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Z-boson production in association with heavy flavor in the k_T-factorization

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Abstract. We present the calculations of associated production of Z bosons and heavy (charm or beauty) quarks at the LHC energies in the framework of k_T-factorization QCD approach. Our consideration is mainly based on the $O(\alpha\alpha_s^2)$ off-shell gluon-gluon fusion subprocess $g^* g^* \to ZQ\bar{Q}$, where produced Z boson subsequently decays into a lepton pair. Several subleading contributions from the $O(\alpha\alpha_s^2)$ and $O(\alpha\alpha_s^3)$ subprocesses are taken into account. Contributions from the double parton scattering mechanism are estimated. The transverse momentum dependent (or unintegrated) gluon densities in a proton are determined using the Catani-Ciafaloni-Fiorani-Marchesini evolution equation. We achieve reasonably good agreement of our predictions and latest experimental data taken by the CMS and ATLAS Collaborations, discuss the theoretical uncertainties of our calculations and demonstrate the importance of subleading quark contributions in description of the LHC data in the whole kinematical region.

Very recently, the CMS Collaboration has presented the measurements of Z boson production in association with charm [1] quarks at the LHC energy $\sqrt{s} = 8$ TeV. Also measurements of angular correlations of B-hadrons at $\sqrt{s} = 7$ TeV were presented [2] and the ATLAS Collaboration has reported the experimental data on the total and differential cross sections of Z boson production in association with beauty quark jets at $\sqrt{s} = 7$ TeV [3]. Such processes involve both strong and weak interactions, so are important as global test of the Standard Model (SM). The $Z + b$-jets production is an important background for studies of the associated production of Higgs and Z bosons, where the Higgs boson decays into $b\bar{b}$ pairs [4–6]. Many physics scenarios beyond the Standard Model (SM), for example, new generations of heavy quarks (b', t') decaying into Z bosons and b-quarks [7], supersymmetric Higgs bosons produced in association with beauty quarks [8] and some SM extentions with additional $SU(2)$ doublets with enhanced $Zb\bar{b}$ coupling [9], predict final states with b-quarks and Z bosons. In addition, $Z + b/Z + c$ cross sections ratio is highly sensitive to the charm content of the proton [10]. Finally, such processes may serve as potential indicators of the Double Parton Scattering (DPS) mechanism [11–13].

In the present work we analyse recent CMS [1, 2] and ATLAS [3] data using so called k_T-factorization QCD approach [14]. This approach is based on the famous Balitsky-Fadin-Kuraev-Lipatov (BFKL) or Ciafaloni-Catani-Fiorani-Marchesini (CCFM) gluon evolution equations and pro-

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vides solid theoretical grounds for the effects of gluon radiation in initial state and intrinsic gluon transverse momentum. The certain advantages are connected with the fact that, even with the LO partonic amplitudes, one can include a large piece of higher-order corrections (namely, part of NLO + NNLO + ... terms containing leading $\log 1/x$ enhancement of cross sections due to real initial state gluon emissions) taking them into account in the form of CCFM-evolved transverse momentum dependent (TMD) gluon densities. In particular, it gives better agreement with Tevatron data on the associated production of prompt photons and charm or beauty quarks compared to the NLO pQCD predictions [15], that is an additional motivation of our present study.

Let us start from a short review of calculation steps. Our consideration is mainly based on the off-shell gluon-gluon fusion subprocess:

$$g^*(k_1) + g^*(k_2) \to Z(p) + Q(p_1) + Q(p_2), \tag{1}$$

where Q denotes the produced charm or beauty quark, and four-momenta of the particles are given in parentheses. The corresponding gauge-invariant off-shell amplitude was calculated earlier [16, 17]. To fully reproduce the experimental setup [1–3] we simulate the subsequent decay of the produced Z boson into lepton pair according to the electroweak theory, that has not been made in the previous calculations [16, 17]. So, the off-shell gluon-gluon fusion gives the $O(\alpha\alpha_s^2)$ contribution to the production cross-section. Additionally, we take into account several subprocesses involving quarks in the initial state, namely

$$q(k_1) + Q(k_2) \to Z(p) + q(p_1) + Q(p_2), \tag{2}$$

$$q(k_1) + \bar{q}(k_2) \to Z(p) + Q(p_1) + \bar{Q}(p_2), \tag{3}$$

$$q(k_1) + g(k_2) \to Z(p) + q(p_1) + Q(p_2) + \bar{Q}(p_3), \tag{4}$$

where the produced Z bosons also decay to lepton pairs. These subprocesses give $O(\alpha\alpha_s^2)$ (subprocesses (2) and (3)) and $O(\alpha\alpha_s^3)$ (subprocess (4)) contributions. Subprocess (4) is suppressed by the additional degree of strong coupling QCD constant, however, we include it into the consideration because it can give sizeble contribution to the production cross section due to large gluon flux at the LHC energies, especially at low $\eta - \phi$ distances between the produced heavy quarks. Moreover, while the gluon-gluon fusion (1) contributes mainly at low and moderate transverse momenta, the subprocesses (2) — (4) become important at high transverse momenta, which correspond to the region of relatively large x, where the standard Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) parton evolution works perfectly and contribution of the $\log 1/x$-enhanced terms is negligible. Therefore, the subprocesses (2) — (4) can be safely taken into account in the usual collinear QCD factorization. Thus, we rely on a combination of two techniques with each of them being used where it is most suitable. For the subprocesses (2) and (3) we use here the on-shell limit of formulas obtained earlier [15] supplementing them with the Z boson decays.

According to the collinear QCD factorization theorem, to calculate the contributions of subleading subprocesses (2) — (4) one has to convolute the corresponding partonic cross sections $d\hat{\sigma}_{ab}$ and conventional parton distribution functions $f_a(x_1, \mu^2)$ in a proton:

$$\sigma = \int dx_1 dx_2 d\hat{\sigma}_{ab}(x_1, x_2, \mu^2) f_a(x_1, \mu^2) f_b(x_2, \mu^2), \tag{5}$$

where indices a and b denote quark and/or gluon, x_1 and x_2 are the fractions of longitudinal momenta of colliding protons and μ^2 is the hard scale. In the case of the leading off-shell gluon-gluon fusion subprocess (1) we employ the k_T-factorization approach:

$$\sigma = \int dx_1 dx_2 d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 d\hat{\sigma}_{gg}^*(x_1, x_2, \mathbf{k}_{1T}^2, \mathbf{k}_{2T}^2, \mu^2) f_g(x_1, \mathbf{k}_{1T}^2, \mu^2) f_g(x_2, \mathbf{k}_{2T}^2, \mu^2), \tag{6}$$

where $\mathbf{k}_{1T}^2$ and $\mathbf{k}_{2T}^2$ are the transverse momenta of the initial off-shell gluons, and $f_g(x_1, \mathbf{k}_T^2, \mu^2)$ is the TMD distribution in a proton.The essential point of our consideration is that we use the numerical solution of the CCFM evolution equation to determine the distribution. Numerically, we adopt the latest JH'2013 set, where two density functions, namely JH'2013 set 1 and JH'2013 set 2, were released [18]. The input parameters of corresponding initial gluon distributions were fitted from the best description of the precision DIS data on the inclusive F_2 data (set 1) or both F_2 and F_2^c data (set 2). Both these fits are based on the TMD matrix elements (which are directly related with the resummation of DIS coefficient functions) and include two-loop strong coupling constant, kinematic consistency constraint [19, 20] and non-singular terms in the CCFM gluon splitting function [21]. However, the inclusive structure function F_2 receives significant contributions from quark channels, whereas charm production is dominated by the gluon distribution. Therefore, below we will use JH'2013 set 2 gluon density as a default choice. For the conventional quark and gluon distributions in a proton we apply the LO MSTW'2008 set [22].

To calculate the DPS contributions one commonly makes use of a simple factorization formula (for details see the reviews [11–13] and references therein):

$$\sigma_{\text{DPS}}(Z + Q + \bar{Q}) = \frac{\sigma_{\text{SPS}}(Z)\sigma_{\text{SPS}}(Q + \bar{Q})}{\sigma_{\text{eff}}}, \tag{7}$$

where $\sigma_{\text{eff}} \simeq 15$ mb is a normalization constant which incorporates all "DPS unknowns" into a single phenomenological parameter. The numerical value of σ_{eff} was earlier obtained from fits to pp and $p\bar{p}$ data. This will be taken as default value throughout the paper. Deriving the formula (7) relies on two simplifying approximations: the double parton distribution functions can be decomposed into longitudinal and transverse components and the longitudinal component reduces to the diagonal product of independent single parton densisites. The inclusive SPS cross sections for the individual partonic subrocesses can be derived in a usual way (in the collinear QCD approximation or k_T-factorization). To calculate them below we strictly follow the approaches described earlier [23–25].

We now are in a position to present our numerical results. First we describe the input parameters and kinematic conditions. The predicted cross sections depend on the renormalization (μ_R) and factorization (μ_F) scales. As it is often done in the collinear QCD factorization, which we apply for the subprocesses (2) — (4), we set both these scales to be equal to transverse mass of produced Z bosons. In the k_T-factorization approach, employed for the gluon-gluon fusion subprocess (1), we set $\mu_F^2 = \hat{s} + \mathbf{Q}_T^2$ with $\hat{s}$ and $\mathbf{Q}_T^2$ being the energy of scattering subprocess and transverse momentum of the incoming off-shell gluon pair, respectively. The special choice of μ_F is connected with the CCFM evolution [18]. We set charm and beauty masses $m_c = 1.4$ GeV and $m_b = 4.75$ GeV, Z boson mass $m_Z = 91.1876$ GeV, its total decay width $\Gamma_Z = 2.4952$ GeV and $\sin^2\theta_W = 0.23122$. For the subprocesses (2) — (4) we used LO formula for the strong coupling constant with $n_f = 4$ massless quark flavours and $\Lambda_{QCD} = 200$ MeV, so that $\alpha_s(m_Z^2) = 0.1232$. According to the fit [18], we apply two-loop strong coupling constant for the off-shell gluon-gluon fusion subprocess. The multidimensional integration everywhere was performed by means of a Monte Carlo technique, using the VEGAS routine [26].

Let us consider first the associated production of Z bosons and b-jets [27]. The ATLAS Collaboration collected the data [3] at $\sqrt{s} = 7$ TeV. Both leptons originating from the Z boson decay were required to have $p_T^l > 20$ GeV and $|\eta^l| < 2.4$, the lepton pair invariant mass lied in the interval $76 < M_{ll} < 106$ GeV, the beauty jets were required to have $p_T^b > 20$ GeV and $|\eta^b| < 2.4$. We confront our predictions with the available data in Figs. 1 and 2. To estimate the theoretical uncertainties in the quark-involving subprocesses (2) — (4), calculated using the collinear QCD factorization, we have varied the scales μ_R and μ_F by a factor of 2 around their default values. In the k_T-factorization approach, employed for off-shell gluon-gluon fusion subprocess (1), the scale uncertainties have been

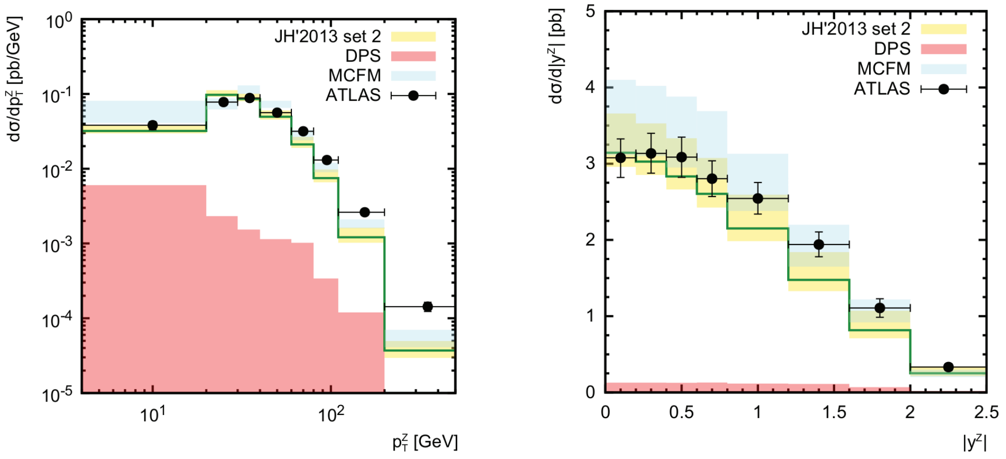

Figure 1. Associated $Z + b$ production cross section at $\sqrt{s} = 7$ TeV presented as a function of the Z boson transverse momentum (left panel) or rapidity (right panel). Solid histograms show our predictions at the default scale while shaded bands correspond to scale variations described in the text. The estimated DPS contributions and MCFM [28] predictions (taken from [3]) are shown additionally. The data are from ATLAS [3]

estimated by using the gluon densities JH'2013 set 2+ and JH'2013 set 2- instead of default density JH'2013 set 2. These two sets refer to the varied hard scales in the strong coupling constant α_s in the off-shell amplitude: JH'2013 set 2+ stands for $2\mu_R$, while JH'2013 set 2- refers to $\mu_R/2$ (see [18] for more information). The estimated scale uncertainties are shown as shaded bands. As one can see, we achieve reasonably good agreement with the ATLAS data [3] within the experimental and theoretical uncertainties, although we observe some underestimation of these data at high p_T^Z and slight overestimation at small transverse momenta. The slight overestimation of the data at low p_T^Z can probably be attributed to the TMD gluon density used, since the region $p_T^Z < 100$ GeV is fully dominated by off-shell gluon-gluon fusion, as it is demonstrated in Fig. 2. The rapidity distribution is well described practically everywhere. The NLO pQCD calculations, performed using MCFM routine [28], tend to slightly overestimate our predictions and better decribe the data at large transverse momenta.

We find that the quark-initiated subprocesses (2) — (4) become important only at high transverse momenta, where the typical x values are large, and that supports using of the DGLAP quark and gluon dynamics for these subprocesses (see Fig. 2). The subprocesses (2) — (4) are important to achieve an adequate description of the data in the whole p_T^Z region. The estimated DPS contributions are found to be small in the considered kinematic region. Some reasonable variations in $\sigma_{\rm eff} \simeq 15 \pm 5$ mb would affect DPS predictions, though without changing our basic conclusion. We note also that scale uncertainties of the CCFM-based predictions are comparable with the ones of NLO pQCD calculations.

Now we turn to the associated production of Z bosons and two beauty jets [27]. The data provided by the ATLAS Collaboration [3] refer to the same energies and kinematic restrictions as in the previous subsection. The observables shown by the ATLAS Collaboration are the Z boson transverse momentum p_T^Z and rapidity y^Z, invariant mass of the b-jet pair M^{bb} and angular separation in $\eta - \phi$ plane between the jets ΔR^{bb}. The latter is useful to identify the contributions where scaterring amplitudes are dominated by terms involving gluon splitting $g \to Q + \bar{Q}$.

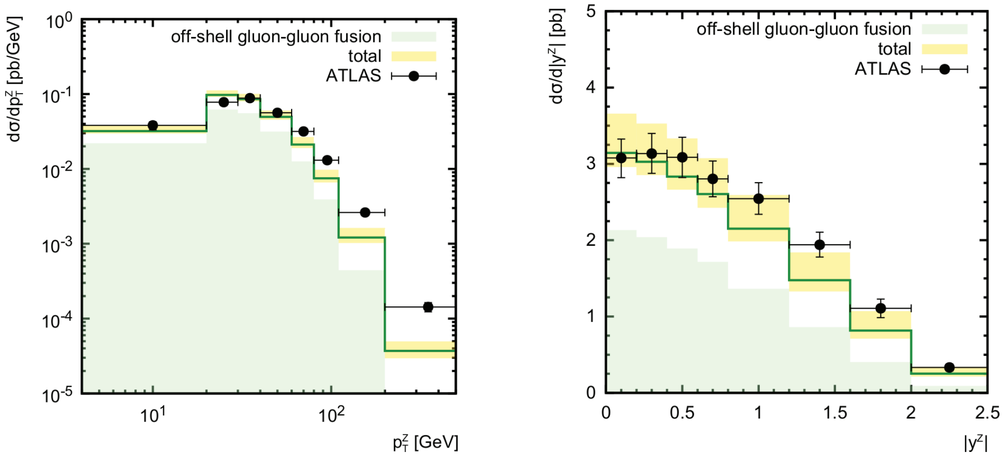

Figure 2. The off-shell gluon-gluon fusion contribution to the associated $Z + b$ production at $\sqrt{s} = 7$ TeV. The data are from ATLAS [3]

The results of our calculations are shown in Fig. 3 in comparison with the ATLAS data [3]. As one can see, our results describe the data reasonably well within the experimental and theoretical uncertainties, although some tendency to slightly underestimate the data at high transverse momentum p_T^Z and large M^{bb} can be seen. The role of off-shell gluon-gluon fusion subprocess is a bit enhanced here compared to the case of $Z + b$ production because the quark-antiquark annihilation subprocess (3) gives a negligible contribution and gluon splitting subprocess (4) populates mainly at low $\eta - \phi$ distances ΔR^{bb}. The estimated DPS contribution is small and can play a role at low p_T^Z only. The NLO pQCD calculations, performed using MCFM program, tend to slightly underestimate the ATLAS data at low ΔR^{bb} and M^{bb}, although provide better description of the data at large transverse momentum p_T^Z and invariant mass M^{bb}.

In the measurements reported by CMS Collaboration [2], both b-hadrons were identified explicitly by their full decay reconstruction. This data sample allows to study the production properties of a $Zb\bar{b}$ system even in the region of small angular seperation between the b quarks (where the usual jet analysis is not possible as the jets would overlap). In a specific subsample, an additional cut on the Z boson transverse momentum is applied, $p_T^Z > 50$ GeV. The CMS Collaboration described the angular configuration of the $Zb\bar{b}$ system in terms of spatial (in $\eta - \phi$ plane) and azimuthal separation between the b-hadrons ΔR^{bb} and $\Delta\phi^{bb}$, spatial separation $\min \Delta R^{Zb}$ between the Z boson and closest b-hadron and the asymmetry in the $Zb\bar{b}$ system defined as

$$A^{Zbb} = \frac{\max \Delta R^{Zb} - \min \Delta R^{Zb}}{\max \Delta R^{Zb} + \min \Delta R^{Zb}}, \quad (8)$$

where $\max \Delta R^{Zb}$ is the distance between the Z boson and remote b-hadron. The correlation observables are useful to identify the different production mechanisms (or specific higher-order corrections). For example, low $\min \Delta R^{Zb}$ identifies Z bosons in the vicinity of one of the b-hadrons (Z bosons promptly radiated from b-quarks), small $\Delta\phi^{bb}$ indicates gluon to quark splitting $g \to Q + \bar{Q}$. Moreover, while the configurations where the two b-hadrons are emitted symmetrically with respect to the Z directions leads to a zero value of A^{Zbb} assymetry, the additional final-state gluon radiation results in a non-zero one, that provides us with the possibility to test the high-order pQCD corrections.

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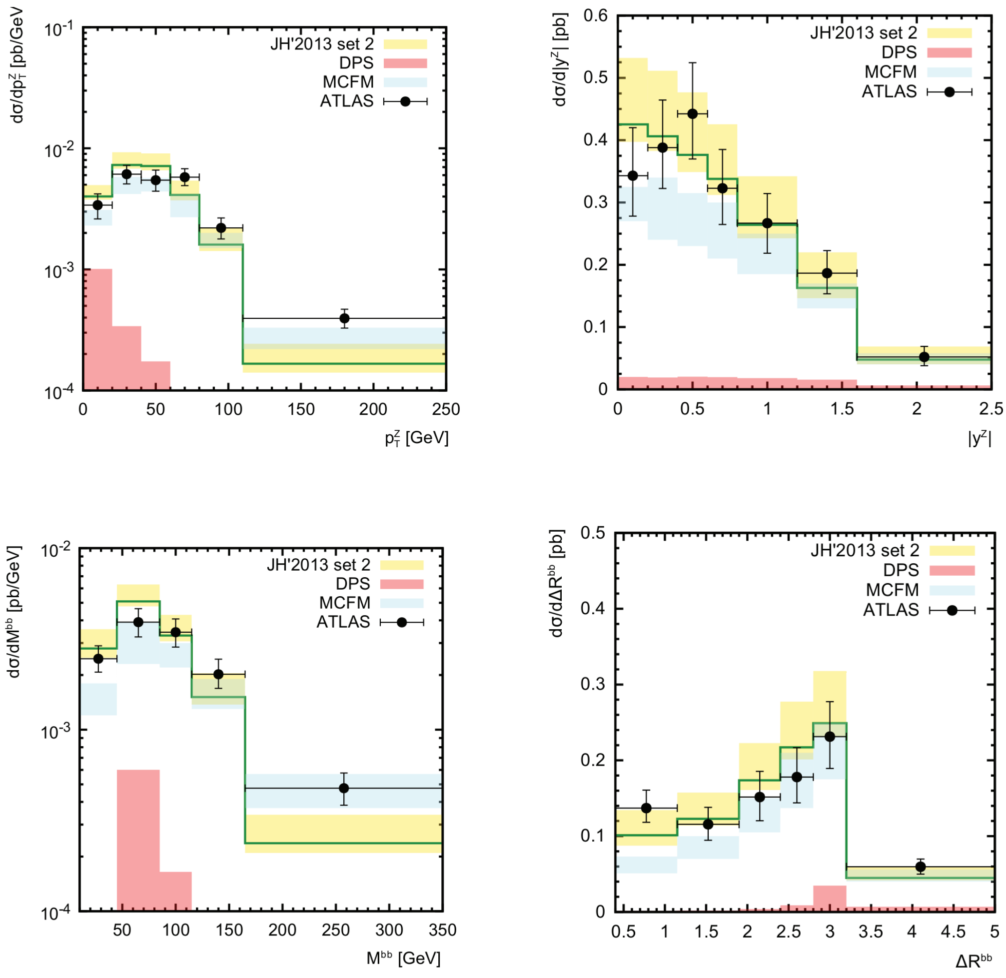

Figure 3. Associated production of a Z boson with two beauty jets at $\sqrt{s} = 7$ TeV calculated as a function of the Z boson transverse momentum, rapidity, invariant mass of the b-jet pair and angular separation between the jets. Notation of the histograms is the same as in Fig. 1. The data are from ATLAS [3]. The MCFM [28] predictions are taken from [3].

Our predictions [27] are shown in Figs. 4 and 5 in comparison with the CMS data [2]. As one can see, our results with default b-quark fragmentation parameters reasonably well describe the data within the theoretical and experimental uncertainties. To estimate an additional uncertainty coming from the b-quark fragmentation, we repeated our calculations with varied shape parameter $\epsilon_b = 0.003$ (not shown), which is often used in NLO pQCD calculations. We find that the predicted cross sections (in the considered p_T region) are larger for smaller ϵ_b values. However, the typical dependence of numerical predictions on the fragmentation scheme is much smaller than the scale uncertainties of our calculations. The NLO pQCD predictions, obtained using the aMC@NLO [29] event generator, are rather close to our results.

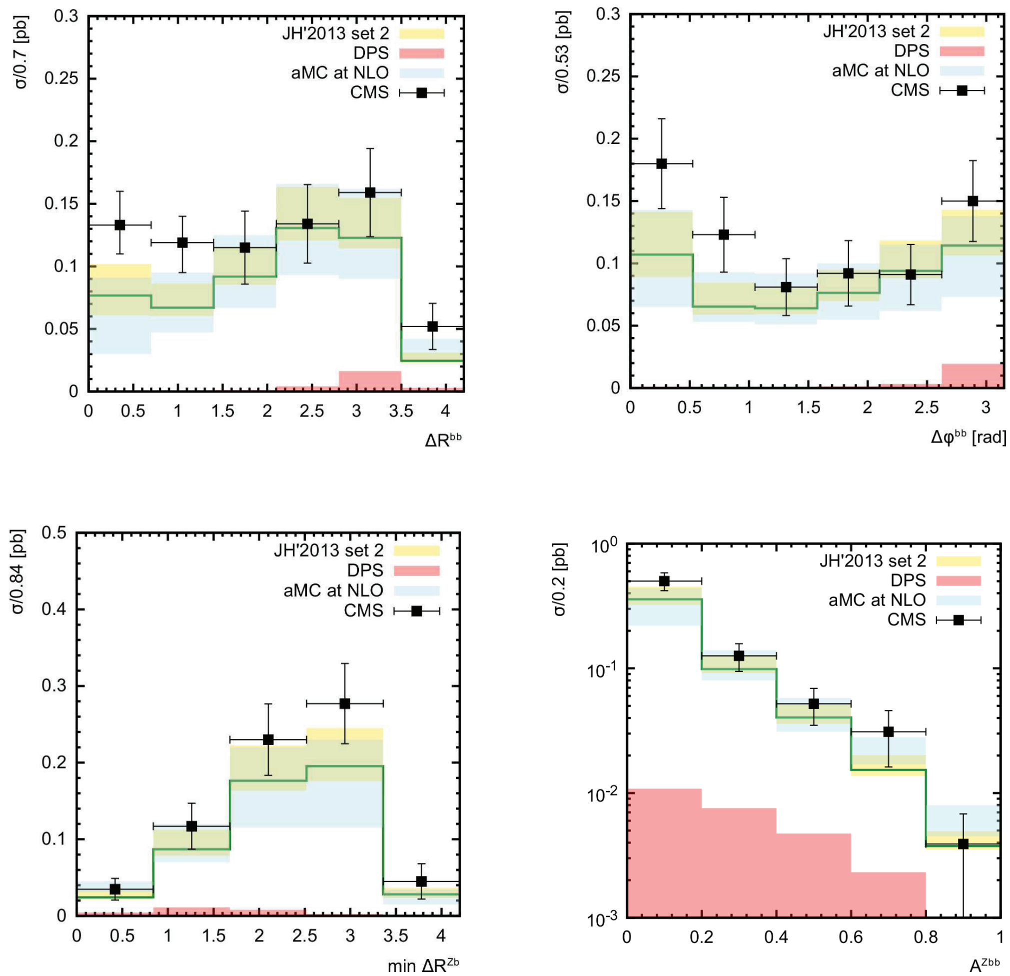

Figure 4. Associated production of a Z boson and two b-hadrons at $\sqrt{s}$ = 7 TeV. Notation of the histograms is the same as in Fig. 1. The data are from CMS [2]. The aMC@NLO [29] predictions are taken from [2].

Finally, let us consider measurements of the associated $Z+c$-jet production performed by the CMS Collaboration at $\sqrt{s}$ = 8 TeV [1]. Similar to $Z + b$ production, leptons originating from the Z boson decay have transverse momenta $p_T > 20$ GeV and pseudo-rapidities $|\eta| < 2.1$. The invariant mass of the lepton pair must lie within the 71 — 111 GeV interval and c-jets are required to have $p_T > 25$ GeV and $|\eta| < 2.5$. The cross sections ratios $\sigma(Z+c)/\sigma(Z+b)$ was also measured as a functions of Z boson and jet transverse momenta. Our predictions are shown in Figs. 6 and 7 in comparison with the CMS data. We have achieved good description of the Z boson transverse momentum distribution, whereas the measured cross section as a function of leading c-jet is above the predicted one at $p_T^{c-\text{jet}} < 40$ GeV. Taking into account the DPS contributions only slightly increases the calculated cross sections. The NLO pQCD predictions, obtained using the MCFM program [28], are lower than the data in the first and second Z boson p_T bins and in the first $p_T^{c-\text{jet}}$ bin. The LO pQCD predictions matched with parton

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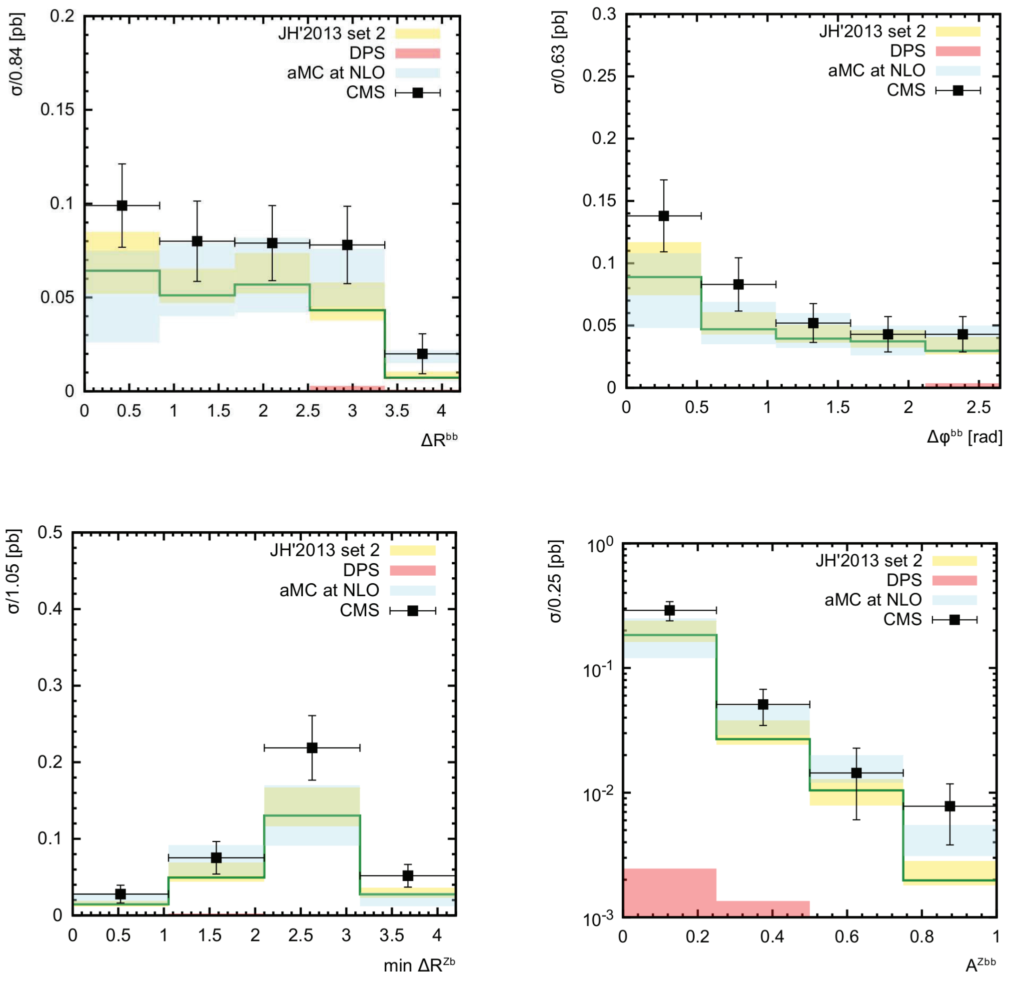

Figure 5. Associated production of a Z boson and two b-hadrons at $\sqrt{s} = 7$ TeV under additional kinematical cut on the Z boson transverse momentum $p_T^Z > 50$ GeV. Notation of the histograms is the same as in Fig. 1. The data are from CMS [2]. The aMC@NLO [29] predictions are taken from [2].

showering, obtained with MadGraph event generator, are in better agreement with the data than the NLO ones, although they still slightly underestimate the CMS data. The differences between these predictions and the data are reduced in the $\sigma(Z + c)/\sigma(Z + b)$ ratios. The results of our calculations underestimate these ratios at high Z-boson and jet transverse momenta. However, they are rather close to the CMS data within the large experimental uncertainties.

To conclude, we have considered the associated production of Z bosons and charmed or beauty quarks at the LHC energies in the framework of the k_T-factorization approach. Our consideration has been based on the $O(\alpha\alpha_s^2)$ off-shell gluon-gluon fusion subprocess $g^*g^* \to ZQ\bar{Q}$, where the produced Z boson subsequently decays into the lepton pair. Several subleading $O(\alpha\alpha_s^2)$ and $O(\alpha\alpha_s^3)$ contributions from the quark-involved subprocesses, which come into play at high transverse momenta, were

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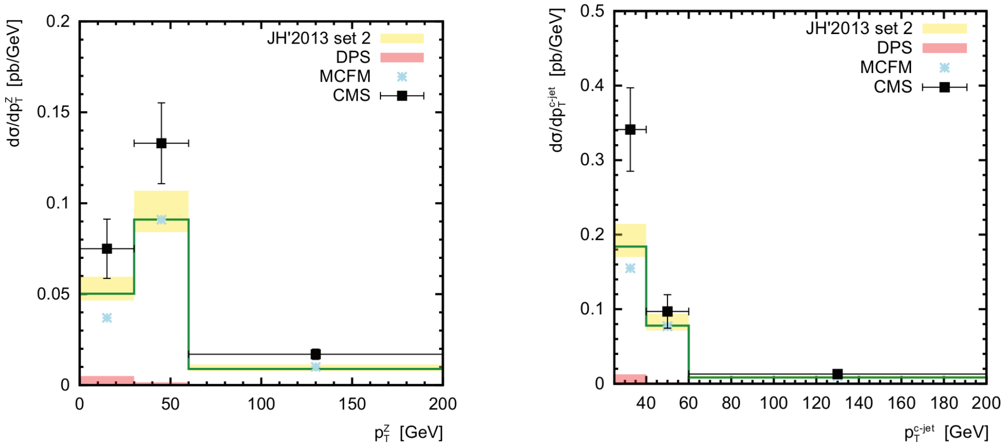

Figure 6. The associated $Z + c$ production calculated as a function of Z boson and c-jet transverse momenta at $\sqrt{s}$ = 8 TeV. Notation of histograms is the same as in Fig. 1. The experimental data are from CMS [1].

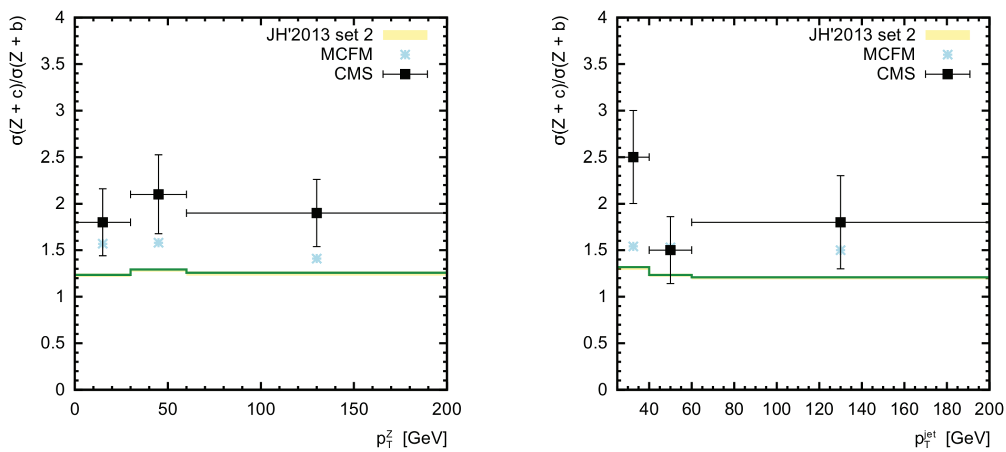

Figure 7. The ratio $\sigma(Z+c)/\sigma(Z+b)$ calculated as a function of Z boson and c-jet transverse momenta at $\sqrt{s}$ = 8 TeV. Notation of histograms is the same as in Fig. 1. The experimental data are from CMS [1].

taken into account in the collinear approximation of QCD. Additionally, we have estimated the contributions from the double parton scattering mechanism. Using the TMD gluon densities in a proton derived from the CCFM evolution equation, we have analyzed the latest experimental data taken by the ATLAS and CMS Collaborations at different energies $\sqrt{s}$ = 7 and 8 TeV. We have achieved reasonably good agreement between our predictions and the LHC data in the whole kinematical region and demonstrated that the subleading quark contributions are important to describe the LHC data, especially at high transverse momenta. The double parton scattering contributions are estimated to be small.

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