DOI: 10.1051/epjconf/201715805007

Neutrino-electron scattering in a dense strongly magnetized plasma

Alexander Kuznetsov[1,⋆] and *Vasiliy* Savin[1,⋆⋆]

[1] *Division of Theoretical Physics, Department of Physics, Yaroslavl State P. G. Demidov University, Sovietskaya 14, 150000 Yaroslavl, Russian Federation*

Abstract. We investigate the process of neutrino-electron scattering in a dense plasma and magnetic field of arbitrary strength, where electrons can occupy the states corresponding to excited Landau levels. We calculate the total probability of this process, summarized over all initial states of the plasma electrons which is only physically meaningful. Possible astrophysical manifestations of the process are briefly discussed.

1 Introduction

Astroparticle physics has manifested itself in recent decades as a vigorously growing and prospective line of investigation at the junction of particle physics, astrophysics, and cosmology; see e.g. [1–3]. An important stimulus of its development is an understanding of the essential role of quantum processes in the dynamics of astrophysical objects and of the early Universe. A decisive role in astrophysical cataclysms such as supernova explosions and coalescence of neutron stars, as well as in the early Universe, belongs to neutrino physics. Consequently, studies of neutrino interactions and in particular neutrino-electron processes in an external active medium are of considerable interest. At the same time, an investigation of neutrino processes under such extreme physical conditions is interesting from the conceptual viewpoint since it affects fundamental problems of quantum field theory.

For self-consistent analysis of the neutrino propagation process in a hot dense plasma in the presence of a strong magnetic field, consideration is required of a complete set of neutrino-electron processes. Then only the probability of the process summarized over all initial states of the plasma electrons is physically meaningful. In Ref. [4], numerical calculations of the differential cross-section of the neutrino-electron scattering in dense magnetized plasma were performed in the limit of rather weak magnetic field B, $eB < \mu E$, where μ is the plasma chemical potential, E is the typical neutrino energy (we use natural units in which $c = \hbar = 1$, $e > 0$ is the elementary charge). In Refs. [5, 6], the probability was evaluated of the total sum of νe processes ($\nu \to \nu e^- e^+$, $\nu e^\mp \to \nu e^\mp$, $\nu e^- e^+ \to \nu$) and the volume density of the neutrino energy and momentum losses, integrated over the momenta of the plasma electrons, in a strong magnetic field, $eB \gg (\mu^2, T^2, E^2) \gg m_e^2$, ($T$ is the plasma temperature); in such conditions electrons and positrons occupied only the ground Landau levels. In Refs. [7, 8], the probability of the $\nu e \to \nu e$ process and the volume density of the neutrino energy and momentum losses, summarized over all initial states of the plasma electrons, were evaluated under the physical

⋆e-mail: avkuzn@uniyar.ac.ru
⋆⋆e-mail: vs_post07@mail.ru

DOI: 10.1051/epjconf/201715805007

situation where the magnetic field is moderate, while the density of plasma is large, so the conditions are satisfied: $\mu^2 > eB \gg (T^2, E^2) \gg m_e^2$, where T is the plasma temperature, and $eB \gg \mu E$. Calculations were performed for the case when both initial and final electrons occupy the same Landau levels, because, as it was concluded in Refs. [7, 8], such transitions were dominating. The purpose of the present research is to calculate analytically the probability of this process for a more general case when the initial and final electrons could occupy any physically allowed Landau levels. Some details of the calculation technique can be found e.g. in Refs. [9–12].

2 Solutions of the Dirac equation for an electron in a magnetic field

There exist several descriptions of the procedure of obtaining the electron wave functions in the presence of an external magnetic field by solving the Dirac equation, see e.g. Refs. [13–19] and also Refs. [9, 10]. In the most cases, the solutions are presented in the form with the upper two components of the bispinor corresponding to the electron states with the spin projections 1/2 and -1/2 on the magnetic field direction. As the analysis has shown, it is optimal to use the electron wave functions as the eigenstates of the covariant operator of a magnetic polarization $\hat{\mu}_z$ [15, 16]:

$$\hat{\mu}_z = m_e \Sigma_z - \mathrm{i}\gamma_0\gamma_5 [\boldsymbol{\Sigma} \times \boldsymbol{P}]_z \,, \tag{1}$$

where $\boldsymbol{P} = -\mathrm{i}\,\boldsymbol{\nabla} + e\boldsymbol{A}$. We take the frame where the field is directed along the z axis, and the Landau gauge where the four-potential is: $A^\lambda = (0, 0, xB, 0)$. In this approach, the electron wave functions have the form

$$\Psi^s_{p,n}(X) = \frac{1}{\sqrt{4\varepsilon_n M_n(\varepsilon_n + M_n)(M_n + m_e) L_y L_z}}\, \mathrm{e}^{-\mathrm{i}(\varepsilon_n t - p_y y - p_z z)}\; U^s_n(\xi)\,, \tag{2}$$

where $\varepsilon_n = \sqrt{M_n^2 + p_z^2}$, $M_n = \sqrt{m_e^2 + 2\beta n}$, $\beta = eB$, $\xi = \sqrt{\beta}\left(x + p_y/\beta\right)$. The functions $\Psi^s_{p,n}(X)$ satisfy the equation: $\hat{\mu}_z\, \Psi^s_{p,n}(X) = s\, M_n\, \Psi^s_{p,n}(X)\, (s = \pm 1)$. The bispinors $U^s_n(\xi)$ in Eq. (2) take the form:

$$U^-_n(\xi) = \begin{pmatrix} -\mathrm{i}\sqrt{2\beta n}\, p_z V_{n-1}(\xi) \\ (\varepsilon_n + M_n)(M_n + m_e) V_n(\xi) \\ -\mathrm{i}\sqrt{2\beta n}(\varepsilon_n + M_n) V_{n-1}(\xi) \\ -p_z (M_n + m_e) V_n(\xi) \end{pmatrix}, \quad U^+_n(\xi) = \begin{pmatrix} (\varepsilon_n + M_n)(M_n + m_e) V_{n-1}(\xi) \\ -\mathrm{i}\sqrt{2\beta n}\, p_z V_n(\xi) \\ p_z (M_n + m_e) V_{n-1}(\xi) \\ \mathrm{i}\sqrt{2\beta n}(\varepsilon_n + M_n) V_n(\xi) \end{pmatrix}. \tag{3}$$

Here, $V_n(\xi)\, (n = 0, 1, 2, \dots)$ are the well-known normalized harmonic oscillator functions, which are expressed in terms of the Hermite polynomials $H_n(\xi)$:

$$V_n(\xi) = \frac{\beta^{1/4} \mathrm{e}^{-\xi^2/2}}{\sqrt{2^n n!\, \sqrt{\pi}}}\, H_n(\xi)\,, \tag{4}$$

With using the solutions of the Dirac equation for an electron in a magnetic field in the form (2), (3), the amplitude of any process will have an explicit Lorentz invariant structure. It is in contrast to the case when the solutions are presented in the form with the upper two components of the bispinor corresponding to the electron states with the spin projections 1/2 and -1/2 on the magnetic field direction.

3 The process of the $\nu e \to \nu e$ scattering

The effective local Lagrangian of the neutrino-electron interaction is:

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}}[\bar{e}\gamma_\alpha(C_V - C_A\gamma_5)e]\,[\bar{\nu}\gamma^\alpha(1-\gamma_5)\nu], \quad C_V = \pm\frac{1}{2} + 2\sin^2\theta_W\,,\; C_A = \pm\frac{1}{2}\,, \tag{5}$$

where the upper signs correspond to ν_e and the lower signs correspond to $\nu_{\mu,\tau}$, θ_W is the Weinberg angle.

The S matrix element of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$ takes the form

$$\begin{aligned} S &= \mathrm{i}\,\frac{G_F}{\sqrt{2}}\,\frac{(2\pi)^3\,\delta(\varepsilon'_n - \varepsilon_\ell - q_0)\,\delta(p'_y - p_y - q_y)\,\delta(p'_z - p_z - q_z)}{\sqrt{2EV\,2E'V\,2\varepsilon_\ell L_y L_z\,2\varepsilon'_n L_y L_z}}\,\mathrm{e}^{-q_\perp^2/4eB - \mathrm{i}q_x(p_y+p'_y)/2eB} \\ &\times\; [\bar{u}(p')\,(j\gamma)\,(C_V - C_A\gamma_5)\,u(p)]\,, \end{aligned} \tag{6}$$

where $q = P - P' = p' - p$ is the 4-momentum transferred, ε_ℓ and ε'_n are the energies of the initial and final electrons, $q_\perp$ is the projection of the vector $\boldsymbol{q}$ on the plane perpendicular to the vector $\boldsymbol{B}$, $q_\perp^2 = q_x^2 + q_y^2$, and $j_\alpha = \bar{\nu}(P')\gamma_\alpha(1-\gamma_5)\nu(P)$ is the Fourier transform of the current of the left-handed neutrinos.

The total process probability per unit time can be presented in the form

$$W(\nu e^- \to \nu e^-) = \frac{1}{\mathcal{T}}\sum_\ell \sum_n \sum_{s,s'} \int |\, S\,|^2\; \mathrm{d}n_{e^-}\,\mathrm{d}n'_{e^-}\,\frac{\mathrm{d}^3P'\,V}{(2\pi)^3}\,(1 - f(E'))\,, \tag{7}$$

where $\mathrm{d}n_{e^-} = f(\varepsilon_\ell)\,\mathrm{d}p_y\,\mathrm{d}p_z\,L_y\,L_z/(2\pi)^2\,,\; \mathrm{d}n'_{e^-} = (1 - f(\varepsilon'_n))\,\mathrm{d}p'_y\,\mathrm{d}p'_z\,L_y\,L_z/(2\pi)^2\,,\; f(\varepsilon_\ell)$ is the fermion distribution function, e.g. for initial electrons it is: $f(\varepsilon_\ell) = [\mathrm{e}^{(\varepsilon_\ell-\mu)/T} + 1]^{-1}$; $\mathcal{T}$ is the total time of interaction, $V = L_x\,L_y\,L_z$ is the total volume of the interaction region.

For the probability per unit time of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$ one obtains

$$\begin{aligned} W_{\ell n} &= \frac{\beta}{(2\pi)^4 16E}\int \frac{\mathrm{d}^3P'}{E'}\,(1 - f(E')) \\ &\times \int \frac{\mathrm{d}p_z}{\varepsilon'_n\,\varepsilon_\ell}\,\delta(\varepsilon'_n - \varepsilon_\ell - q_0)\,f(\varepsilon_\ell)\,(1 - f(\varepsilon'_n))\sum_{s,s'}|\mathcal{M}^{ss'}_{\ell n}|^2\,. \end{aligned} \tag{8}$$

Taking into account possible polarisation states of the initial and final electrons, there exist four invariant polarization amplitudes $\mathcal{M}^{ss'}_{\ell n}$, where $s, s' = \pm 1$, which can be presented in the form:

$$\mathcal{M}^{ss'}_{\ell n} = \eta^{ss'}\,\frac{G_F}{2\sqrt{2}}\left\{A^{ss'}_{\ell n}\,\frac{(j\tilde{\varphi}q)}{q_\parallel^2} + B^{ss'}_{\ell n}\,\frac{(j\widetilde{\Lambda}q)}{q_\parallel^2} + C^{ss'}_{\ell n}\,\frac{(j\Lambda q)}{q_\perp^2} + D^{ss'}_{\ell n}\,\frac{\mathrm{i}\,(j\varphi q)}{q_\perp^2}\right\}, \tag{9}$$

where $\eta^{ss'}$ is an inessential phase factor, $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ is the dimensionless tensor of the external magnetic field, $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\,\varepsilon_{\alpha\beta\mu\nu}\varphi^{\mu\nu}$ is the dual dimensionless tensor; the four-vectors with the indices $\perp$ and $\parallel$ belong to the Euclidean $\{1, 2\}$-subspace and the pseudo-Euclidean $\{0, 3\}$-subspace, correspondingly, and $\Lambda_{\alpha\beta} = (\varphi\varphi)_{\alpha\beta} = \mathrm{diag}(0, 1, 1, 0)$, $\widetilde{\Lambda}_{\alpha\beta} = (\tilde{\varphi}\tilde{\varphi})_{\alpha\beta} = \mathrm{diag}(1, 0, 0, -1)$. In Eq. (9), auxiliary functions are introduced: A, B, C, D. We give here their explicit forms for all possible sets of polarization.

For the case $s = s' = -1$, one obtains:

$$\begin{aligned} A^{--}_{\ell n} &= \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\mathcal{I}_{\ell,n}\,(C_V\,\tau + C_A\,\kappa) \\ &+ \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\mathcal{I}_{\ell-1,n-1}(C_V\,\tau - C_A\,\kappa)\,, \end{aligned} \tag{10}$$

$$\begin{aligned} B^{--}_{\ell n} &= \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\mathcal{I}_{\ell,n}\,(C_V\,\kappa + C_A\,\tau) \\ &+ \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\mathcal{I}_{\ell-1,n-1}(C_V\,\kappa - C_A\,\tau)\,, \end{aligned} \tag{11}$$

$$\begin{aligned} C^{--}_{\ell n} &= \sqrt{q_\perp^2}\,\Bigg[-\sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\mathcal{I}_{\ell,n-1}\,(C_V\,u + C_A\,v) \\ &- \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\mathcal{I}_{\ell-1,n}\,(C_V\,u - C_A\,v)\Bigg]\,, \end{aligned} \tag{12}$$

$$\begin{aligned} D^{--}_{\ell n} &= \sqrt{q_\perp^2}\,\Bigg[\sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\mathcal{I}_{\ell,n-1}\,(C_V\,u + C_A\,v) \\ &- \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\mathcal{I}_{\ell-1,n}\,(C_V\,u - C_A\,v)\Bigg]\,, \end{aligned} \tag{13}$$

where, for $n \geqslant \ell$

$$\mathcal{I}_{n,\ell}(x) = \sqrt{\frac{\ell!}{n!}}\;\mathrm{e}^{-x/2}x^{(n-\ell)/2}L_\ell^{n-\ell}(x)\,,\quad \mathcal{I}_{\ell,n}(x) = (-1)^{n-\ell}\mathcal{I}_{n,\ell}(x)\,, \tag{14}$$

$L_n^k(x)$ are the generalized Laguerre polynomials, and also:

$$\kappa = u\,(M_n - M_\ell)\,,\ \tau = v\,(M_n + M_\ell)\,,\ u = \sqrt{(M_n + M_\ell)^2 - q_\parallel^2}\,,\ v = \alpha\,\zeta\,\sqrt{(M_n - M_\ell)^2 - q_\parallel^2}\,.$$

Here, $\alpha = q_0/|q_0|$ is the sign of q_0, $\zeta = \pm 1$ is the sign factor associated with the two roots of the equation, corresponding to the zeros of the δ function argument in Eq. (8). In the frame where $q_z = 0$, ζ is the sign of the p_z component, which is not fixed by the equation.

For the case $s = s' = +1$, one obtains:

$$\begin{aligned} A^{++}_{\ell n} &= \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\mathcal{I}_{\ell,n}\,(C_V\,\tau + C_A\,\kappa) \\ &+ \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\mathcal{I}_{\ell-1,n-1}(C_V\,\tau - C_A\,\kappa)\,, \end{aligned} \tag{15}$$

$$\begin{aligned} B_{\ell n}^{++} &= \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n}\,(C_V\,\kappa + C_A\,\tau) \\ &+ \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n-1}(C_V\,\kappa - C_A\,\tau)\,, \end{aligned} \tag{16}$$

$$\begin{aligned} C_{\ell n}^{++} &= \sqrt{q_\perp^2}\,\Bigg[-\sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n-1}\,(C_V\,u + C_A\,v) \\ &- \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n}\,(C_V\,u - C_A\,v)\Bigg], \end{aligned} \tag{17}$$

$$\begin{aligned} D_{\ell n}^{++} &= \sqrt{q_\perp^2}\,\Bigg[\sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n-1}\,(C_V\,u + C_A\,v) \\ &- \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n}\,(C_V\,u - C_A\,v)\Bigg]. \end{aligned} \tag{18}$$

For the case $s = +1,\ s' = -1$, one obtains:

$$\begin{aligned} A_{\ell n}^{+-} &= \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n}\,(C_V\,\kappa + C_A\,\tau) \\ &- \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n-1}(C_V\,\kappa - C_A\,\tau)\,, \end{aligned} \tag{19}$$

$$\begin{aligned} B_{\ell n}^{+-} &= \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n}\,(C_V\,\tau + C_A\,\kappa) \\ &- \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n-1}(C_V\,\tau - C_A\,\kappa)\,, \end{aligned} \tag{20}$$

$$\begin{aligned} C_{\ell n}^{+-} &= \sqrt{q_\perp^2}\,\Bigg[\sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n-1}\,(C_V\,v + C_A\,u) \\ &- \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n}\,(C_V\,v - C_A\,u)\Bigg], \end{aligned} \tag{21}$$

$$
\begin{aligned}
D^{+-}_{\ell n} &= \sqrt{q_\perp^2}\Big[-\sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n-1}\,(C_V\,v + C_A\,u) \\
&- \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n}\,(C_V\,v - C_A\,u)\Big]. \qquad (22)
\end{aligned}
$$

For the case $s=-1,\ s'=+1$, one obtains:

$$
\begin{aligned}
A^{-+}_{\ell n} &= \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n}\,(C_V\,\kappa + C_A\,\tau) \\
&- \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n-1}(C_V\,\kappa - C_A\,\tau)\,, \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
B^{-+}_{\ell n} &= \sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n}\,(C_V\,\tau + C_A\,\kappa) \\
&- \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n-1}(C_V\,\tau - C_A\,\kappa)\,, \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
C^{-+}_{\ell n} &= \sqrt{q_\perp^2}\Big[\sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n-1}\,(C_V\,v + C_A\,u) \\
&- \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n}\,(C_V\,v - C_A\,u)\Big], \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
D^{-+}_{\ell n} &= -\sqrt{q_\perp^2}\Big[\sqrt{\left(1+\frac{m}{M_\ell}\right)\left(1+\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell,n-1}\,(C_V\,v + C_A\,u) \\
&+ \sqrt{\left(1-\frac{m}{M_\ell}\right)\left(1-\frac{m}{M_n}\right)}\,\mathcal{I}_{\ell-1,n}\,(C_V\,v - C_A\,u)\Big]. \qquad (26)
\end{aligned}
$$

Calculations of the process probability per unit time (7) with Eqs. (8)–(26) should be performed numerically for different values of the physical parameters B, T, μ, E etc., and these calculations now are in progress. It should be noted, that, in contrast to calculations of Refs. [11, 12], in the present case no upper limits arise on the Landau level numbers ℓ, n from kinematics, and the suppression of the large number contributions is provided by the distribution functions of initial and final electrons.

To illustrate, we present in the figure 1 the results of numerical calculations of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$ probability per unit time (8) as the function of Landau level numbers of the initial and final electrons, for fixed values of the physical parameters B, T, μ, E.

As the analysis shows, the assumption made in Refs. [7, 8], that the subprocesses were dominating where both initial and final electrons occupied the same Landau levels, is not confirmed. This means that the results for the probability of the $\nu e \to \nu e$ process obtained in Refs. [7, 8], were underestimated.

DOI: 10.1051/epjconf/201715805007

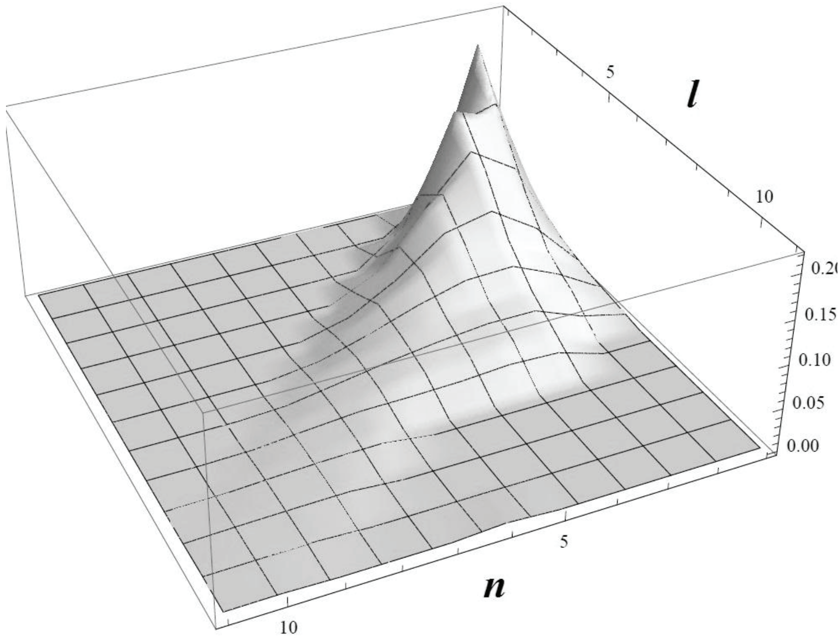

Figure 1. The dependence of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$ probability per unit time on the Landau level numbers of the initial and final electrons

4 Conclusions

The probability of the $\nu e \to \nu e$ process in a dense magnetized plasma is calculated analytically, for a general case when the initial and final electrons could occupy any physically allowed Landau levels. The analysis shows, that the assumption made in previous calculations, that the subprocesses were dominating where both initial and final electrons occupied the same Landau levels, is not confirmed.

In astrophysical applications, the mean values of the neutrino energy and momentum losses could be more interesting:

$$Q^\alpha \; = \; E \int (P - P')^\alpha \, \mathrm{d}W = -E \, (\mathcal{I}, \boldsymbol{F}) \, , \tag{27}$$

where dW is the total differential probability of the process which can be extracted from Eq. (7). The zeroth component of Q^α is connected with the mean energy lost by a neutrino per unit time due to the process considered, $\mathcal{I} = \mathrm{d}E/\mathrm{d}t$. The space components of the four-vector Q^α are similarly connected with the mean neutrino momentum loss per unit time, $\boldsymbol{F} = \mathrm{d}\boldsymbol{P}/\mathrm{d}t$. An analysis of the four-vector Q^α in a general case for the magnetic field of arbitrary strength, where electrons can occupy the states corresponding to excited Landau levels, now is in progress. The force density $\boldsymbol{F}$ could lead to a very interesting consequences if a strong toroidal magnetic field is generated in the supernova envelope, providing an asymmetry of the supernova explosion and, in particular, it can explain the phenomenon of high pulsar kick-velocities, for details see Ref. [10].

Acknowledgement

The paper is done within "YSU Initiative Scientific Researches" (Project No. AAAA-A16-116070610023-3).

References

[1] G. G. Raffelt, *Stars as Laboratories for Fundamental Physics* (Univ. Chicago Press, Chicago, 1996)
[2] M. Yu. Khlopov, *Cosmoparticle Physics* (World Scientific, Singapore, 1999)
[3] H. V. Klapdor-Kleingrothaus and K. Zuber, *Particle Astrophysics* (Inst. of Phys., Bristol, 2000)
[4] V. G. Bezchastnov and P. Haensel, Phys. Rev. D **54**, 3706 (1996)
[5] A. V. Kuznetsov and N. V. Mikheev, Mod. Phys. Lett. A **14**, 2531 (1999)
[6] A. V. Kuznetsov and N. V. Mikheev, Zh. Eksp. Teor. Fiz. **118**, 863 (2000) [JETP **91**, 748 (2000)]
[7] N. V. Mikheev and E. N. Narynskaya, Mod. Phys. Lett. A **15**, 1551 (2000)
[8] N. V. Mikheev and E. N. Narynskaya, Centr. Eur. J. Phys. **1**, 145 (2003)
[9] A. V. Kuznetsov and N. V. Mikheev, *Electroweak Processes in External Electromagnetic Fields* (Springer-Verlag, New York, 2003)
[10] A. V. Kuznetsov and N. V. Mikheev, *Electroweak Processes in External Active Media* (Springer-Verlag, Berlin, Heidelberg, 2013)
[11] A. V. Kuznetsov, D. A. Rumyantsev, and V. N. Savin, Int. J. Mod. Phys. A **29**, 1450136 (2014)
[12] A. V. Kuznetsov, D. A. Rumyantsev, and V. N. Savin, J. Phys.: Conf. Ser. **675**, 032019 (2016)
[13] M. H. Johnson and B. A. Lippmann, Phys. Rev. **76**, 828 (1949)
[14] A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics* (Wiley, New York 1965)
[15] A. A. Sokolov and I. M. Ternov, *Synchrotron Radiation* (Pergamon, Oxford, 1968)
[16] D. B. Melrose and A. J. Parle, Aust. J. Phys. **36**, 755 (1983)
[17] A. A. Sokolov and I. M. Ternov, *Radiation from Relativistic Electrons* (American Institute of Physics, New York 1986)
[18] K. Bhattacharya and P. B. Pal, Pramana J. Phys. **62**, 1041 (2004)
[19] I. A. Balantsev, Yu. V. Popov, and A. I. Studenikin, J. Phys. A **44**, 255301 (2011)