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Annihilation-Type Semileptonic B-Meson Decays

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Abstract. The rare semileptonic decay $B^0 \to \phi\ell^+\ell^-$, where ℓ is an electron or muon, is considered in the QCD Factorization approach. The differential branching fraction, partially integrated over the interval $q^2 = \left[1\ \mathrm{GeV}^2, 8\ \mathrm{GeV}^2\right]$ of dilepton invariant mass squared, is estimated. The systematic uncertainty up to 10% resulting from the choice of the B-meson distribution amplitude is determined. The purely perturbative contribution to the $B^0 \to \phi\ell^+\ell^-$ total branching fraction is found to be $\mathcal{B} \sim 10^{-12}$ which suggests searches of the $B^0 \to \phi\mu^+\mu^-$ decay in the four-muon events at the LHC collider.

1 Introduction

Heavy quark physics, in particular related to the bottom (or b) quark, has been an important aspect of the high energy physics research carried out in experiments at hadron colliders at CERN and Fermilab and at the so-called B-meson factories operated at SLAC and KEK. Currently, the LHCb Collaboration working at the CERN proton-proton collider LHC is at the experimental forefront in flavor physics. These researches will be set forth at the Super-KEKB factory at KEK which is close to start its operation now. From a theoretical point of view, one of basic research goals in B-meson physics is a search for physics beyond the Standard Model (BSM) in rare B-meson decays. Here, the main question is whether all experimental observations can be consistently described in the context of the Standard Model. This requires a controlled theoretical framework to understand the strong interaction aspects of flavor physics.

In the heavy quark limit, $m_b \to \infty$, matrix elements of the four-quark operators entering the Effective Weak Hamiltonians [1–4] can be factorized in terms of theoretically more tractable hadronic quantities, then the problem can, in principle, be solved. Of course, one has still to calculate the factorized parts of the decay amplitudes using the QCD perturbation theory (see, for example, [5, 6]) and non-perturbative techniques, such as the QCD Sum Rules and Lattice QCD. In some cases, factorization theorems have been rigorously proved, and in these cases one has, in principle, a well-defined and controlled theory for heavy-hadron decays [1–4].

Phenomenologically, the factorization framework works remarkably well in accounting for the observed non-leptonic decays involving the current-current operators, inducing the $b \to c$ transition [7]. The $B \to h_1 h_2$ decays induced by the charged $b \to u$ and neutral $b \to s$ and $b \to d$ transitions [1–4] are more complex as they involve, in general, in addition to the current-current contributions, also the loop-induced QCD- and electroweak-penguin amplitudes. Here, h_1 and h_2 are light hadrons, such as π-, K-, η-, η'-, ρ-, ω-, ϕ-, and K^*-mesons. During two last decades, these decays have been studied

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in two specific factorization methods, the so-called perturbative QCD (pQCD) approach by Keum, Li and Sanda [8, 9] and the generalized QCD factorization approach by Beneke, Buchalla, Neubert and Sachrajda (BBNS) [10, 11]. A difference between the pQCD and the BBNS approaches is now understood in terms of the so-called Sudakov behavior of the non-leptonic decay amplitudes, required to alleviate some of the technical problems from the end-point regions of momentum distributions in the perturbative QCD kernels. An outgrowth of this work is the Soft-Collinear Effective Theory (SCET) [12–16], which is developed to deal with problems having different scales, such as obtained by separating the soft, semi-hard and hard regions in decay processes.

Precise data from the B-factory experiments BELLE and BaBar in the decays $B \to \pi\pi$, πK, KK, ηK, $\eta' K$, and related decays in which one of the pseudoscalar particles is replaced by a vector particle, show several mismatches between the predictions of the theories based on the QCD factorization and measurements (see, for example, [17]). This has been the outcome of several theoretical investigations using data from the B-factories.

Within the QCD Factorization approach, the phenomenological analysis of the $B^\pm \to \pi^\pm \ell^+ \ell^-$ decays, where ℓ is an electron or muon, has been completed [18, 19]. These decays are very similar to the exclusive $B \to (K, K^*)\, \ell^+ \ell^-$ decays which have received a lot of theoretical attention in the past and are under intensive experimental and phenomenological analysis at present (see [20, 21]). The theory of the $B \to (K, K^*)\, \ell^+ \ell^-$ processes — decay rates and a number of distributions — has been developed to the next-to-next-to-leading logarithmic (NNLL) accuracy in the SM [20, 21]. The decay rates have recently been measured by the experiments at the B-factories and the LHC, and are found to be basically in agreement with the corresponding SM-based theoretical estimates. However, the CKM-suppressed semileptonic decays involving the $b \to d\ell^+\ell^-$ transition at the quark level, which can be measured in the exclusive processes such as $B \to (\pi, \eta, \rho, \omega)\, \ell^+ \ell^-$ and their $SU(3)_F$-symmetric analogues $B_s \to (K, K^*)\, \ell^+ \ell^-$, have not yet been studied to the same theoretical accuracy. On the experimental front, there exist a measurement of the $b \to d\ell^+\ell^-$ transition in the $B^\pm \to \pi^\pm \mu^+ \mu^-$ decay presented by the LHCb collaboration first as the total branching ratio $\mathcal{B}(B^\pm \to \pi^\pm \mu^+ \mu^-) = (2.3 \pm 0.6 \pm 0.1) \times 10^{-8}$ [22] which was updated on the larger statistics to the smaller but consistent with the previous one value $\mathcal{B}(B^\pm \to \pi^\pm \mu^+ \mu^-) = (1.83 \pm 0.24 \pm 0.05) \times 10^{-8}$ [23] and the first measurement of the direct CP-asymmetry $\mathcal{A}_{\rm CP}(B^\pm \to \pi^\pm \mu^+ \mu^-) = -0.11 \pm 0.12 \pm 0.01$ [23] was also presented. Our prediction for the branching fraction of the $B^\pm \to \pi^\pm \mu^+ \mu^-$ decay coincides with the LHCb measurement [22] within large experimental errors. The $B^+ \to \pi^+ \mu^+ \mu^-$ dilepton invariant-mass spectrum was measured by the LHCb collaboration recently [23] and is also in a good agreement with theoretical predictions [18, 19] except the first bin $q^2 < 1$ GeV^2. This part of the spectrum is sizably enhanced by the long-distance contributions from the two-body B-meson decays like $B \to \pi(\rho, \omega, \phi)$ and was worked out in [24] which correct our predictions. The predictions for the $B^0 \to \pi^0 \ell^+ \ell^-$ decay also presented by us [18, 19] can be precisely tested at the forthcoming Super-KEKB factory at KEK.

There exist more rare Semileptonic and radiative decays of B-mesons which are of the annihilation type and consequently they are not observed at present. Examples of pure annihilation-type decays are as follows: $B^0 \to \phi \ell^- \ell^+$, $B_s^0 \to \rho^0 \ell^- \ell^+$, $B_s^0 \to \omega \ell^- \ell^+$, $B^0 \to \phi\gamma$, $B_s^0 \to \rho^0\gamma$, and $B_s^0 \to \omega\gamma$. On the best of our knowledges, there are experimental restrictions on one decay mode only: $\mathcal{B}(B^0 \to \phi\gamma) < 8.5 \times 10^{-7}$ at 90% CL, by the BaBar Collaboration [25] which has been superseded by the more stringent upper limit: $\mathcal{B}(B^0 \to \phi\gamma) < 1.0 \times 10^{-7}$, obtained by the Belle Collaboration [26] recently.

In this paper, we present some results concerning the analysis of the rare semileptonic $B^0 \to \phi \ell^- \ell^+$ decays obtained within the QCD Factorization approach. This is the extended version of [27] and the details will be published elsewhere [28]. This decay is can be experimentally searched by the LHCb Collaboration at the LHC collider and by the Belle-II Collaboration at the forthcoming Super-KEKB factory at KEK.

2 Theory of Rare Semileptonic Annihilation-Type B-Meson Decays

From theoretical side, the description of such decays is convenient to do within the method of Effective Weak Hamiltonians [1] when the heavy degrees of freedom — top-quark, W- and Z-boson and Higgs boson are integrated out as well as photons and gluons of energies larger than the W-boson mass. The corresponding effective Lagrangian consists of the usual QED and QCD Lagrangians responsible for the electromagnetic and strong interactions in the effective theory and the effective Lagrangian of the neutral $b \to s$ or $b \to d$ transitions [1]:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED} + \mathcal{L}_{\rm weak}^{b\to s} + \mathcal{L}_{\rm weak}^{b\to d}. \tag{1}$$

For us, the later one is of interest for its application to the $B \to \phi\ell^+\ell^-$ decay [1]:

$$\mathcal{L}_{\rm weak}^{b\to d} = -\frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \sum_j C_j(\mu)\mathcal{P}_j(\mu) + \text{h. c.}, \tag{2}$$

where G_F is the Fermi constant, $\lambda_p^{(d)} = V_{p,d}^* V_{p,b}$ is the product of the elements of the Cabibbo-Kobayashi-Maskawa matrix, and $O_j(\mu)$ is a set of local operators accompanied by the corresponding Wilson coefficients $C_j(\mu)$. The local operators can be divided into several groups [1]: tree, strong- and electroweak-penguins, electromagnetic and chromomagnetic dipole and semileptonic operators. For the decay $B \to \phi\ell^+\ell^-$, we are interested in the strong-penguin, electromagnetic dipole and semileptonic operators which give non-trivial contributions to the decay amplitude

In calculations we adopt the following set of operators [29]:

Tree operators ($p = u, c$):

$$\mathcal{P}_1^{(p)} = (\bar{d}\gamma_\mu L T^a p)(\bar{p}\gamma^\mu L T^a b), \quad \mathcal{P}_2^{(p)} = (\bar{d}\gamma_\mu L p)(\bar{p}\gamma^\mu L b), \tag{3}$$

where $L = (1-\gamma_5)/2$ is the left-handed projection of the fermionic fields and T^a ($a = 1, \dots, 8$) are the generators of the color $SU(3)_C$-group in the fundamental representation [6].

QCD-penguin operators:

$$\mathcal{P}_3 = (\bar{d}\gamma_\mu L b) \sum_q (\bar{q}\gamma^\mu q), \quad \mathcal{P}_4 = (\bar{d}\gamma_\mu L T^a b) \sum_q (\bar{q}\gamma^\mu T^a q),$$
$$\mathcal{P}_5 = (\bar{d}\gamma_\mu\gamma_\nu\gamma_\rho L b) \sum_q (\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho q), \quad \mathcal{P}_6 = (\bar{d}\gamma_\mu\gamma_\nu\gamma_\rho L T^a q) \sum_q (\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho T^a q). \tag{4}$$

Electromagnetic and chromomagnetic dipole operators:

$$\mathcal{P}_{7\gamma} = \frac{e}{16\pi^2}\left[\bar{d}\sigma^{\mu\nu}(m_b R + m_d L)\, b\right] F_{\mu\nu}, \quad \mathcal{P}_{8g} = \frac{g_{\rm st}}{16\pi^2}\left[\bar{d}\sigma^{\mu\nu}(m_b R + m_d L)\, T^a b\right] G^a_{\mu\nu}, \tag{5}$$

where $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the electromagnetic and gluonic field strength tensors, respectively.

Semileptonic operators:

$$\mathcal{P}_{9\ell(10\ell)} = \frac{\alpha}{2\pi}(\bar{d}\gamma_\mu L b) \sum_\ell (\bar{\ell}\gamma^\mu(\gamma^5)\ell), \tag{6}$$

where $\alpha = e^2/(4\pi)$ is the fine-structure constant.

As for the Wilson coefficients, they are scale-dependent quantities and their evolution is determined by the renormalization group [29]:

$$C_j(\mu_W) = \sum_{k=0}^{\infty}\left[\frac{\alpha_s(\mu_W)}{4\pi}\right]^k C_j^{(k)}(\mu_W), \tag{7}$$

Table 1. The values of the Wilson coefficients accompanied the strong- and electroweak-penguin operators as well as the electromagnetic and chromomagnetic dipole operators at the scale of the b-quark pole mass $\mu_b = m_b = 4.8$ GeV.

$C_1(m_b)$	1.080	$C_3(m_b)$	0.011	$C_7(m_b)$	4.9×10^{-4}
$C_2(m_b)$	−0.177	$C_4(m_b)$	−0.033	$C_8(m_b)$	4.6×10^{-4}
$C_{7\gamma}(m_b)$	−0.317	$C_5(m_b)$	0.010	$C_9(m_b)$	-9.8×10^{-3}
$C_{8g}(m_b)$	0.149	$C_6(m_b)$	−0.040	$C_{10}(m_b)$	1.9×10^{-3}

where $\mu_W \sim m_W$ is a typical scale of order of the W-boson mass where the effective theory is matched into the exact theory — the Standard Model. Their values at the scale of the b-quark mass are presented in Table 1.

3 $B^0 \to \phi \ell^+ \ell^-$ Decay: Differential Branching Fraction

We are discussing the B-meson decay into the ϕ-meson and charged lepton pair, where the lepton ℓ is either an electron or muon. The calculations of the branching fraction were done in the QCD Factorization approach and the ϕ-meson is assumed to be an energetic particle. Kinematics of the process is restricted by the condition $m_\phi^2 \lesssim q^2 \lesssim m_{J/\psi}^2$, where q^2 is the invariant mass squared of the lepton pair and $m_\phi = 1019.46$ MeV and $m_{J/\psi} = 3096.9$ MeV [7] are the ϕ- and $J/\psi(1S)$-meson masses, respectively. This q^2-region is free from the long-distance contributions from the two-body B-meson decays and dominates by the perturbative contribution. To the leading order in the b-quark mass, the differential branching fraction can be presented in the form [27]:

$$\begin{aligned} \frac{d\mathcal{B}}{dq^2} &= \tau_B \, \frac{G_F^2 |V_{td}^* V_{tb}|^2 \alpha^2}{216\pi} \, M_B f_B^2 f_\phi^2 Q_d^2 \lambda^3(1, m_\phi/M_B, \sqrt{q^2}/M_B) \, |C_3 + 4C_4|^2 \\ &\times \left[\left| \frac{1}{\lambda_-^B(q^2)} \right|^2 + \frac{m_\phi^2}{q^2 \left(1 - q^2/M_B^2\right)^2} \left| \frac{1}{\lambda_+^B(q^2)} \right|^2 \right], \end{aligned} \tag{8}$$

where M_B and τ_B are the B-meson mass and life-time, respectively, m_ϕ is the ϕ-meson mass, $\alpha \simeq 1/137$ is fine-structure constant, $Q_d = -1/3$ is the relative charge of the d-quark, and $\lambda^2(a, b, c) = (a + b + c)\,(a - b + c)\,(a + b - c)\,(a - b - c)$ is the kinematical function. The differential branching fraction also depends on several non-perturbative quantities of which f_B and f_ϕ are the leptonic decay constants of the B- and ϕ-meson, respectively, and $\lambda_\pm^B(q^2)$ are the first inverse moments of the B-meson distribution amplitudes. These inverse moments should be known as functions of the momentum squared, q^2, and are dependent on the choice of the B-meson distribution amplitudes $\tilde{\varphi}_\pm^B(t)$ [30]. In the position space, they are defined as scalar functions entering the transition matrix element from the B-meson state to the vacuum one [30]. In the limit of the static heavy quark and massless light quark, the later quark is situated on the light cone (the separation z^μ between the quarks is assumed to be light-like, $z^2 = 0$) and the matrix element can be parameterized as follows [30, 31]:

$$\langle 0 | q_\alpha(z) \, E(0, z) \, h_{v,\beta}(0) | \bar{B}(v) \rangle = -\frac{i f_B M_B}{4} \left[(1 + \hat{v}) \left\{ \tilde{\varphi}_+^B(t) - \left[\tilde{\varphi}_+^B(t) - \tilde{\varphi}_-^B(t) \right] \frac{\hat{z}}{2t} \right\} \gamma_5 \right]_{\beta\alpha} . \tag{9}$$

Here, v^μ is the four-velocity of the heavy quark and $t = (zv)$ is the time in the rest frame of the B-meson. To calculate the first inverse moments, one needs the Fourier transforms of the distribution

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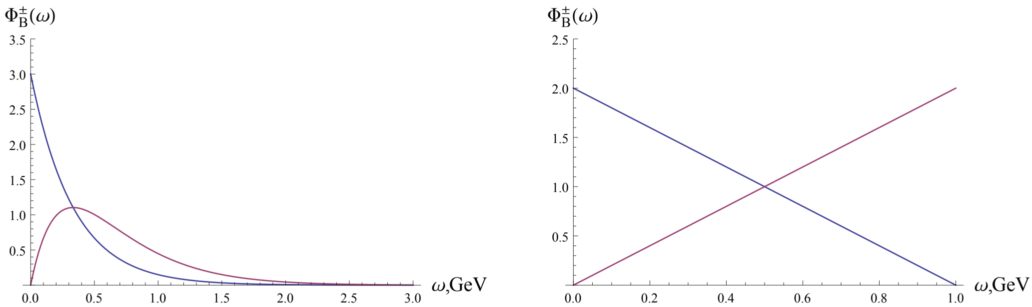

Figure 1. The exponential (the left panel) and linear (the right panel) model of the *B*-meson distribution amplitudes.

amplitudes defined above in the momentum space, $\varphi_\pm^B(\omega)$. Note that these two distribution amplitudes are not independent but can be related to each other by the Wandzura-Wilczek-type equation [30, 31]:

$$\phi_-^B(\omega) = \int_\omega^\infty \frac{\phi_+^B(\omega')}{\omega'}\, d\omega', \tag{10}$$

which is applicable when the three-particle contribution in the *B*-meson wave-function is neglected. This approximation is accepted in calculations. From the existing models suggested for the *B*-meson distribution amplitudes, $\varphi_\pm^B(\omega)$, we consider only two: the exponential one by Grozin and Neubert [30] and the linear model suggested by Kawamura et al. [32]. The explicit shapes of the distribution amplitudes are presented in Fig. 1.

4 First Inverse Moments

First inverse moments (ordinary and logarithmic) are determined as follows [30, 31, 33]:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\, \phi_+^B(\omega;\mu), \qquad \frac{\sigma_{B,n}(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\, \phi_+^B(\omega;\mu) \ln^n \frac{\mu}{\omega}. \tag{11}$$

The former one can be generalized to include the q^2-dependence [30, 31, 34]:

$$\lambda_{B,\pm}^{-1}(q^2) = \int_0^\infty \frac{\phi_\pm^B(\omega)\, d\omega}{\omega - q^2/M_B - i\epsilon}. \tag{12}$$

Note that $\lambda_{B,-}^{-1}(q^2)$ has the logarithmic divergence at $q^2 \to 0$.

Within the representative models selected by us, the moments (12) can be calculated analytically. For the exponential model, the first inverse moments (ordinary and logarithmic with $n = 1$) have the forms [30, 34]:

$$\lambda_B(\mu) = \omega_0(\mu) = \frac{2}{3}\,\bar{\Lambda}; \quad \sigma_B(\mu) = \ln \frac{\mu}{\lambda_B(\mu)} + \gamma_E, \tag{13}$$

where $\bar{\Lambda} = M_B - m_b$ is the effective *B*-meson mass with m_b being the *b*-quark mass, $\gamma_E \simeq 0.577$ is the Euler constant. The q^2-dependent first inverse moments (12) can be also calculated rather easily [27, 34]:

$$\lambda_{B,+}^{-1}(q^2) = \lambda_B^{-1} + \zeta\, \lambda_{B,-}(q^2), \quad \lambda_{B,-}^{-1}(q^2) = \lambda_B^{-1}\, e^{-\zeta}\left[-\mathrm{Ei}(\zeta) + i\pi\right], \tag{14}$$

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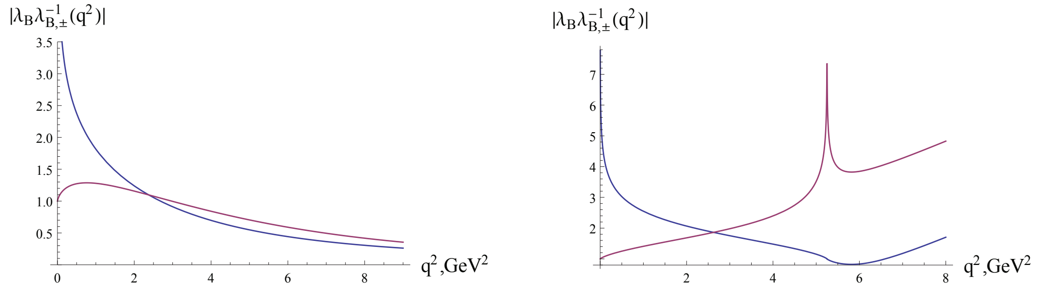

Figure 2. The q^2-dependent first inverse moments in the interval $q^2 \in [1\ \text{GeV}^2, 8\ \text{GeV}^2]$. The moments in the exponential model are shown on the left panel and corresponding curves in the linear model are presented on the right panel.

where $\zeta = q^2/(M_B\lambda_B)$ is the reduced momentum squared and $\mathrm{Ei}(x)$ is the exponential integral function.

For the linear model [32], one can make similar calculations with the following results for first inverse (ordinary and $n = 1$ logarithmic) moments [27, 32]:

$$\lambda_B(\mu) = \bar{\Lambda} = m_B - m_b(\mu), \qquad \sigma_B(\mu) = \ln\frac{\mu}{2\lambda_B(\mu)} + 1, \tag{15}$$

and for the q^2-dependent first inverse moments [27]:

$$\begin{aligned} \lambda_{B,+}^{-1}(q^2) &= \lambda_B^{-1}\left[\xi\ln|1/\xi - 1| + 1 + i\pi\xi\,\Theta(1-\xi)\right], \\ \lambda_{B,-}^{-1}(q^2) &= \lambda_B^{-1}\left[(1-\xi)\ln|1/\xi - 1| - 1 + i\pi(1-\xi)\,\Theta(1-\xi)\right], \end{aligned} \tag{16}$$

where $\xi = q^2/(2M_B\lambda_B)$ is the reduced momentum squared and $\Theta(x)$ is the unit-step function. The dependence of $|\lambda_B\,\lambda_{B,\pm}^{-1}(q^2)|$ in the interval $q^2 \in [1\ \text{GeV}^2, 8\ \text{GeV}^2]$ is shown in Fig. 2: the exponential- and linear-model curves are presented in the left and right panels, respectively.

5 Numerical results

For experimentalists, the partially-integrated branching fractions over some q^2-interval are of interest:

$$\Delta\mathcal{B}(q^2_{\text{min}} < q^2 < q^2_{\text{max}}) = \int_{q^2_{\text{min}}}^{q^2_{\text{max}}} \frac{d\mathcal{B}}{dq^2}\,dq^2 \tag{17}$$

The differential branching fractions (8) involves the q^2-dependent first inverse moments. To work out the dependence on the choice of the B-meson distribution amplitudes, the branching fraction is evaluated in the interval $q^2 \in [1\ \text{GeV}^2, 8\ \text{GeV}^2]$, where the perturbative calculations are applicable. For the exponential (GN) and linear (KKQT) models, the branching fractions are [27]:

$$\begin{aligned} &\Delta\mathcal{B}^{\text{GN}}(1\ \text{GeV}^2 < q^2 < 8\ \text{GeV}^2) = 5.70\times 10^{-13}, \\ &\Delta\mathcal{B}^{\text{KKQT}}(1\ \text{GeV}^2 < q^2 < 8\ \text{GeV}^2) = 5.25\times 10^{-13}. \end{aligned} \tag{18}$$

One can see, that the branching fraction obtained is approximately 5×10^{-13} and the choice of the B-meson model results into the 10% error. This uncertainty can be accumulated into the total theoretical error with a sizable impact. Note that a dominant contribution into the total error is from the uncertainty in the energy scale $\mu \sim m_b$ which is the typical scale of the process considered. It is difficult to quantify all uncertainties numerically at present but we plan to perform such an analysis soon [28]. The last comment is about a possibility to observe such a decay. Assuming that this decay is dominated by the perturbative contribution, the total branching fraction can be estimated as [27]:

$$\mathcal{B}\left(B^0 \to \phi\ell^+\ell^-\right) \sim 10^{-12}. \tag{19}$$

With such a probability, the $B^0 \to \phi\ell^+\ell^-$ decay can not be observed experimentally at the LHC at present but after several years of data taking, it could be possible to get an evidence or strong limitation on this process.

6 Conclusions

The annihilation-type semileptonic decay $B \to \phi\ell^-\ell^+$ is considered in the QCD Factorization approach. Its branching fraction is sensitive to the choice of the B-meson distribution amplitude in the form of the momentum-dependent first inverse moments. Two models for the B-meson distribution amplitudes are considered and their q^2-dependent first inverse moments are calculated. The partially integrated branching fraction is calculated in interval $q^2 \in [1\ \text{GeV}^2, 8\ \text{GeV}^2]$, where the perturbative contribution dominates, and the dependence on the choice of the distribution amplitude model results in the uncertainty of order 10%. The total branching fractions of the $B^0 \to \phi\ell^-\ell^+$ decays, where $\ell = e, \ ,\mu$, are too small to be observed experimentally at the LHC at present but the LHCb Collaboration can, at least, get a strong limit on this decay probability after several years of the LHC run.

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