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Inter-brane distance stabilization by bulk Higgs field in RS model

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Abstract. An extension of the Standard Model based on the Randall-Sundrum model with two branes is considered. The size of the extra dimension is stabilized by a five-dimensional two-component complex scalar field, the bulk Higgs field. All the matter fields, which are supposed to be localized on the brane with negative tension, acquire their masses due to the interaction with the boundary value of this field. The gauge invariance of the theory demands that the electroweak gauge fields also propagate in the bulk. The equations of motion for the background fields and their fluctuations against a background solution are found. The second variation Lagrangian is derived. The interactions of the bulk Higgs field with the multidimensional gauge and fermion fields are studied and a possible background solution is obtained.

1 Introduction

Models with extra space-time dimensions are widely discussed in theoretical physics in these latter days. One of the most interesting models of such type is the Randall-Sundrum model. Within its framework one considers two four-dimensional branes with tension interacting with gravity in a five-dimensional space-time [1]. The extra dimension forms the orbifold S^1/Z_2 and the branes are located at its fixed points. It is assumed that our world is located on one of the branes. The background metric is not flat, it has an exponential warp factor, which provides a solution to the hierarchy problem of the gravitational interaction.

In case the inter-brane distance is not stabilized, i.e. it is arbitrary, the model predicts the existence of a massless scalar mode, the radion, which couples to matter much stronger than gravity. It is not **in agreement** already with the classical gravity. In order to make the RS model phenomenologically acceptable, the Goldberger-Wise field is introduced into it [2, 3]. It is a 5D scalar field with a bulk potential and additional potentials on the branes. This allows one to stabilize the extra dimension size, giving a mass to the radion. The stabilization is achieved due to boundary conditions for the Goldberger-Wise field on the branes.

The idea of this report is to stabilize the size of extra dimension by a two-component complex scalar field propagating in the bulk and carrying the same representation of the gauge group $SU(2) \times U(1)$ as the usual Higgs field. It will act as the Higgs field on the brane, where our world is supposed to be located, providing spontaneous symmetry breaking. All the Standard Model fermion fields,

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localized on the brane, will acquire their masses through the interaction with the boundary value of this field. Thus, we introduce an object, which can be called the bulk Higgs field.

The stabilization of the extra dimension size by the 5D Higgs field was discussed earlier in papers by L. Vecchi [4] and M. Geller et al. In the latter paper a perturbative background solution for gravity and the bulk scalar field was considered. Furthermore, the fact that the 5D theory is gauge invariant implies that there are corresponding gauge fields in the bulk. These fields have not been taken into consideration in the mentioned papers. Here we try to find an exact vacuum solution for gravity and the bulk Higgs field, while taking the 5D gauge fields into account in the linearized theory.

2 Equations of motion for background fields

Let us consider gravity interacting with two branes, a two-component scalar field $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ and bulk gauge fields A_M and B_M in a five-dimensional space-time $E = M_4 \times S^1/Z_2$. The action of the model can be written as follows:

$$S = S_g + S_\phi + S_{gauge} + S_{brane+SM}, \tag{1}$$

where the gravitational action S_g is

$$S_g = 2M^3 \int d^4x \int\limits_{-L}^{L} dy R \sqrt{g}, \tag{2}$$

S_ϕ stands for the action of the two-component complex scalar field,

$$S_\phi = M \int d^4x \int\limits_{-L}^{L} dy \left[(D_M\phi)^+ D^M\phi - V(\phi^+\phi) \right] \sqrt{g}, \tag{3}$$

the action S_{gauge} of the gauge fields is given by

$$S_{gauge} = -\int d^4x \int\limits_{-L}^{L} dy \left[\frac{1}{4p^2} A^a_{MN} A^{a,MN} + \frac{1}{4q^2} B_{MN} B^{MN} \right] \sqrt{g}, \tag{4}$$

and $S_{brane+SM}$ is the action of the branes and the Standard Model,

$$S_{brane+SM} = -\int_{y=0} d^4x \lambda_1 (\phi^+\phi) \sqrt{-\tilde{g}} + \int_{y=L} d^4x \left[-\lambda_2 (\phi^+\phi) + L_{SM-HP} (\phi, \phi^+) \right] \sqrt{-\tilde{g}}. \tag{5}$$

Here D_M is the covariant derivative, $M = \frac{1}{2}(4\pi\hat{G})^{-\frac{1}{3}}$ stands for the fundamental five-dimensional energy scale of the theory, where $\hat{G}$ is the five-dimensional gravitational constant; p and q denote the gauge coupling constants of the groups $SU(2)$ and $U(1)$, respectively. L_{SM-HP} is the Lagrangian of the Standard model without the Higgs potential and the electroweak gauge field Lagrangian. The scalar field Lagrangian is chosen in the way that the field ϕ has the dimension of mass, i.e. the same dimension as a four-dimensional scalar field.

A background solution is sought in the standard form, which preserves the Poincaré invariance in any four-dimensional subspace $y = const$:

$$ds^2 = \gamma_{MN} dx^M dx^N = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \tag{6}$$

$$\phi(x, y) = \phi(y)$$

and

$$A_\mu(x, y) = 0, \qquad A_4(x, y) = A_4(y), \qquad B_\mu(x, y) = 0, \qquad B_4(x, y) = B_4(y).$$

Variation of the action gives the equations of motion for the background fields:

$$\frac{1}{2}\left(\phi'^{+}\phi' + V + \frac{\lambda_1}{M}\delta(y) + \frac{\lambda_2}{M}\delta(y-L)\right) = 2M^2\left(3A'' - 6(A')^2\right), \tag{7}$$

$$12M^2(A')^2 + \frac{1}{2}\left(V - \phi'^{+}\phi'\right) = 0, \tag{8}$$

$$\frac{dV}{d\phi} + \frac{1}{M}\frac{d\lambda_1}{d\phi}\delta(y) + \frac{1}{M}\frac{d\lambda_2}{d\phi}\delta(y-L) = \phi''^{+} - 4A'\phi'^{+}, \tag{9}$$

$$\frac{dV}{d\phi^+} + \frac{1}{M}\frac{d\lambda_1}{d\phi^+}\delta(y) + \frac{1}{M}\frac{d\lambda_2}{d\phi^+}\delta(y-L) = \phi'' - 4A'\phi', \tag{10}$$

$$A_M = 0, \tag{11}$$

$$B_M = 0. \tag{12}$$

The requirement of the Poincare invariance leads to the vanishing of the background gauge field configurations. Thus, they do not affect the vacuum solution for the metric and the Higgs field.

3 Equations for the field fluctuations

Suppose we have a solution to these equations. In order to construct the linearized theory we represent the metric, the scalar field and the vector fields in the form "background solution + fluctuation":

$$g_{MN}(x, y) = \gamma_{MN}(y) + \frac{1}{\sqrt{2M^3}} h_{MN}(x, y), \tag{13}$$

$$\phi(x, y) = \phi_0(y) + f(x, y).$$

The gauge fields can be treated as fluctuations, because their vacuum values are zero.

Substituting this representation into the Lagrangian of the model and retaining only the terms of the second order in the deviations, we derive the so-called the second variation Lagrangian for the field fluctuations against a vacuum solution:

$$
\begin{aligned}
\frac{L^{(2)}}{\sqrt{\gamma}} &= \frac{1}{4}\left(\nabla_S h_{MN}\nabla^S h^{MN} + 2\nabla_M h^{MN}\nabla_N h - 2\nabla_M h^{MN}\nabla^S h_{SN} - \nabla_S h \nabla^S h\right) - \\
&- A''\left(h_{MN}h^{MN} - \frac{1}{2}h\tilde{h} + \frac{1}{2}h_{M\nu}h^{M\nu}\right) + (A')^2\left(\frac{7}{2}h_{MN}h^{MN} - hh\right) + \\
&+ \frac{1}{4M^2}\Bigg[\frac{V}{2}\left(h_{MN}h^{MN} - \frac{1}{2}hh\right) + \frac{1}{2M}\left[\lambda_1\delta(y) + \lambda_2\delta(y-L)\right]\left(h_{\mu\nu}h^{\mu\nu} - \frac{1}{2}\tilde{h}\tilde{h}\right) + \\
&\qquad + \phi_0'^{+}\phi_0'\left(-\frac{1}{4}hh + \frac{1}{2}h_{MN}h^{MN} - hh_{44} + 2h_{M4}h^{M4}\right)\Bigg] + \\
&+ M\Bigg[\partial_M f^+\partial^M f + \phi_0^+\left(\frac{1}{4}A^a_M A^{a,M} + B_M B^M + A^a_M b^M \tau^a\right)\phi_0 + \\
&\qquad + iA^a_M\left(\phi_0^+\frac{\tau^a}{2}\partial^M f - \partial^M f^+\frac{\tau^a}{2}\phi_0\right) - iA^a_4\left(f^+\frac{\tau^a}{2}\phi_0' - \phi_0'^{+}\frac{\tau^a}{2}f\right) + \\
&\qquad + iB_M\left(\phi_0^+\partial^M f - \partial^M f^+\phi_0\right) - iB_4\left(f^+\phi_0' - \phi_0'^{+}f\right) - \\
&\qquad - \frac{1}{2}\left(\frac{d^2V}{(d\phi)^2}ff + 2f^+\frac{d^2V}{d\phi^+ d\phi}f + f^+f^+\frac{d^2V}{(d\phi^+)^2}\right)\Bigg] - \\
&- \frac{1}{2}\left[\frac{1}{p^2}\left(\partial_M A^a_N - \partial_N A^a_M\right)\partial^M A^a_K + \frac{1}{q^2}\left(\partial_M B_N - \partial_N B_M\right)\partial^M B_K\right]\gamma^{NK} - \\
&- \frac{1}{2}\Bigg[\left(\frac{d^2\lambda_1}{(d\phi)^2}ff + 2f^+\frac{d^2\lambda_1}{d\phi^+ d\phi}f + f^+f^+\frac{d^2\lambda_1}{(d\phi^+)^2}\right)\delta(y) + \\
&\qquad + \left(\frac{d^2\lambda_2}{(d\phi)^2}ff + 2f^+\frac{d^2\lambda_2}{d\phi^+ d\phi}f + f^+f^+\frac{d^2\lambda_2}{(d\phi^+)^2}\right)\delta(y-L)\Bigg] + \\
&+ \frac{1}{\sqrt{2M}}\left[\left(\phi_0'^{+}\partial^M f + \partial^M f^+\phi_0'\right)h_{M4} - \frac{1}{2}\left(\phi_0'^{+}f' + f'^{+}\phi_0' + \frac{dV}{d\phi}f + f^+\frac{dV}{d\phi^+}\right)h\right] - \\
&- \frac{1}{2\sqrt{2M^3}}\left[\left(\frac{d\lambda_1}{d\phi}f + f^+\frac{d\lambda_1}{d\phi^+}\right)\delta(y) + \left(\frac{d\lambda_2}{d\phi}f + f^+\frac{d\lambda_2}{d\phi^+}\right)\delta(y-L)\right]\tilde{h}. \qquad (14)
\end{aligned}
$$

Here $\gamma = \det\gamma_{MN}$, $h = h_{MN}\gamma^{MN}$, $\tilde{h} = h_{\mu\nu}\gamma^{\mu\nu}$ and ∇_M denotes the covariant derivative with respect to the background metric γ_{MN}. The indices are raised using the metric γ^{MN}.

Varying the action built with this Lagrangian one obtains the equations of motion for the fluctuations. Choosing a vacuum solution $\phi_0(y)$ having only a real lower component $v(y)/\sqrt{2}$, which breaks the gauge group $SU(2)\times U(1)$ to $U(1)_{em}$, and passing to the gauge, where the scalar field $f(x,y)$ also has only a real lower component, we get the equations for the scalar degrees of freedom in the following form:

1. $\mu\nu$-component of the metric fluctuation:

$$
\begin{aligned}
&\frac{1}{4}\left(\partial_\mu\partial_\nu\tilde{h} - 2\partial_\mu\partial_\nu h_{44}\right) + \frac{1}{4}\gamma_{\mu\nu}\left(-\partial_\sigma\partial^\sigma\tilde{h} + 2\partial_\sigma\partial^\sigma h_{44} + \frac{3}{2}\partial_4\partial_4\tilde{h}\right) - \\
&-\frac{1}{2}\gamma_{\mu\nu}A'\left(4\partial_4\tilde{h} + 3\partial_4 h_{44}\right) + \frac{1}{2}\gamma_{\mu\nu}(A')^2\left(12h_{44} + \tilde{h}\right) - \\
&-\frac{1}{4}\gamma_{\mu\nu}A''\left(\tilde{h} + 6h_{44}\right) + \frac{1}{\sqrt{2M}}\gamma_{\mu\nu}\left[\phi_0'^{+}f' + \left(\phi_0''^{+} - 4A'\phi_0'^{+}\right)f\right] = 0; \qquad (15)
\end{aligned}
$$

2. μ4-component of the metric fluctuation:

$$\frac{3}{4}\partial_4\partial_\mu\tilde{h} - 3A'\partial_\mu h_{44} + \sqrt{\frac{2}{M}}\phi_0'^{+}\partial_\mu f = 0; \tag{16}$$

3. 44-component of the metric fluctuation:

$$\frac{3}{4}\partial_\mu\partial^\mu\tilde{h} + 3A'\partial_4\tilde{h} + \frac{1}{2M^2}Vh_{44} + \sqrt{\frac{2}{M}}\left[\phi_0'^{+}f' - \frac{1}{2}\left(\frac{dV}{d\phi}f + f^+\frac{dV}{d\phi^+}\right)\right] = 0; \tag{17}$$

4. equation for the field f:

$$\begin{aligned}
&M\left(\partial_M\partial^M f + 4A'f' + \frac{d^2V}{d\phi^+ d\phi}f + f^+\frac{d^2V}{(d\phi^+)^2}\right) - \\
&-\frac{1}{\sqrt{2M}}\left[\frac{1}{2}\phi_0'\left(\partial_4\tilde{h} + \partial_4 h_{44}\right) + \left(\phi_0'' - 4A'\phi_0'\right)h_{44}\right] + \\
&+\left(\frac{d^2\lambda_1}{d\phi^+ d\phi}f + f^+\frac{d^2\lambda_1}{(d\phi^+)^2}\right)\delta(y) + \left(\frac{d^2\lambda_2}{d\phi^+ d\phi}f + f^+\frac{d^2\lambda_2}{(d\phi^+)^2}\right)\delta(y-L) + \\
&+\frac{1}{2\sqrt{2M^3}}\left[\frac{d\lambda_1}{d\phi^+}\delta(y) + \frac{d\lambda_2}{d\phi^+}\delta(y-L)\right]h_{44} = 0.
\end{aligned} \tag{18}$$

The remarkable result here is that the fluctuations of the scalar components of the 5D gauge fields, namely A_4 and B_4, vanish due to the equations of motion and the gauge condition choice. Hence there are no extra light scalars, which allows one to correctly reproduce the electroweak sector of the Standard Model in the effective four-dimensional theory.

4 Radion-like interactions of the Higgs boson

Since the bulk Higgs field in the theory is the stabilizing Goldberger-Wise field, the Higgs boson here is the radion at the same time. The radion as a separate particle does not exist. The Higgs boson inherits the interaction with the energy-momentum tensor, which is intrinsic to the radion. The interaction term can be written as [6]

$$S \supset -\frac{1}{\sqrt{8M^3}}\int dx\int_{-L}^{L} dy T^{MN}h_{MN}\sqrt{\gamma} = \frac{1}{\sqrt{8M^3}}\int dx\int_{-L}^{L} dy\left(-\frac{1}{2}T^\mu_\mu - T_{44}\right)e^{-2A}g, \tag{19}$$

where $g = e^{-2A(y)}h_{44}(x,y)$. The field g, in fact, represents the Higgs field f because h_{44} and f are connected by the gauge condition choice and thereby represent the same degree of freedom.

Let us consider the bulk vector field Z_M interacting with the field g. Its Lagrangian is as follows:

$$L_Z = -\frac{1}{4}Z_{MN}Z^{MN} + \frac{1}{2}\phi_0^+\phi_0 Z_M Z^M. \tag{20}$$

Here we have ϕ_0 squared instead of m_Z^2 since we consider the situation previous to the electro-weak symmetry breaking. Performing the mode decomposition

$$Z_\mu(x,y) = \sum_n z_\mu^n(x)\,Z_n(y) \tag{21}$$

(where the gauge condition $Z_4(x, y) = 0$ is chosen), one obtains the interaction term as follows:

$$S \supset \frac{1}{\sqrt{8M^3}} \sum_n \int dx \int_{-L}^{L} dy \Bigg(\frac{1}{4} \eta^{\rho\sigma} z^n_{\mu\rho}(x) z^n_{\nu\sigma}(x) - z^n_\mu(x) \sum_m z^m_\nu(x) B_{nm} + \\ + m^2_{Zn} z^n_\mu(x) z^n_\nu(x) \Bigg) \eta^{\mu\nu} e^{-2A} Z^2_n(y) g, \tag{22}$$

where $B_{nm} = \int_{-L}^{L} dy e^{-2A} \phi_0^+ \phi_0 Z_n(y) Z_m(y)$, $Z^n_{\mu\nu}(x, y) = z^n_{\mu\nu}(x) Z_n(y)$. Note that the interaction Lagrangian turns out to be non-diagonal, which means that there is KK number non-conservation. Moreover, the couplings of the excited bulk states to the Higgs boson are not characterized by one coupling constant, as in the case of brane localized fields, but depend on the overlap integrals of their wave functions and the Higgs wave function. A similar interaction Lagrangian can be obtained for the W-boson field.

5 Specific example of a non-perturbative background solution

Let us choose the following ansatz:

$$V = \frac{1}{4} \frac{dW}{d\phi} \frac{dW}{d\phi^+} - \frac{1}{24M^2} \left(W(\phi^+\phi)\right)^2, \tag{23}$$

$$\phi'(y) = \mathrm{sign}(y) \frac{1}{2} \frac{dW}{d\phi^+}, \quad \phi'^+(y) = \mathrm{sign}(y) \frac{1}{2} \frac{dW}{d\phi}, \tag{24}$$

$$A'(y) = \mathrm{sign}(y) \frac{1}{24M^2} W(\phi^+\phi). \tag{25}$$

In this case the equations of motion (7)–(10) are valid everywhere, except the branes. In order the equations of motion be valid everywhere, one needs to make a consistent choice of the brane potentials $\lambda_{1,2}(\phi^+\phi)$ and the bulk potential $V(\phi^+\phi)$.

Let us take the function W to be

$$W = 24M^2 k - 2u\phi^+\phi, \tag{26}$$

then the brane potentials $\lambda_{1,2}$ should be chosen as follows:

$$\lambda_1(\phi^+\phi) = MW(\phi^+\phi) + \beta_1 \left(\phi^+\phi - \frac{v_1^2}{2}\right)^2, \tag{27}$$

$$\lambda_2(\phi^+\phi) = -MW(\phi^+\phi) + \beta_2 \left(\phi^+\phi - \frac{v_2^2}{2}\right)^2.$$

Here k, u, $\beta_{1,2}$ and $v_{1,2}$ are the model parameters. Note that the potential λ_2 chosen in this simplest form has the same structure as the standard Higgs potential.

The corresponding solution looks like

$$\phi(y) = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} e^{-u(|y|-L)} \end{pmatrix}, \tag{28}$$

$$A(y) = k\left(|y| - L\right) + \frac{v^2}{96M^2}\left(e^{-2u(|y|-L)} - 1\right).$$

Here $v = 246$ GeV is the vacuum value of the Higgs field. In this case all the fermion fields localized on the brane will get the same masses as in the interaction with the usual Higgs field.

The inter-brane distance is defined by the boundary conditions for the scalar field and is expressed in terms of the model parameters by the relation:

$$L = \frac{1}{u}\ln\left(\frac{v_1}{v}\right). \tag{29}$$

The size of the extra dimension becomes stabilized.

6 Estimate of the model parameters

Taking the mode decomposition of the previously defined field g we obtain the equations for g_n with the boundary conditions on the branes:

$$\frac{d}{dy}\left(\frac{e^{2A}}{\left(\phi_2'\right)^2} g_n'\right) - \frac{e^{2A}}{6M^2} g_n = -\mu_n^2 g_n \frac{e^{4A}}{\left(\phi_2'\right)^2}, \tag{30}$$

$$\left(\frac{1}{4M}\frac{d^2\lambda_1}{d\phi_2^2} - \frac{\phi_2''}{\phi_2'}\right) g_n' + \mu_n^2 e^{2A} g_n \Big|_{y=+0} = 0,$$

$$\left(\frac{1}{4M}\frac{d^2\lambda_2}{d\phi_2^2} + \frac{\phi_2''}{\phi_2'}\right) g_n' - \mu_n^2 e^{2A} g_n \Big|_{y=L-0} = 0.$$

Here ϕ_2 is the lower component of a background solution for the Higgs field. These equations define a mass spectrum of the infinite Kaluza-Klein tower of the field ϕ.

In the case $uL \ll 1$ we can get the following expression for the mass of the lowest excitation of the scalar field, which is identified with the Higgs boson [7]:

$$m_H^2 = \frac{v^2u^2}{3M^2}\frac{\beta_2 v^2 - uM}{\beta_2 v^2 + uk}. \tag{31}$$

The correct value of the Higgs mass can be obtained for the following values of the model parameters: $M = 2$ TeV and $\beta_2 \to \infty$, consequently, $u \simeq 1.76$ TeV, $\phi_1 = 345$ TeV, $k \simeq 186$ TeV, $L = 0.2\ \text{TeV}^{-1} \simeq 2 \cdot 10^{-18}$ cm. The coupling of the Higgs boson to the energy-momentum tensor of the Standard Model, given by

$$\epsilon_H = -\sqrt{\frac{k}{24M^3}}, \tag{32}$$

turns out to be of the order of 1 TeV^{-1}. It must significantly affect the properties of the Higgs boson in this model.

Further excitations of the field ϕ have masses of the order of hundreds of TeV and cannot be observed at the existing colliders.

Unfortunately, this model turns out to be a toy model: it does not allow one to reproduce the Standard Model Higgs interactions. Another background solution is needed, this is the matter of future research.

7 Conclusion

In the present paper it has been shown that the bulk scalar field stabilizing the extra dimension size in the Randall-Sundrum model can act simultaneously as the Higgs field on the brane, where our world is supposed to be located, providing the spontaneous symmetry breaking there. The equations of motion for the background field configuration and for the field fluctuations against a vacuum solution are derived.

An important point is that the bulk electroweak gauge fields that necessarily appear in the model do not give rise to extra light scalars in the effective four-dimensional theory. The Higgs boson here is the radion at the same time, so it interacts with the energy-momentum tensor. These interactions with the bulk fields are studied. A remarkable result is that the coupling constants of these interactions depend on a particular excitation and there is no KK number conservation in such interactions.

A specific example of a non-perturbative vacuum solution is found. Based on it, the values of the model parameters are estimated, which give the correct value of the Higgs boson mass in the approximation of a small deviation of the metric of the stabilized model from the metric of the unstabilized one. Unfortunately, this background solution is not good due to its inability to correctly reproduce the Standard Model interactions of the Higgs boson, so it should be treated only as a toy model. Another exact vacuum solution is needed, which is the matter of further research.

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