EPJ Web of Conferences **158**, 05005 (2017)
QFTHEP 2017
DOI: 10.1051/epjconf/201715805005

Radiative decay of keV-mass sterile neutrino in magnetized electron plasma

Alexandra Dobrynina[1,⋆], *Nicolay* Mikheev, and *Georg* Raffelt[2,]

[1] *P. G. Demidov Yaroslavl State University, Sovietskaya 14, 150003 Yaroslavl, Russia*
[2] *Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany*

Abstract. The radiative decay of sterile neutrinos with typical masses of 10 keV is investigated in the presence of an external magnetic field and degenerate electron plasma. Full account is taken of the modified photon dispersion relation relative to vacuum. The limiting cases of relativistic and nonrelativistic plasma are analyzed. The decay rate calculated in a strongly magnetized plasma, as a function of the electron number density, is compared with the unmagnetized plasma limit. It is found that the presence of the strong magnetic field in the electron plasma suppresses the catalyzing influence of the plasma by itself on the sterile-neutrino decay rate.

1 Introduction

It is not a secret that stars as laboratories for fundamental physics complement an effort from the existing accelerator facilities on particle physics. The properties and dynamics of astrophysical objects require detailed understanding of quantum processes under an influence of an external active medium. A dense electron-positron and nucleon plasma as well as an external electromagnetic field can play this role both in the interior and envelope of compact astrophysical objects.

An influence of an external magnetic field on properties of leptons and their interactions becomes especially important when the field strength reaches the critical Schwinger value of $B_e = m_e^2/e = 4.41 \times 10^{13}$ Gauss, where m_e is the electron mass and e is the elementary charge. Such fields are not accessible in accelerator experiments but can exist in strongly magnetized neutron stars called "magnetars" [1–3]. In astrophysics, "magnetars" are usually identified with objects like Soft Gamma-ray Repeaters (SGR) and Anomalous X-ray Pulsars (AXP) [4]. Recent simulations of core-collapse supernovae [5–7] and the formation of accretion discs at the merger of compact objects in a close binary system [8, 9] demonstrate that the magnetic field strength could reach the value of $B \sim 10^{15}$ Gauss.

Although all these phenomena are different, they are connected with a powerful neutrino emission. In a supernova explosion, the neutrino radiation source is the protoneutron star that is formed as a result of the core collapse of a massive star at the final stage of its evolution. A merger of compact objects in a close binary system leads to the formation of a central object and the hot accretion disk around it. In this case, the disk is a source of a powerful neutrino radiation. The emission of neutrinos is a basic channel of magnetar cooling.

⋆e-mail: dobrynina@uniyar.ac.ru

The plasma and magnetic field manifest themselves as optically active medium and can change dispersion relations of particles [10, 11]. As a result, the processes which are kinematically forbidden or suppressed in vacuum, could be opened or significantly enhanced in plasma and magnetic field. In particular, they can significantly influence the photon-neutrino interaction that arises in vacuum at the loop level and turns out to be extremely weak. Good example is the effect of neutrino luminosity by a plasma in the $\gamma \to \nu\bar{\nu}$ decay which is observed experimentally.

Studies of the radiative decay of a neutrino discussed in this paper have a long history. This process has been considered by several groups in the different cases: vacuum [12], electron plasma [13–18], magnetic fields of different configurations [19–23] and magnetized plasma [24–26]. The main point of the work we have done [26] was to extend the previous analysis by including the modified photon dispersion relation. The results obtained in [26] are briefly presented in this contribution.

2 Strongly magnetized plasma

In a strongly magnetized plasma the neutrino-photon interaction is mainly determined by electrons occupying the lowest Landau level. We assume that the magnetic field is directed along the third axis, $\vec{B} = (0, 0, B)$, and suppose the following hierarchy $2eB > \mu_e^2 - m_e^2 \gg T^2$ of plasma parameters, where m_e is the electron mass, e is the elementary charge, μ_e is the electron chemical potential, and T is the plasma temperature. In the presence of the strong magnetic field and degenerate plasma the photon is a particle with an effective mass [27]:

$$\Omega_0^2 = \frac{2\alpha}{\pi} \frac{eB\, p_F}{\sqrt{p_F^2 + m_e^2}} \,. \tag{1}$$

Here, α is the fine-structure constant and p_F is the electron Fermi momentum. The effective mass Ω_0 is often called the plasma frequency. The electron number density in the strong magnetic field is defined by the expression:

$$n_e = \frac{eB\, p_F}{2\pi^2} \,. \tag{2}$$

This relation allows us to express the plasma frequency in terms of the electron number density and magnetic field as follows [26]:

$$\Omega_0 \simeq 37.1 \text{ keV} \left[\frac{n_{30}^2 b^2}{b^2 + 1.3\, n_{30}^2} \right]^{1/4}, \tag{3}$$

where $b = B/B_e$ and $n_{30} = n_e / \left(10^{30} \text{ cm}^{-3}\right)$. The benchmark number density corresponds to the value of the mass density, where degenerate electrons would still be nonrelativistic. We can see that the typical mass scale of plasma frequency is in the keV region.

Standard neutrinos have sub-eV masses [28] and their radiative decays are forbidden. It is clear that radiative decays would be of interest only for sterile neutrinos with keV masses and above. The sterile neutrinos with keV-masses are very popular as candidates for a dark matter in the Universe [29]. Under minimal assumptions, the mass of DM sterile neutrinos should be in the range of 0.4–50 keV [30–32].

Sterile neutrinos can mix with massive standard ones and in this way interact with a matter. Let us consider one sterile neutrino which is mixing with one active neutrino of a definite flavor. In this case, the neutrino mass states are the mixture of the active and sterile ones:

$$|\nu_1\rangle = \cos\theta_s\, |\nu_a\rangle - \sin\theta_s\, |\nu_s\rangle, \qquad |\nu_2\rangle = \sin\theta_s\, |\nu_a\rangle + \cos\theta_s\, |\nu_s\rangle, \tag{4}$$

where θ_s is the mixing angle. When the mixing angle is negligibly small, the first mass state practically coincides with the active neutrino, $|\nu_1\rangle \simeq |\nu_a\rangle$, and the second mass state is close to sterile neutrino, $|\nu_2\rangle \simeq |\nu_s\rangle$. The smallness of the mixing angle is confirmed by the existing bound [31]:

$$\theta_s^2 \lesssim 1.8 \times 10^{-5} \left(\frac{1\ \mathrm{keV}}{m_s}\right)^5 , \tag{5}$$

which is obtained from analysis of the radiative decay $\nu_s \to \nu_a \gamma$ of sterile neutrino in cosmology. Note that this bound depends substantially on the sterile neutrino mass m_s.

In presence of an electron plasma and magnetic field, the neutrino-photon interaction is induced by real electrons from the magnetized plasma. The neutrino-electron interaction can be described by the effective local Lagrangian:

$$\mathcal{L}_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}} \left[\bar{\Psi}_e \gamma^\alpha \left(C_V - C_A \gamma_5\right) \Psi_e\right] j_\alpha . \tag{6}$$

Here, G_F is the Fermi constant, $C_V = 1/2 + 2\sin^2\theta_W$ and $C_A = 1/2$ are the vector and axial-vector constants of the electron, θ_W is the Weinberg angle, and Ψ_e is the electron quantum field. The neutrino current j_α of our interest is responsible for the transition of a sterile neutrino to an active one [26]:

$$j_\alpha = \cos\theta_s \sin\theta_s \left[\bar{\nu}_a \gamma_\alpha \left(1 - \gamma_5\right) \nu_s\right] , \tag{7}$$

where the dependence on the mixing angle is organized in the product $\sin\theta_s \cos\theta_s$. Further, we fix the flavor of the active neutrino to be the electron one. In addition, we assume that this neutrino is massless and neglect the modification of its dispersion relation by the magnetized plasma.

In the strong magnetic field limit, we may transform the axial-vector current of electrons in the Lagrangian (6) to the vector current [26]:

$$\bar{\Psi}_e \gamma_\alpha \gamma_5 \Psi_e = \bar{\Psi}_e \Pi_- \gamma_\alpha \gamma_5 \Pi_- \Psi_e = \bar{\Psi}_e (\tilde{\varphi}\gamma)_\alpha \Psi_e , \tag{8}$$

where $\Pi_- = (1 - i\gamma^1\gamma^2)/2$ is the projection operator [10, 11], $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}/2$ are dimensionless tensor of the external magnetic field and its dual, and $(\tilde{\varphi}\gamma)_\alpha = \tilde{\varphi}_{\alpha\beta}\gamma^\beta$. Taking into account Eq. (8), the effective Lagrangian (6) becomes:

$$\mathcal{L}_{\mathrm{eff}} = e\left(\bar{\Psi}_e \gamma^\alpha \Psi_e\right) V_\alpha , \tag{9}$$

where the following local operator (the effective neutrino current) is introduced [26]:

$$V_\alpha = -\frac{G_F}{e\sqrt{2}} \left[C_V \left(\tilde{\varphi}\tilde{\varphi} j\right)_\alpha - C_A \left(\tilde{\varphi} j\right)_\alpha\right] . \tag{10}$$

The effective Lagrangian (9) has the same form as the usual Lagrangian of the electromagnetic interaction of electrons (the QED Lagrangian) [33, 34]:

$$\mathcal{L}_{\mathrm{EM}} = e\left(\bar{\Psi}_e \gamma^\alpha \Psi_e\right) A_\alpha . \tag{11}$$

Therefore, the decay amplitude of the sterile neutrino in the magnetized plasma is described by two Feynman diagrams shown in Fig. 1. These diagrams are identical to the ones presented in Fig. 2, after one of the photon lines is replaced by the neutrino current V_α (10). Note that the diagrams in Fig. 2 determine the amplitude of the Compton scattering of photons on plasma electrons.

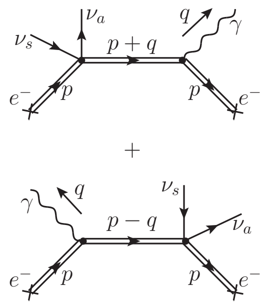

Figure 1. The Feynman graphs of the $\nu_s \to \nu_a + \gamma$ decay in a strongly magnetized electron plasma. The crosses at the ends of the electron lines specify that electrons pertain to plasma. Double lines indicate that the effect of an external magnetic field on the electron fields is taken into account exactly.

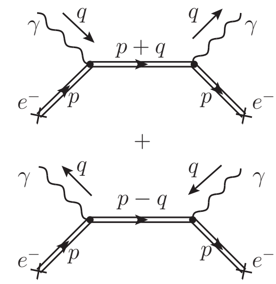

Figure 2. The diagrams of the photon forward scattering on electrons of the strongly magnetized plasma. The crosses at the ends of the electron lines are shown to indicate that electrons pertain to plasma. Double lines specify that the influence of an external magnetic field on electron fields is taken into account exactly.

It is well known [10, 11, 35] that the amplitude of the Compton scattering of photons on plasma electrons, $\gamma \to \gamma$, shown in Fig. 2 contributes to the photon polarization operator $\Pi_{\alpha\beta}$:

$$M_{\gamma\to\gamma} = -\mathcal{E}^*_\alpha \, \Pi^{\alpha\beta} \, \mathcal{E}_\beta \, . \tag{12}$$

In strongly magnetized plasma, the correspondence of the diagrams presented in Figs. 1 and 2 allows to express the sterile-neutrino decay amplitude in terms of the photon polarization operator $\Pi_{\alpha\beta}$:

$$M_{\text{pl+f}} = -\mathcal{E}^*_\alpha \, \Pi^{\alpha\beta} \, V_\beta \, . \tag{13}$$

In such plasma, there are only two physical photon states with the polarization vectors [36]:

$$\mathcal{E}^{(1)}_\alpha \approx \frac{(q\varphi)_\alpha}{\sqrt{q^2_\perp}} \, , \qquad \mathcal{E}^{(2)}_\alpha \approx \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q^2_\parallel}} \, , \tag{14}$$

where $q^2_\parallel = (q\tilde{\varphi}\tilde{\varphi}q)$ and $q^2_\perp = (q\varphi\varphi q)$, which practically coincide with the eigenvectors of the photon polarization operator in the pure magnetic field [10, 11]. It is also possible to determine the set of eigenvalues Π_λ ($\lambda = 1, 2$) from the equation:

$$\Pi_{\alpha\beta} \, \mathcal{E}^{(\lambda)}_\beta = \Pi_\lambda \, \mathcal{E}^{(\lambda)}_\alpha . \tag{15}$$

In the strong magnetic field limit, the eigenvalues Π_1 and Π_2 under the kinematical conditions $\omega \lesssim m_s \ll m_e$ can be written in the form [36]:

$$\Pi_1 \approx -\frac{2\alpha}{\pi} \, \frac{\omega V_F \mu_e \sqrt{q^2}}{\sqrt{q^2_\parallel}} \, , \quad \Pi_2 \approx \frac{2eB\alpha}{\pi} \, \frac{q^2_\parallel V_F}{\omega^2 - V_F^2 k_3^2} , \tag{16}$$

where $\alpha = e^2/(4\pi)$ is the fine-structure constant, μ_e is the electron chemical potential, and V_F is the Fermi velocity. From the comparison of these eigenvalues with each other under the conditions on the plasma parameters, $2eB > \mu_e^2 - m_e^2 \gg T^2$, one can easily see that the contribution of Π_2 to the decay amplitude dominates [26].

With account of the discussion presented above, the sterile-neutrino decay amplitude in the conditions of the strongly magnetized plasma can be written as follows [26]:

$$M_{\text{pl+f}} = \frac{G_F\,\Omega_0^2}{e\sqrt{2}}\,\sqrt{q_{\parallel}^2}\,\frac{C_V\,(q\tilde{\varphi}j) + C_A\,(q\tilde{\varphi}\tilde{\varphi}j)}{\omega^2 - V_F^2 k_3^2}, \tag{17}$$

where j^μ is the neutrino current in the momentum space. The decay width is convenient to calculate in the rest frame of the sterile neutrino, $p_s^\mu = (m_s, \vec{0})$. As all the particles participating in the decay are electrically neutral, the standard procedure — an integration over the phase space of the final-state particles, including their appropriate dispersion properties — can be used for the width evaluation:

$$W_{\text{pl+f}} = \frac{1}{32\pi^2 m_s}\int \frac{d^3 p_a}{E_a}\,\frac{d^3 k}{\omega}\,\delta^{(4)}(p_s - p_a - q)\,\left|M_{\text{pl+f}}\right|^2, \tag{18}$$

where $p_a^\mu = (E_a, \vec{p}_a)$ and $q^\mu = (\omega, \vec{k})$ are the four-momenta of the active neutrino and photon, respectively. In addition, the active neutrino is assumed to be massless, $E_a = |\vec{p}_a|$.

As it was mentioned earlier, the presence of the strongly magnetized plasma is changing the photon dispersion properties. To get the modified dispersion for the photon of the second mode $\mathcal{E}_\alpha^{(2)}$ (14) one needs to resolve the following dispersion relation:

$$q^2 = \Pi_2, \tag{19}$$

where the explicit value of Π_2 is presented in Eq. (16), or the equivalent relation written in terms of the photon energy and momentum components:

$$\omega^2 = k_3^2 + k_\perp^2 + \Omega_0^2\,\frac{\omega^2 - k_3^2}{\omega^2 - V_F^2 k_3^2}\,. \tag{20}$$

Because the photon dispersion is non-trivial, the integrand in the decay width (18) is complicated and becomes a relatively simple function only in limiting cases, for example, in the nonrelativistic and relativistic limits where the integral in the r.h.s. of (18) can be taken analytically.

We adopt the following parameter ranges $m_s = 2 - 20$ keV and $B = 1 - 100\,B_e$ in numerical analysis. In the nonrelativistic limit the decay width can be presented in the form [26]:

$$\begin{aligned} W_{\text{pl+f}}^{\text{n-rel}} = W_{\text{vac}}\,\frac{32\,\pi^2}{2835\,\alpha^2}\left(C_V^2 + C_A^2\right)\Big[\theta(1 - 2x_0)\,4x_0^4\left(21 + 6x_0^2 - 8x_0^4\right) \\ + \theta(2x_0 - 1)\left(-\frac{11}{x_0} + 129x_0 - 210x_0^2 + 168x_0^3 - 84x_0^4 - 24x_0^6 + 32x_0^8\right)\Big], \end{aligned} \tag{21}$$

where $x_0 = \Omega_0/m_s$ is the dimensionless plasma frequency and, as the normalization scale, the vacuum rate of the sterile neutrino is accepted:

$$W_{\text{vac}} = \frac{9\alpha G_F^2}{2048\pi^4}\,m_s^5\,\sin^2(2\theta_s)\,. \tag{22}$$

Note that at $C_V = C_A = 1/2$ and in the limit $x_0 \ll 1$ we reproduce the result of Ref. [25].

Under the conditions of relativistic strongly-magnetized plasma, the plasma frequency can be written as [26]:

$$\Omega_0 \simeq 34.7\ \text{keV}\,\sqrt{B/B_e}\,. \tag{23}$$

One can relatively easy obtain the analytical results for the sterile-neutrino decay width in two limiting cases [26]: the plasma frequency is small ($x_0 \ll m_e/\mu_e$):

$$W^{\rm rel}_{\rm pl+f} \simeq \frac{(G_F \Omega_0^2)^2}{64\pi^2\alpha}\, m_s\, \sin^2(2\theta_s) \left(C_V^2 + C_A^2\right) \frac{\ln(2\mu_e/m_e) - 5/4}{1 - e^{-m_s/(2T)}}\,, \tag{24}$$

and in the opposite limit — of large x_0 ($m_e/\mu_e \ll x_0 < 1$):

$$W^{\rm rel}_{\rm pl+f} \simeq \frac{(G_F m_s^2)^2}{64\pi^2\alpha}\, m_s \sin^2(2\theta_s)\, \frac{\left(C_V^2 + C_A^2\right) x_0^4}{1 - e^{-m_s\left(1+x_0^2\right)/(2T)}} \left[\left(1 + x_0^2\right) \ln\frac{1}{x_0} - \frac{1}{8}\left(1 - x_0^2\right)\left(3 + x_0^2\right)\right]. \tag{25}$$

It is interesting to compare the decay width in the strongly magnetized electron plasma with a width in the unmagnetized plasma, which is presented in the next section.

3 Unmagnetized electron plasma

In the unmagnetized case, the neutrino-electron interaction is defined by the same effective Lagrangian (6). The vector current of electrons in this Lagrangian is similar to the electromagnetic one in QED Lagrangian (11). Let us also apply the procedure explained in Sec. 2 that was successful in the case of the strongly magnetized plasma. After the replacement of the photon polarization vector by the neutrino current, one can express the vector part of the sterile-neutrino decay amplitude through the photon polarization operator [26]:

$$M^V_{\rm pl} = \frac{C_V\, G_F}{e\sqrt{2}}\, (j \Pi \varepsilon^*)\ . \tag{26}$$

The corresponding axial-vector contribution to the decay amplitude is much smaller [26]. In a nonrelativistic plasma, one finds explicitly that it is suppressed by the factor $(C_A/C_V)\,(m_s/m_e) \ll 1$. In a relativistic plasma, the electron mass should be replaced by the chemical potential μ_e and the condition above remains correct again.

As mentioned earlier, photons in plasma have an effective mass called the plasma frequency. In the nonrelativistic limit, the plasma frequency can be expressed in terms of an electron number density [35]:

$$\omega_0^2 = \frac{4\pi\alpha n_e}{m_e}\,, \tag{27}$$

where the number density of degenerate electrons is defined as $n_e = p_F^3/(3\pi^2)$. Photons in plasma have three polarization states, one longitudinal ε^ℓ_α and two transverse ε^t_α, where $t = 1, 2$. Eigenvalues of the polarization operator for longitudinal and transverse modes are different. For a nonrelativistic plasma, the eigenvalues are well known [35]:

$$\Pi_t \approx \omega_0^2\,, \qquad \Pi_\ell \approx \omega_0^2\Big(1 - \frac{k^2}{\omega^2}\Big). \tag{28}$$

These eigenvalues determine dispersion relations of photon modes through the equation:

$$q^2 = \omega^2 - k^2 = \Pi_\lambda\,, \tag{29}$$

which should be included in the integration over the photon momentum.

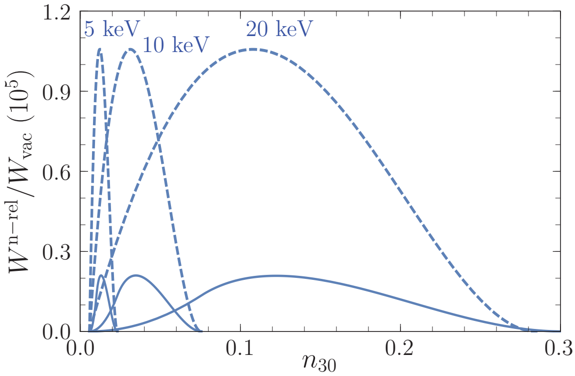

Figure 3. The decay probability $W^{\text{n-rel}}$ (in units of $10^5\, W_{\text{vac}}$) in the strongly magnetized (solid lines) and unmagnetized (dashed lines) non-relativistic plasma as a function of the electron number density $n_{30} = n_e/(10^{30}\ \text{cm}^{-3})$. The solid curves correspond to the magnetic field strength $B = B_e = 4.41\times10^{13}$ Gauss. Three values of the sterile-neutrino mass, $m_s = 5,\ 10,\ 20$ keV, (explicitly specified) are used to demonstrate the corresponding dependence of the decay width.

The decay width for longitudinal and transverse modes are known analytically [26]:

$$W^{t}_{\text{pl}} = W_{\text{vac}} \frac{16\pi^2}{9\alpha^2}\, C_V^2\, x_0^4 (1 - x_0^2)^2 \,, \qquad W^{\ell}_{\text{pl}} = W_{\text{vac}} \frac{32\pi^2}{9\alpha^2}\, C_V^2\, x_0 (1 - x_0)^2 \,, \tag{30}$$

where $x_0 = \omega_0/m_s$ is the scaled plasma frequency. The decay width W^{t}_{pl} and W^{ℓ}_{pl} are expressed in terms of the vacuum rate (22). The lifetime of the sterile neutrino with the mass of 10 keV is equal to 1.2×10^{26} years under the assumption that the mixing angle squared is $\theta_s^2 \simeq 10^{-11}$. Comparing this result with the age of the our Universe, $\tau_{\text{Univ}} = 1.37 \times 10^{10}$ years [28], we see that the sterile neutrino is stable to this decay even if we take into account the influence of the electron plasma or external magnetic field.

In Fig. 3 we compare the decay rate in the unmagnetized (dashed lines) and strongly magnetized (solid lines) plasma as a function of the electron number density for several fixed sterile-neutrino mass values. The strong catalyzing effect of the plasma is clearly seen with an enhancement of up to 5 orders of magnitude compared with vacuum. When the neutrino mass increases, the distribution over the electron number density becomes wider and maximum of this function moves to larger value of n_e. For the chosen field strength, $B = B_e = 4.41 \times 10^{13}$ Gauss, the decay rate is strongly suppressed as compared with the case of unmagnetized plasma, but it is still much larger than in vacuum. It is clear from this figure that the influence of plasma plays an important role in the relatively narrow range of plasma parameters, in particular, the electron number density. This could be explained by the fact that we have two opposite effects of the plasma influence on the decay width of sterile neutrino. On one hand, when the value of n_e increases, the amplitude and width of the decay in both unmagnetized and strongly magnetized plasma grow up. On the other hand, when the value of n_e increases, the plasma frequency grows up and can reach the sterile neutrino mass or becomes even larger. In this case the radiative decay of sterile neutrino will be kinematically suppressed or even forbidden.

DOI: 10.1051/epjconf/201715805005

4 Conclusions

We study the radiative decay of sterile neutrinos with keV-masses in presence of a strongly magnetized and unmagnetized electron plasma. The modified photon dispersion relation is included in the analysis. The limiting cases of relativistic and nonrelativistic magnetized plasma are considered. The decay rate in a strongly magnetized plasma as a function of the electron number density is compared with the one in the unmagnetized plasma. We found that the strong magnetic field suppresses by the order of magnitude the catalyzing influence of the plasma by itself on the decay rate.

Acknowledgments

A.D. acknowledges financial support by the *Russian Foundation for Basic Research* (Project No. 16-32-00066 мол-а and 15-02-06033-a), and by the *Dynasty Foundation*. Partial support by the Deutsche Forschungsgemeinschaft (DFG) under Grant No. EXC-153 (Cluster of Excellence "Origin and Structure of the Universe") and by the European Union under Grant No. PITN-GA-2011-289442 (FP7 Initial Training Network "Invisibles") is acknowledged.

References

[1] R.C. Duncan, C. Thompson, Astrophys. J. **392**, L9 (1992)
[2] C. Thompson, R.C. Duncan, Mon. Not. Roy. Astron. Soc. **275**, 255 (1995)
[3] C. Thompson, R.C. Duncan, Astrophys. J. **473**, 322 (1996)
[4] S.A. Olausen, V.M. Kaspi, Astrophys. J. Suppl. **212**, 6 (2014), 1309.4167
[5] A. Heger, S.E. Woosley, H.C. Spruit, Astrophys. J. **626**, 350 (2005), astro-ph/0409422
[6] H. Sawai, S. Yamada, Astrophys. J. **817**, 153 (2016), 1504.03035
[7] G.S. Bisnovatyi-Kogan, S.G. Moiseenko, N.V. Ardeljan, Acta Polytech. Proc. **1**, 181 (2014), 1408.2395
[8] M.V. Barkov, S.S. Komissarov, Mon. Not. Roy. Astron. Soc. **401**, 1644 (2010), 0908.0695
[9] I. Zalamea, A.M. Beloborodov, Mon. Not. Roy. Astron. Soc. **410**, 2302 (2011), 1003.0710
[10] A.V. Kuznetsov, N.V. Mikheev, *Electroweak Processes in External Electromagnetic Fields*, Vol. 197 of *Springer Tracts in Modern Physics* (Springer-Verlag, New York, 2004)
[11] A.V. Kuznetsov, N.V. Mikheev, *Electroweak Processes in External Active Media*, Vol. 252 of *Springer Tracts in Modern Physics* (Springer-Verlag, New York, 2013)
[12] P.B. Pal, L. Wolfenstein, Phys. Rev. **D25**, 766 (1982)
[13] V.N. Tsytovich, Sov. Phys. JETP **18**, 816 (1964)
[14] V.N. Oraevsky, V.B. Semikoz, Y.A. Smorodinsky, JETP Lett. **43**, 709 (1986)
[15] J.C. D'Olivo, J.F. Nieves, P.B. Pal, Phys. Rev. Lett. **64**, 1088 (1990)
[16] R.F. Sawyer, Phys. Rev. **D46**, 1180 (1992)
[17] J.C. D'Olivo, J.F. Nieves, P.B. Pal, Phys. Lett. **B365**, 178 (1996), hep-ph/9509415
[18] S.J. Hardy, D.B. Melrose, Phys. Rev. **D54**, 6491 (1996)
[19] D.V. Gal'tsov, N.S. Nikitina, Sov. Phys. JETP **35**, 1047 (1972)
[20] V.V. Skobelev, Sov. Phys. JETP **44**, 660 (1976)
[21] A.N. Ioannisian, G.G. Raffelt, Phys. Rev. **D55**, 7038 (1997), hep-ph/9612285
[22] A.A. Gvozdev, N.V. Mikheev, L.A. Vasilevskaya, Phys. Rev. **D54**, 5674 (1996), hep-ph/9610219

[23] A.A. Gvozdev, N.V. Mikheev, L.A. Vasilevskaya, Phys. Lett. **B410**, 211 (1997), hep-ph/9702285

[24] M.V. Chistyakov, N.V. Mikheev, Phys. Lett. **B467**, 232 (1999), hep-ph/9907345

[25] A.I. Ternov, P.A. Eminov, Phys. Rev. **D87**, 113001 (2013)

[26] A.A. Dobrynina, N.V. Mikheev, G.G. Raffelt, Phys. Rev. **D90**, 113015 (2014), 1410.7915

[27] H. Perez Rojas, A.E. Shabad, Annals Phys. **121**, 432 (1979)

[28] C. Patrignani et al. (Particle Data Group), Chin. Phys. **C40**, 100001 (2016)

[29] M. Drewes et al., JCAP **1701**, 025 (2017), 1602.04816

[30] S. Tremaine, J.E. Gunn, Phys. Rev. Lett. **42**, 407 (1979)

[31] A. Boyarsky, O. Ruchayskiy, M. Shaposhnikov, Ann. Rev. Nucl. Part. Sci. **59**, 191 (2009), 0901.0011

[32] M. Laine, M. Shaposhnikov, JCAP **0806**, 031 (2008), 0804.4543

[33] V.B. Berestetsky, E.M. Lifshitz, L.P. Pitaevsky, *Course of Theoretical Physics. Vol. 4. Quantum Electrodynamics* (Pergamon, Oxford, 1982)

[34] M.E. Peskin, D.V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Massachusetts, 1995)

[35] G.G. Raffelt, *Stars as Laboratories for Fundamental Physics* (University of Chicago Press, Chicago & London, 1996)

[36] M.V. Chistyakov, D.A. Rumyantsev, J. Exp. Theor. Phys. **107**, 533 (2008), 0811.4526