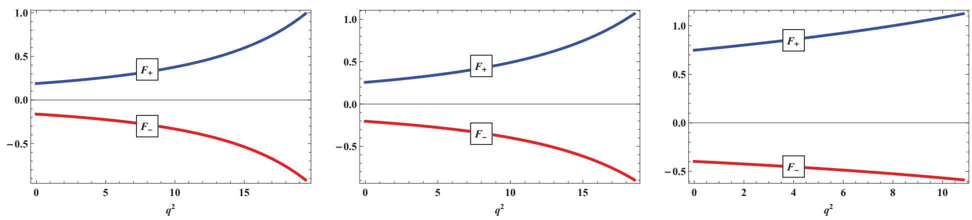

Figure 1. The $F_+(q^2)$ and $F_-(q^2)$ form factors for $B_c \to D, B_c \to D_s$ and $B_c \to \eta_c$ transitions, respectively.

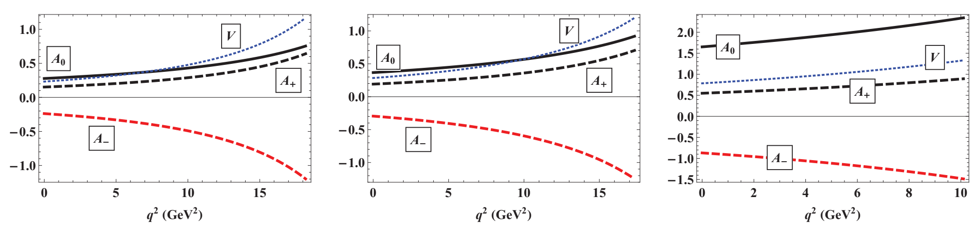

Figure 2. The A_0, A_-, A_+ and V form factors for $B_c \to D^*$, $B_c \to D_s^*$ and $B_c \to J/\psi$ transitions, respectively.

Table 1. Leptonic decay constants f_H (MeV).

	This work	Other	Ref.
f_{B_c}	489	$489 \pm 4 \pm 3$	LAT [9]
		395 ± 15	[10]
f_D	206	$222.6 \pm 16.7^{+2.8}_{-3.4}$	CLEO [11]
		$201 \pm 3 \pm 17$	MILC LAT [12]
		$235 \pm 8 \pm 14$	LAT [13]
		$210 \pm 10^{+17}_{-16}$	UKQCD LAT [14]
		$211 \pm 14^{+2}_{-12}$	LAT [15]
		204.6 ± 5.0	PDG [16]
f_{D^*}	244	$245 \pm 20^{+3}_{-2}$	LAT [15]
		$278 \pm 13 \pm 10$	LAT [17]
		$252.2 \pm 22.3 \pm 4$	QCD SR [18]
f_{D_s}	257	257.5 ± 4.6	PDG [16]
		$249 \pm 3 \pm 16$	MILC LAT [12]
		$266 \pm 10 \pm 18$	LAT [13]
		$290 \pm 20 \pm 29 \pm 29 \pm 6$	LAT [19]
		$236 \pm 8^{+17}_{-14}$	UKQCD LAT [14]
		$231 \pm 12^{+8}_{-1}$	LAT [15]
$f_{D_s^*}$	272	311 ± 9	LAT [17]
		$272(16)^{+3}_{-20}$	LAT [15]
		$305.5 \pm 26.8 \pm 5$	QCD SR [18]
$\frac{f_{D_s}}{f_D}$	1.25	1.258 ± 0.038	PDG [16]
		$1.24 \pm 0.01 \pm 0.07$	MILC LAT [12]
		$1.13 \pm 0.03 \pm 0.05$	LAT [13]
		$1.13 \pm 0.02^{+0.04}_{-0.02}$	UKQCD LAT [14]
		1.10 ± 0.02	LAT [15]
f_{η_c}	628	420 ± 52	[20]
		337.7 ± 18.2	pQCD[21]
$f_{J/\psi}$	415	405 ± 14	pQCD[22]
		416.2 ± 7.4	pQCD[21]

Table 2. Form factors for $B_c \to D(D^*)$, $B_c \to D_s(D_s^*)$ and $B_c \to \eta_c(J/\psi)$ transitions. Form factors are *approximated* by the form $F(q^2) = F(0)/(1 - a\,\hat{s} + b\,\hat{s}^2)$ with $\hat{s} = q^2/m_{B_c}^2$.

	$B_c \to D(D^*)$	$B_c \to D_s(D_s^*)$	$B_c \to \eta_c(J/\psi)$
$F_+(0)$	0.186	0.254	0.74
$F_-(0)$	-0.160	-0.202	-0.39
$A_0(0)$	0.276	0.365	1.65
$A_+(0)$	0.151	0.190	0.55
$A_-(0)$	-0.236	-0.293	-0.87
$V(0)$	0.230	0.282	0.78

Table 3. Branching ratios (in %) of semileptonic B_c decays into ground state charmonium states.

Mode	This work	[23]	[7]	[24, 25]	[26]	[27]	[28]
$B_c^- \to \eta_c \ell \nu$	0.95	0.81	0.98	0.75	0.97	0.59	0.44
$B_c^- \to \eta_c \tau \nu$	0.24	0.22	0.27	0.23		0.20	0.14
$B_c^- \to J/\psi \ell \nu$	1.67	2.07	2.30	1.9	2.35	1.20	1.01
$B_c^- \to J/\psi \tau \nu$	0.40	0.49	0.59	0.48		0.34	0.29
$B_c^- \to \overline{D}^- \ell \nu$	0.0033	0.0035	0.018		0.004	0.006	0.0032
$B_c^- \to \overline{D}^- \tau \nu$	0.0021	0.0021	0.0094	0.002			0.0022
$B_c^- \to \overline{D}^{*-} \ell \nu$	0.006	0.0038	0.034		0.018	0.018	0.011
$B_c^- \to \overline{D}^{*-} \tau \nu$	0.0034	0.0022	0.019	0.008			0.006

Table 4. Ratios of semileptonic decays of the B_c meson

Decay rate	This work	[23]	[28]
$R_{\eta_c} = \frac{B_c^- \to \eta_c \ell \nu}{B_c^- \to \eta_c \tau \nu}$	3.96	3.68	3.2
$R_{J/\psi} = \frac{B_c^- \to J/\psi \ell \nu}{B_c^- \to J/\psi \tau \nu}$	4.18	4.22	3.4
$R_D = \frac{B_c^- \to D^- \ell \nu}{B_c^- \to D^- \tau \nu}$	1.57	1.67	1.42
$R_{D^*} = \frac{B_c^- \to D^{*-} \ell \nu}{B_c^- \to D^{*-} \tau \nu}$	1.76	1.72	1.66

The results of our evaluation of the branching ratios of the semileptonic B_c decays appear in Table 3, which contains our predictions for the semileptonic B_c decays into ground state charmonium states and charm meson states. We compare our ratios of semileptonic decays of the B_c meson with those of other models in Table 4.

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