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Symanzik approach in modeling of bound states of Dirac particle in singular background

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Abstract. In the model of interaction of spinor field with homogeneous isotropic material plane constructed in framework of Symanzik approach, the bound states are studied. For localized near plane Dirac particle the expression for current, charge and density are presented. For bound state with massless dispersion law the current, charge and density are calculated for simplified model with 2 parameter exactly.The model can find application to a wide class of phenomena arising by the interaction of fields of quantum electrodynamics with two-dimensional materials.

Investigation of macroscopic quantum phenomena is an important direction in experimental and theoretical physics which has attracted recently the attention of many researchers. In this area, the main problem of modelling employing the principles of quantum field theory is that the space-time homogeneity and isotropy, which is typical for usual quantum field theory models of elementary particles [1], must be broken. The symmetry properties of the vacuum may change essentially as a result of interactions of quantum fields with macroscopic objects. Therefore quantum macro effects may appear in dynamics of material bodies which cannot be explained both in classical physics and in framework of non relativistic quantum mechanics.

Theoretically, this problem was first considered in 1948 by Casimir [2]. He showed that the fluctuations of the quantum vacuum generate attraction between ideally conducting plates of a planar uncharged capacitor. This phenomenon, called the Casimir effect (CE), is observed experimentally [3–6], and the empirical results obtained for materials with high conductivity, with a high degree of accuracy, are in agreement with theoretical ones.

At typical for CE distances of 10 to 1000 nm both classical and quantum properties of the system turn out to be essential. Their combination forms a special nanophysics. Investigation of it is not solely of theoretical interest, understanding its particular feature is also important for developing new technological devices, in view of the increasing trend to their miniaturization.

Although theoretical investigations of the CE are the subject of numerous works [6, 7], they are often based on simplified models ignoring usually specificity of quantum electrodynamics. Such models are not suitable for comprehensively describing a wide range of nanophysical phenomena occurring in a system as a result of the interaction of its quantum degrees of freedom with material bodies of a given shape (classic defect).

In our work we used the the Symanzik's approach [8], modelling the interaction of quantized fields with a spatial inhomogeneity with the aid of an additional action functional (defect action), concentrated in the spatial domain where this inhomogeneity -a macroscopic object -is located. It is important also that the standard requirements for quantum field models (locality, renormalisability, symmetry properties) are assumed to be fulfilled.

In this work we use Symanzik's approach for modification of the standard QED in (3+1)-dimensional space-time for description of interaction of a two-dimensional material object with a Dirac field. Such a problem for a spinor field was considered in [15]. In accordance with the basic principles of QED, the defect action functional is the sum of the photon Chern-Simons action and a purely Dirac field defect action [17]. It is possible in this model framework, to investigate both scattering of spinor particles on a material plane and also the properties bound states localised in its vicinity. For simple model with 4 parameter in the Dirac defect action it was studied in [17]. The most general model with 8 parameters, describing interaction of Dirac field with homogeneous isotropic material plane was considered in [18]. For this model the dispersion relation for bound state was obtained, and it was shown that by a special choosing parameters the Dirac particle being massive in volume is massless in the bound state moving along the material plane with arbitrary Fermi-velocity less then velocity of light. In this work we study in more details the properties of such bound state and present results of estimation of theirs characteristics which was calculated exactly for a simplified model with 2 independent parameters.

1 Formulation of model

Within the Symanzik approach, the general form of the action functional modelling the interaction of the quantum field with material object (defect) can be written as:

$$S(\varphi) = S_V(\varphi) + S_{def}(\varphi).$$

Here, S_V is the action of the basic quantum field model, and S_{def} is the defect action:

$$S_V(\varphi) = \int L(\varphi(x))d^D x,\ S_{def}(\varphi) = \int_\Gamma L_{def}(\varphi(x))d^{D'}x,$$

where Γ is a subspace of dimension $D' < D$ in a D-dimensional space [8]. In the quantum electrodynamic (QED) the action $S(\varphi)$ is the functional of photon vector field $A_\mu(x)$, Dirac spinor fields $\bar{\psi}(x), \psi(x)$, and

$$S_V(\varphi) = S(\bar{\psi}, \psi, A) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\hat{\partial} - m + ie\hat{A})\psi,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The gauge invariance, locality, and renormalisability as the basic principles of QED impose strong constraints on the possible form of the defect action $S_{def}(\varphi)$. Taking them into account by modelling the interaction of the QED fields with a two-dimensional surface without charges and currents, one obtains the following result: $S_{def}(\varphi) = S_{def}(A) + S_{def}(\bar{\psi}, \psi)$. If the surface is defined by equation $\Phi(x) = 0$, $x = (x^0, x^1, x^2, x^3)$, then $S_{def}(A)$ is the Chern-Simons action

$$S_{def}(A) = \frac{a}{2}\int \varepsilon^{\lambda\mu\nu\rho}\partial_\lambda\Phi(x)A_\mu(x)F_{\nu\rho}(x)\delta(\Phi(x))\, d^4x,$$

where $\varepsilon^{\lambda\mu\nu\rho}$ is the totally antisymmetric tensor ($\varepsilon^{0123} = 1$), and the parameter a is a dimensionless coupling constant. This expression is the most general form of a gauge-invariant functional of A_μ concentrated on a defect surface being invariant in respect to reparameterisation of one and not having any negative-dimension parameters. The the most general form of the defect action $S_{def}(\bar{\psi}, \psi)$ is written as

$$S_{def}(\bar{\psi}, \psi) = \sum_{j=1}^{16}\int \alpha_j\bar{\psi}(x)\Gamma_j\psi(x)\delta(\Phi(x))\, d^4x,$$

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where Γ_j are the 16 basis Dirac matrices and α_j are dimensionless coupling constants.

We consider the plane $x^3 = 0$ as the material defect. In this case

$$S(\overline{\psi}, \psi) = \int \overline{\psi}(x)(i\hat{\partial} - m + \Omega(x^3))\psi(x)dx. \tag{1}$$

The matrix $\Omega(x_3) = Q\delta(x^3)$ describes the coupling of Dirac fields with the plane. Because $\Omega(x^3)$ and $\delta(x^3)$ have the dimension of mass, the matrix Q is dimensionless. We suppose that the defect plane is isotropic and homogeneous. It means that $S(\overline{\psi}, \psi)$ must be be invariant under translation in x^0-, x^1-, x^2-directions and rotation around x^3-axe. The most general form of the matrix Q fulfilling this condition is

$$Q = r_1\mathbf{1} + ir_2\gamma_5 + r_3\gamma_3 + r_4\gamma_5\gamma_3 + r_5\gamma_0 + r_6\gamma_5\gamma_0 + ir_7\gamma_0\gamma_3 + ir_8\gamma_1\gamma_2, \tag{2}$$

where $\mathbf{1}$ is the unit 4×4 matrix, $i = \sqrt{-1}$, γ^j, $j = 0, 1, 2, 3$ are the Dirac matrices and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. For the γ matrices we use the following representation

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

Taking the functional derivatives of the action (1) with respect to $\overline{\psi}(x)$ and $\psi(x)$ we obtain the Euler-Lagrange equations:

$$(i\hat{\partial} - m + \Omega(x^3))\psi(x) = 0, \ (\partial_\mu\overline{\psi}(x))\gamma^\mu + \overline{\psi}(x)(m - \Omega(x^3)) = 0. \tag{3}$$

It can be easily verified that $\overline{\psi}(x) = \psi^*(x)\gamma_0$, if $\gamma_0\Omega^+(x^3) = \Omega(x^3)\gamma_0$. This condition is fulfilled if parameters r_j, $1 \le j \le 8$ are real. In what follows we assume that this is the case.

Let us introduce the following notations

$$\overline{x} = (x^0, x^1, x^2), \ \overline{p} = (p^0, p^1, p^2), \ \overline{px} = p^0x^0 - p^1x^1 - p^2x^2,$$

for parts of 4-component vectors, and represent $\psi = \psi(\bar{x}, x^3)$ as $\psi(x) = \psi_s(x) + \psi_a(x)$, where

$$\psi_s(x) = \frac{1}{2}(\psi(\bar{x}, x^3) + \psi(\bar{x}, -x^3)), \ \psi_a(x) = \frac{1}{2}(\psi(\bar{x}, x^3) - \psi(\bar{x}, -x^3)).$$

Analysis of the possible singularities of spinor $\psi(x)$ at the plane $x^3 = 0$, made in [17], yields: for $x^3 \neq 0$, the field $\psi(x)$ satisfies the Dirac equation

$$(i\hat{\partial} - m)\psi(x) = 0, \tag{4}$$

and the equation

$$i\gamma^3\psi_a(\bar{x}) + \frac{Q}{2}\psi_s(\bar{x}) = 0 \tag{5}$$

is fulfilled at $x^3 = 0$, within $\psi_a(\bar{x}) = \lim_{x^3\to 0}\psi_a(x)$ and $\psi_s(\bar{x}) = \lim_{x^3\to 0}\psi_s(x)$.

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It is shown in [17] that the general solution of (4) by $x^3 \neq 0$ can be represented as

$$\psi(x) = \frac{1}{(2\pi)^3}\int e^{i\overline{px}}\psi_+(\bar{p},x^3)d\bar{p}, \ \ \text{for } x^3>0, \ \ \psi(x) = \frac{1}{(2\pi)^3}\int e^{i\overline{px}}\psi_-(\bar{p},x^3)d\bar{p}, \ \ \text{for } x^3<0$$

where

$$\psi_\pm(\bar{p},x^3) = U(\bar{p},x^3)\chi_\pm(\bar{p}), \ U(\bar{p},x^3) = e^{i\gamma^3(\hat{\bar{p}}+m)x^3} = e^{i\kappa(\bar{p})x^3}P^+(\bar{p}) + e^{-i\kappa(\bar{p})x^3}P^-(\bar{p}).$$

Here, $\chi_\pm(\bar{p})$ are arbitrary spinors depending only on $\bar{p}$, and we used the notations

$$\kappa(\bar{p}) = \sqrt{\bar{p}^2-m^2}, \ \hat{\bar{p}} = \overline{\gamma p} = \gamma^0 p^0 - \gamma^1 p^1 - \gamma^2 p^2, \ P^\pm(\bar{p}) = \frac{1}{2}\left(\mathbf{1} \pm \frac{\gamma^3(\hat{\bar{p}}+m)}{\kappa(\bar{p})}\right).$$

It follows from (5) that $\chi_+(\bar{p})$, $\chi_-(\bar{p})$ satisfy the equation

$$i\gamma^3(\chi_+ - \chi_-) + \frac{Q}{2}(\chi_+ + \chi_-) = 0, \tag{6}$$

which implies that

$$\chi_- = S\chi_+, \tag{7}$$

where

$$S = \left(1 - \frac{i\gamma^3 Q}{2}\right)^{-1}\left(1 + \frac{i\gamma^3 Q}{2}\right). \tag{8}$$

The matrices $P^\pm(\bar{p})$ are projectors:

$$P^\pm(\bar{p})P^\pm(\bar{p}) = P^\pm(\bar{p}), \ P^+(\bar{p}) + P^-(\bar{p}) = 1, \ P^+(\bar{p})P^-(\bar{p}) = P^-(\bar{p})P^+(\bar{p}) = 0.$$

Each of them has two-dimensional eigensubspaces L^+ and L^-, which are represented as linear combinations of two mutually orthogonal eigenvectors $v_1^\pm$ and $v_2^\pm$:

$$L^\pm(a_1^\pm, a_2^\pm) = a_1^\pm v_1^\pm + a_2^\pm v_2^\pm,$$

$$P^\pm(\bar{p})L^\pm(a_1^\pm,a_2^\pm) = L^\pm(a_1^\pm,a_2^\pm), \ P^\pm(\bar{p})L^\mp(a_1^\mp,a_2^\mp) = 0.$$

Here, $a_i^\pm$, $i=1,2$, are complex parameters, and $v_i^\pm$, $i=1,2$, are spinors, which we choose as

$$v_1^+ = n_1^+\left(-h_1, -h_1h_2^-, 1, h_2^-\right), \ v_2^+ = n_2^+\left(h_1, h_1h_2^+, 1, h_2^+\right),$$
$$v_1^- = n_1^-\left(-h_1, h_1h_2^+, 1, -h_2^+\right), \ v_2^- = n_2^-\left(h_1, -h_1h_2^-, 1, -h_2^-\right)$$
$$h_1 = \frac{\omega}{m+p_0}, \ h_2^\pm = \frac{\kappa(\bar{p})\pm\omega}{p_1 - ip_2}, \ \omega = \sqrt{p_0^2-m^2},$$

with normalisation constants $n_1^\pm, n_2^\pm$ defined by conditions $\bar{v}_1^\pm v_1^\pm = \bar{v}_2^\pm v_2^\pm = 1$. These spinors are eigenstates of the helicity operator

$$\sigma(\vec{p}) = \frac{i}{2|\vec{p}|}(\vec{p}\,\vec{s}), \ \vec{s} = (\gamma_2\gamma_3, -\gamma_1\gamma_3, \gamma_1\gamma_2),$$

and satisfy the relations

$$\sigma(\vec{p})|_{p_3=\mp\kappa(\bar{p})}v_1^\pm = -\frac{1}{2}v_1^\pm, \ \sigma(\vec{p})|_{p_3=\mp\kappa(\bar{p})}v_2^\pm = \frac{1}{2}v_2^\pm$$

We represent the spinors $\chi_\pm(\overline{p})$ as

$$\chi_\pm(\overline{p}) = b_\pm^1(\overline{p})v_1^-(\overline{p}) + b_\pm^2(\overline{p})v_2^-(\overline{p}) + c_\pm^1(\overline{p})v_1^+(\overline{p}) + c_\pm^2(\overline{p})v_2^+(\overline{p}). \tag{9}$$

If $\kappa > 0$ the fields $\bar{\psi}, \psi$ describe the scattering processes of Dirac particles on the defect plane. For the model with $r_5 = r_6 = r_7 = r_8 = 0$ they are studied in [17]. In this paper we consider the properties of bound states localized in vicinity of $x_3 = 0$ without this restriction.

2 Bound states

The state is localised nearly of the plane $x_3 = 0$ if $\bar{p}^2 - m^2 < 0$ and $\kappa(\bar{p}) = i|\kappa(\bar{p})|$ is imaginary. In this case the spinors $\chi_\pm(\bar{p})$ (9) must fulfil the conditions $P^-(\bar{p})\chi_+ = 0,\ P^+(\bar{p})\chi_- = 0$. Other case the asymptotic of the fields $\bar{\psi}(x), \psi(x)$ cold be infinite by $x_3 \to \pm\infty$. Thus, we can present $\chi_+,\ \chi_-$ as follows

$$\chi_+(\bar{p}) = c_+^1(\bar{p})\, v_1^+(\bar{p}) + c_+^2(\bar{p})\, v_2^+(\bar{p}),\ \ \chi_-(\bar{p}) = c_-^1(\bar{p})\, v_1^-(\bar{p}) + c_-^2(\bar{p})\, v_2^-(\bar{p}). \tag{10}$$

Substituting in (8) the spinors $\chi_+(\bar{p}), \chi_-(\bar{p})$ presented in (10), we obtain the equation

$$c_-^1(\bar{p})\, v_1^-(\bar{p}) + c_-^2(\bar{p})\, v_2^-(\bar{p}) = S\,(c_+^1(\bar{p})\, v_1^+(\bar{p}) + c_+^2(\bar{p})\, v_2^+(\bar{p})). \tag{11}$$

For any c_+^1, c_+^2, we can present the spinor $d = S\chi_+(\bar{p})$ as $d = d^1 v_1^+ + d^2 v_2^+ + d^3 v_1^- + d^4 v_2^-$ and express the coefficients d^1, d^2, d^3, d^4 in terms of c_+^1, c_+^2 in the following way

$$d^i = \sum_{j=1}^{2} M_j^i(\bar{p}) c_+^j,\ 1 \le j \le 4. \tag{12}$$

Thus, the coefficients $c_+^1,\ c_+^2$ satisfying the equation (12) must fulfil the homogeneous equations $d^1 = 0,\ d^2 = 0$, which have nontrivial solution if

$$M_1^1(\bar{p}) M_2^2(\bar{p}) - M_2^1(\bar{p}) M_1^2(\bar{p}) = 0. \tag{13}$$

It means that the determinants of the 2×2 matrix M with elements $\{M\}_{ij} = M_j^i,\ i, j = 1, 2$ must be equal to zero. In this case, one of the coefficients c_+^1, c_+^2 remains to be arbitrary, and the other one it is expressed in its term. If c_+^1 is considered as an arbitrary amplitude, parameterising the solution of equations (12), then

$$c_+^2 = L(\bar{p})\, c_+^1,\ L(\bar{p}) = -\frac{M_1^2(\bar{p})}{M_2^2(\bar{p})} = -\frac{M_1^1(\bar{p})}{M_2^1(\bar{p})}. \tag{14}$$

The amplitudes c_-^1, c_-^1 expressed in terms of c_+^1 are obtained by the substitution of c_+^1 and $c_+^2 = L(\bar{p})\, c_+^1$ in the right hand sides of equations (12) for $d^3,\ d^4$:

$$c_-^1 = (M_1^3(\bar{p}) + M_2^3 L(\bar{p}))\, c_+^1,\ c_-^2 = (M_1^4(\bar{p}) + M_2^4 L(\bar{p}))\, c_+^1. \tag{15}$$

The solvability condition describes the correlation between components of the moment $\bar{p} = (p^0, p^1, p^2) = (p^0, \vec{p})$ of the bound state. The relation of such a kinds are called the dispersion laws. For the considered model the equation (13) reads

$$(R_1\lambda - 2(m r_{18}^- - p^0 r_{45}^+))(R_2\lambda - 2(m r_{18}^+ + p^0 r_{45}^-)) - R_3 \vec{p}^{\,2} = 0. \tag{16}$$

where we used the notations $r_{jk}^\pm = (r_j \pm r_k)/2$, $\vec{p}^{\,2} = (p^1)^2 + (p^2)^2$ $\lambda = \sqrt{m^2 + \vec{p}^{\,2} - p^{0\,2}} = -i\kappa$, and

$$R_1 = 1 + {r_{18}^-}^2 + {r_{27}^-}^2 + {r_{36}^+}^2 - {r_{45}^+}^2,\ R_2 = 1 + {r_{18}^+}^2 + {r_{27}^+}^2 + {r_{36}^-}^2 - {r_{45}^-}^2,\ R_3 = r_6^2 + r_7^2 + r_8^2 - r_4^2.$$

For bound states, $\lambda = |\kappa| > 0$ and can be considered as an index of its localisation near the plane $x_3 = 0$. Simple essential characteristics of properties of bound states could be also the scalar density $D(\bar{p}, x^3) = \exp\{-|x^3|\lambda\}\, d(\bar{p})$ and the 4-component current $J(\bar{p}, x^3) = \exp\{-|x^3|\lambda\}\, J(\bar{p})$, $J(\bar{p}) = (j^0(\bar{p}), j^1(\bar{p}), j^2(\bar{p}), j^3(\bar{p}))$ with

$$d_\pm(\bar{p}) = \bar{\chi}_\pm(\bar{p})\chi_\pm(\bar{p}),\ j_\pm^\alpha(\bar{p}) = \bar{\chi}_\pm(\bar{p})\gamma^\alpha\chi_\pm(\bar{p}),\ \alpha = 0, 1, 2, 3.$$

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We denoted $d_+(\bar{p})$, $J_+(\bar{p})$ the function $d(\bar{p})$ and the vector $J(\bar{p})$ in subspace $x^3 > 0$, and $d_-(\bar{p})$, $J_-(\bar{p})$ for $x^3 < 0$. They are expressed in terms of amplitudes $c^1_\pm, c^2_\pm$ in the following form:

for $p_0 > m$

$$d_\pm = -\frac{4}{p^0+m}\left\{mC^+_\pm + p^0\lambda\left(\frac{c^1_\pm c^{2\,*}_\pm}{\lambda \mp i\omega} + \frac{c^2_\pm c^{1\,*}_\pm}{\lambda \pm i\omega}\right)\right\}, \tag{17}$$

$$j^0_\pm = \frac{4}{p^0+m}\left\{p^0C^+_\pm + m\lambda\left(\frac{c^1_\pm c^{2\,*}_\pm}{\lambda \mp i\omega} + \frac{c^2_\pm c^{1\,*}_\pm}{\lambda \pm i\omega}\right)\right\}, \tag{18}$$

$$j^1_\pm = \frac{4\omega\,(C^+_\pm\,\omega\,p^1 \pm C^-_\pm\,\lambda\,p^2)}{(p_0+m)(\omega^2+\lambda^2)},\ j^2_\pm = \frac{4\omega\,(C^+_\pm\,\omega\,p^2 \mp C^-_\pm\,\lambda\,p^1)}{(p_0+m)(\omega^2+\lambda^2)},\ j^3_\pm = 0, \tag{19}$$

$$C^+_\pm = |c^1_\pm|^2 + |c^2_\pm|^2,\ C^-_\pm = |c^1_\pm|^2 - |c^2_\pm|^2; \tag{20}$$

for $p_0 < m$

$$d_\pm = -\frac{4}{p^0+m}\left\{m\tilde{C}^+_\pm + p^0\lambda\left(\frac{c^1_\pm c^{1\,*}_\pm}{\lambda \mp i\omega} + \frac{c^2_\pm c^{2\,*}_\pm}{\lambda \pm i\omega}\right)\right\}, \tag{21}$$

$$j^0_\pm = \frac{4}{p^0+m}\left\{p^0\tilde{C}^+_\pm + m\lambda\left(\frac{c^1_\pm c^{1\,*}_\pm}{\lambda \mp i\omega} + \frac{c^2_\pm c^{2\,*}_\pm}{\lambda \pm i\omega}\right)\right\}, \tag{22}$$

$$j^1_\pm = \frac{4\omega\,(\tilde{C}^+_\pm\,\omega\,p^1 \pm \tilde{C}^-_\pm\,\lambda\,p^2)}{(p_0+m)(\omega^2+\lambda^2)},\ j^2_\pm = \frac{4\omega\,(\tilde{C}^+_\pm\,\omega\,p^2 \mp \tilde{C}^-_\pm\,\lambda\,p^1)}{(p_0+m)(\omega^2+\lambda^2)},\ j^3_\pm = 0, \tag{23}$$

$$\tilde{C}^+_\pm = c^1_\pm c^{2\,*}_\pm + c^2_\pm c^{1\,*}_\pm,\ \tilde{C}^-_\pm = c^1_\pm c^{2\,*}_\pm - c^2_\pm c^{1\,*}_\pm. \tag{24}$$

We see that the currents flow along the plane $x_3 = 0$ and are parallel to the moment $\vec{p} = (p_1, p_2)$ if $C^-_\pm = \tilde{C}^-_\pm = 0$. The currents above and below of the plane $x_3 = 0$ are parallel if $C^-_-/C^+_- = C^-_+/C^+_+$ by $p0 > m$, and $\tilde{C}^-_-/\tilde{C}^+_- = \tilde{C}^-_+/\tilde{C}^+_+$ by $p0 < m$. From (17-24) it follows that

$$m\,d_\pm + p^0\,j^0_\pm = p^1\,j^1_\pm + p^2\,j^2_\pm = 4(p^0-m)C^+_\pm,\ p^2\,j^1_\pm - p^1\,j^2_\pm = 4\lambda\sqrt{\frac{p^0-m}{p^0+m}}C^-_\pm \ \text{ for } p^0 > m, \tag{25}$$

$$m\,d_\pm + p^0\,j^0_\pm = p^1\,j^1_\pm + p^2\,j^2_\pm = 4(p^0-m)\tilde{C}^+_\pm,\ p^2\,j^1_\pm - p^1\,j^2_\pm = 4\lambda\sqrt{\frac{p^0-m}{p^0+m}}\tilde{C}^-_\pm \ \text{ for } p^0 < m, \tag{26}$$

and

$$m\,d_\pm + p^0\,j^0_\pm - p^1\,j^1_\pm - p^2\,j^2_\pm = 0. \tag{27}$$

If $R_1 = R_2 = r^+_{18} = r^-_{18} = 0$ by choosing the special parameters in the model , then the dispersion relation (16) has the form

$$p_0^2 - v_F^2\vec{p}^{\,2} = 0 \tag{28}$$

where $\vec{p}$ denotes the two-component vector $\vec{p} = (p^1, p^2)$, $\vec{p}^{\,2} = (p^1)^2 + (p^2)^2$ and v_F is a constant, which is expressed in the term of parameters r_k, $1 \le k \le 8$ of the model. The dispersion relation (28) describe moving of massless particle with Fermi-velocity v_F. The motion of such particles explains numerous effects in graphene.

The constants $R_1, R_2, r^+_{18}, r^-_{18}$ are equal to zero by $r_1 = r_8 = r_3 = 0$ and $r^+_{45} = \pm\sqrt{1 + {r^-_{27}}^2 + r_6^2/4}$, $r^-_{45} = \pm\sqrt{1 + {r^+_{27}}^2 + r_6^2/4}$. For such choosing of parameters, the Fermi-velocity

v_F can obtains two different values: $v_F = v_F^+$ or $v_F = v_F^-$,

$$v_F^\pm = \sqrt{\frac{1}{2}\left(1 \pm \frac{4 + 4r_{27}^+ r_{27}^- - r_6^2}{\sqrt{4 + 4{r_{27}^-}^2 + r_6^2}\sqrt{4 + 4{r_{27}^+}^2 + r_6^2}}\right)}.$$

The Fermi-velocity v_F^+ (v_F^-) corresponds to the positive (negative)value of $r_{45}^+ r_{45}^-$. By changing the values of parameters r_2, r_6, r_7 one can obtain each value of the Fermi-velocity in the interval (0,1). The current flowing along the plane $x_3 = 0$ is parallel to the moment $\vec{p}$ only by $r_6 = 0$.

3 Simple example

To understand better the properties of the bound state with massless dispersion law (28), let us consider a simplified model choosing the parameters r_j in the following way: $r_1 = r_2 = r_3 = r_5 = r_8 = 0$, $r_4 = \sqrt{4 + r_6^2 + r_7^2}$. In this case the dispersion relation has the form (28) with Fermi velocity

$$v_F = \frac{2}{\sqrt{4 + r_6^2 + r_7^2}}.$$

The matrix $i/2\gamma^3 Q$ and S are of the form

$$i/2\gamma^3 Q = \frac{1}{2}\begin{pmatrix} ir_6 - r_7 & 0 & i\frac{2i}{v_F} & 0 \\ 0 & -ir_6 - r_7 & 0 & \frac{2i}{v_F} \\ \frac{2i}{v_F} & 0 & ir_6 - r_7 & 0 \\ 0 & \frac{2i}{v_F} & 0 & -ir_6 - r_7 \end{pmatrix},$$

$$S = \begin{pmatrix} \frac{-ir_7}{r_6+2i} & 0 & -\frac{2}{(r_6+2i)v_F} & 0 \\ 0 & \frac{ir_7}{r_6-2i} & & \frac{2}{(r_6-2i)v_F} \\ -\frac{2}{(r_6+2i)v_F} & 0 & \frac{ir_7}{r_6+2i} & 0 \\ 0 & \frac{2}{(r_6-2i)v_F} & 0 & \frac{-ir_7}{r_6-2i} \end{pmatrix}.$$

Solving the equation (11), and employing equations (17- 24) one can obtain the following result

$$c_+^2 = \frac{c_+^1 p^0 (2\lambda + r_6\omega)}{(i\omega - \lambda)(2m + ir_7\omega\, v_F)},\; c_-^1 = \frac{c_+^1 p^0}{(i\omega - \lambda)\, v_F},\; c_-^2 = \frac{c_+^1 (2\lambda + r_6\omega)\, v_F}{(2m + ir_7\omega\, v_F)}$$

and $d_\pm(\bar{p}) = j_\pm^3(\bar{p}) = 0$. For $p^0 > m$

$$j_\pm^0 = \frac{16|c_+^1|^2 \lambda (2\lambda + \omega r_6)(p^0 - \omega) v_F^2}{(4m^2 + \omega^2 r_7^2 v_F^2) p^0},$$

$$j_\pm^1 = \frac{8|c_+^1|^2 \lambda (p^0 - m)(2p^1 \mp p^2 r_6)}{(2\lambda - \omega r_6)(\lambda^2 + \omega^2)},\; j_\pm^2 = \frac{8|c_+^1|^2 \lambda (p^0 - m)(2p^2 \pm p^1 r_6)}{(2\lambda - \omega r_6)(\lambda^2 + \omega^2)},$$

For $p^0 < m$

$$j_\pm^0 = \frac{16|c_+^1|^2 \lambda (m - p^0)}{(\lambda - i\omega)(2m + i\omega r_7 v_F)},$$

$$j_\pm^1 = -\frac{8|c_+^1|^2 p^0 \lambda (p^0 - m)(2p^1 \mp p^2 r_6)}{(\lambda - i\omega)(2m + i\omega r_7 v_F)(\lambda^2 + \omega^2)},\; j_\pm^2 = -\frac{8|c_+^1|^2 p^0 \lambda (p^0 - m)(2p^2 \pm p^1 r_6)}{(\lambda - i\omega)(2m + i\omega r_7 v_F)(\lambda^2 + \omega^2)}$$

We see that in the considered simplified model the current $\vec{j}(\bar{p})$ can be presented as the sum $\vec{j}_{\pm}(\bar{p}) = \vec{p}f_{\pm}(p^0/m) + \vec{q}g_{\pm}(p^0/m)$, where $\vec{q}$ is orthogonal to $\vec{p}$ and $f_+(p^0/m) = f_-(p^0/m)$, $g_+(p^0/m) = -g_-(p^0/m)$. The charge $j^0_{\pm}$ is also a function of p^0/m, and $j^0_+(p^0/m) = j^0_-(p^0/m)$. Each of the functions $f_{\pm}(p^0/m), g_{\pm}(p^0/m), j^0_{\pm}(p^0/m)$ is continue in the point $p^0 = m$, but not analytical.

4 Conclusions

We considered a model of interaction of spinor field with material plane. It was constructed in framework of Symanzik approach, by the modification the usual spinor action functional which describes the interaction of Dirac field with the homogeneous isotropic material plane. It is realised by taking into account the basic principles of quantum electrodynamic. The most general additional action functional contains 8 real dimensionless parameters, which characterize the property of material of plane. This model enables to investigate both bound states of particles localised near the plane and the scattering processes on it. Analysis of dispersion relation for the bound states shows that by a choosing of the special model parameter it describes the motion of massless particle with Fermi-velocity $0 < v_F < 1$. We studied a simplified model with 2 parameters and calculated some essential characteristic of bound states behaviour. The scalar density appeared to be equal to zero. The current J^3 in the orthogonal to the plane $x^3 = 0$ direction is absent. The current flows parallel to the plane $x_3 = 0$ and has two components. One of them is the current flowing parallel to the moment, in the same direction on both side of the plane $x^3 = 0$. The other one flows orthogonal to the moment direction, and its directions on different sides of the plane $x^3 = 0$ are opposite ones. It disappears only by choosing the special model parameters. The distributions of charge in the space is symmetric in respect to reflection $x^3 \to -x^3$. The charge and intensities of currents are functions the relation p^0/m. They are continuous in the point $p^0 = m$, but not analytical.

The presented results demonstrate the possibility of used approach for theoretical investigation the effects of interaction of Dirac fields with macroscopic objects . One can hope that its development and application in this research region especially for different phenomena in 2-dimensional materials attracted the attention of many researchers [16] appears to be fruitful.

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