Modeling of liquid and gas flows in the horizontal layer with evaporation

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Abstract. Mathematical modeling of two-layer flows in the "ethanol-nitrogen" system on the basis of the exact solutions of a special type is carried out. The influence of the gas flow, temperature and Soret effect on the flow patterns and evaporating processes at the interface is investigated. The results of comparison of the experimental and theoretical data are presented; the dependence of the evaporation intensity at interface of the gas flow rate and temperature is studied.

1 Introduction

The convective fluid flows under action of a gas flow with respect to the evaporation are actively studied experimentally and analytically nowadays. The problem of construction of the exact solutions to describe the two-layer fluid flows in an infinite channel with an interface generates a considerable interest [1]. The solutions, which are studied in this work, can be considered as a generalization of the solution of the convection problem in a horizontal layer with a free boundary [2]. One of the first examples of the exact solutions, that takes into account the mass transfer at the interface of two media, has been obtained in [3]. The solutions of the equations of convection in the two-layer systems subject to the Dufour and Soret effects in the gas layer are studied in [4, 5, 6].

In the present paper the analytical study of the liquid and gas flows with evaporation at the interface and a comparison of the theoretical and experimental results are carried out. Mathematical modeling of the problem is based on the exact solutions of the Navier-Stokes equations in the Oberbeck-Boussinesq approximation [7]. Some characteristics of the liquid and gas flows and the intensity of evaporation are studied with the help of the system like ethanol-air. The influence of gas flow rate and temperature on the velocity and temperature profiles and evaporation intensity at the interface is investigated.

2 Mathematical model of the process

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A viscous incompressible liquid and gas-vapor mixture fill the infinite horizontal layers of thickness l and h, respectively. The coordinate system is chosen so that the vector of the gravity acceleration **g** is directed opposite to the Oy axis. The effects of thermodiffusion and diffusive heat conductivity (the Soret and Dufour effects) are taken into account in the upper layer [7, 8]. The mathematical model in the two-dimensional case represents a system of differential equations of the following form [7]:

$$(\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}p' + \nu\Delta\mathbf{v} + \mathbf{g}\left(\beta T + \underline{\gamma C}\right), \qquad (1)$$

$$div\,\mathbf{v} = 0, \qquad (2)$$

$$\mathbf{v}\cdot\nabla T = \chi\left(\Delta T + \underline{\delta\Delta C}\right), \qquad (3)$$

$$\mathbf{v}\cdot\nabla C = D\left(\Delta C + \alpha\Delta T\right), \qquad (4)$$

The underlined terms and equation (5) are used for simulation of the flow in the upper layer. Here $\mathbf{v} = (u,\upsilon)$ is the velocity vector, p' is the modifed pressure or deviation from the hydrostatic pressure, T is the temperature, C is the vapor concentration, ρ is the density, ν is the kinemtic viscosity, χ is the coefficient of heat diffusivity, D is the coefficient of vapor diffusion, β is the coefficient of thermal expansion, γ is the concentration density coefficient, the coefficients δ and α defines the Dufour and Soret effects in the gas-vapor layer.

Let us seek the solution of the equations (1)-(5) as follows (see [4, 5, 6]):

$$u_i = u_i(y), \quad \upsilon_i = 0, \quad p_i' = p_i'(x,y), \quad T_i = (a_i^1 + a_i^2 y)x + \vartheta_i(y), \quad C = (b_1 + b_2 y)x + \varphi(y). \qquad (5)$$

Here the index i ($i = 1,2$, subscript or superscript) is responsible for belonging to the liquid layer if $i = 1$ or to the gas layer if $i = 2$. In the course of solving the problem (1)-(4) with account (5), the functions u_i are defined as fourth-degree polynomials, ϑ_i and φ is the polynomials of the seventh degree, p' is the eighth degree polynomial on the transverse coordinate y.

The no-slip conditions for the velocity should be imposed on the upper $y = h$ and lower $y = -l$ fixed boundaries and the conditions for temperature are as follows $T_1|_{y=-l} = A_1 x + \vartheta^-$, $T_2|_{y=h} = A_2 x + \vartheta^+$ (here A_1, A_2, ϑ^+, ϑ^- = const). The vapor concentration on the upper boundary is equal to zero.

On the thermocapillary interface $y = 0$ the kinematic condition is fulfilled automatically; the dynamic conditions can be written as follows: $\rho_1\nu_1 u_{1y} = \rho_2\nu_2 u_{2y} + \sigma_T T_{1x}$, $p_1 = p_2$ (here σ_T is the temperature coefficient of the surface tension σ, $\sigma = \sigma_0 + \sigma_T(T - T_0)$). The conditions of the velocity and temperature continuity are set at the interface. The heat transfer condition at interface should be written with respect to the Dufour effect: $\kappa_1 T_{1y} - \kappa_2 T_{2y} - \delta\kappa_2 C_y = -\lambda M$ (here λ is the latent heat of evaporation, κ_i are the thermal conductivity coefficients, M is the evaporation mass flow rate supposed here being constant). A mass balance equation at $y = 0$ has the following form with respect to the Soret effect: $M = -D\rho_2(C_y - \alpha T_{2y})$. The saturated vapor concentration is determined with the help of the relation $C = C_*[1 + \varepsilon(T_{2y} - T_0)]$ that is a result of the Clapeyron-Clausius equation and Mendeleev-Clapeyron equation for the ideal gas. Here C_* is the saturated vapor concentration at $T_2 = T_0$, T_0 is a some initial temperature, $\varepsilon = \lambda\mu / RT_0^2$, μ is the molar mass of the evaporating liquid, R is the universal gas constant.

The gas flow rate in the upper layer is given and defined by the relation

$$Q = \int_0^h \rho_2 u_2(y) dy \, .$$

These boundary conditions allow us to determine the integration constants arising from the finding of exact solutions of the system (1)-(4) and unknown parameters of the problem.

3 Results of analytical calculations. Comparison with physical experiments

We consider the two-layer system, where the lower and upper layers are filled by ethanol and air, respectively. The experimental studies were conducted for different temperature regimes of the system. This fact is reflected in analytical studies, and some values of the physico-chemical parameters of the liquid and gas are selected depending on the initial temperature. The values of ethanol and air are presented in Tab. 1 and [9]-[11]. The Dufour and Soret parameters are $\delta = 10^{-3}$(K) and $\alpha = -5 \cdot 10^{-3}$ (1/K), respectively.

Table 1. Values of some physico-chemical parameters of the liquid and gas.

| Parameter	ethanol		air	
	20 ºC	30 ºC	20 ºC	30 ºC
ρ, kg/m³	789	781	1.20	1.16
v, m²/s	$1.5 \cdot 10^{-7}$	$1.285 \cdot 10^{-7}$	$1.6 \cdot 10^{-5}$	$1.5 \cdot 10^{-5}$
C_*	0.1	0.15	-	-
ε, 1/K	0.059	0.0549	-	-

In Fig. 1 shows the velocity profiles in the case of selecting physical parameters of the system for temperature 20 °C (fig. 1a) and the temperature profiles in the section $x = 0$ for the physical parameters of the system, corresponding to 20 °C (fig. 1b) and 30 °C (fig. 1c) for different values of gas flow rate Q. Increasing the gas flow rate values in the upper layer entails the increase of the flow intensity (see fig. 1a). More intensive cooling of the interface for different temperature regimes is also noticeably (see fig 1b, 1c). Note, that in the case when the physical parameters of the system selected for temperature 30 °C the temperature difference between solid walls and interface much more. However, the qualitative picture of temperature distribution in the system is maintained.
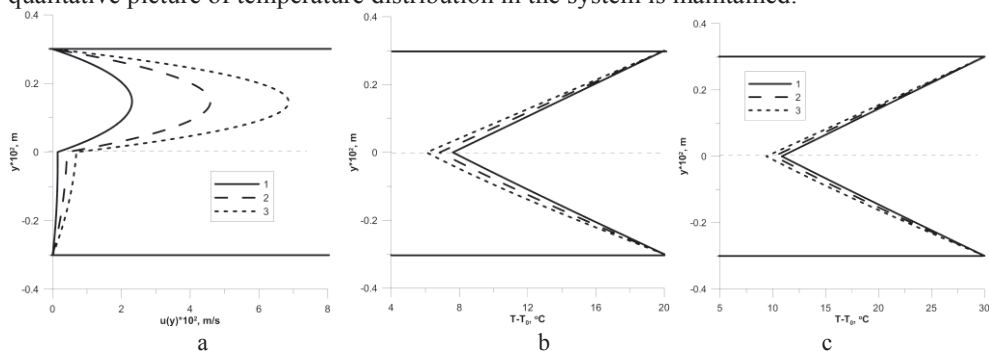
Fig. 1. a) Velocity and b) temperature profiles at different values of the gas flow rate, $T = 20$ ºC; c) temperature profiles at different values of the gas flow rate, $T = 30$ ºC. $A_1 = 10$ K/m, $h = 0.3 \cdot 10^{-2}$ m, $l = 0.3 \cdot 10^{-2}$ m (1 – $Q = 5.625 \cdot 10^{-5}$ kg/(m·s), 2 – $Q = 1.125 \cdot 10^{-4}$ kg/(m·s), 3 – $Q = 1.6875 \cdot 10^{-4}$ kg/(m·s)).

Figures 2 illustrate the results of investigation of the mass flow rate M dependence from the gas flow rate Q obtained experimentally (Fig. 2, lines 1 and 2) and with help of the

analytical formulas (Fig. 2, lines 3 and 4). The intensity of the liquid evaporation increases with increasing of the gas flow rate. This effect occurs by extension of the vapor concentration gradient in the gas phase. The mass flow rate of evaporation increases with the increase of the temperature in the channel, which agrees well with the experiments. Analytical calculations are performed for two-dimensional case and did not take into account all the factors that are present in the experiment. However, for smaller values of temperature (20 ° C) or small values of the gas flow are observed fairly good quantitative agreement between theoretical calculations and experiments. Note, that the analytical results are given for values of the longitudinal temperature gradient at the lower solid boundary A_1 = -100 K/m, whereas the temperature gradients in the experiment are not measured.
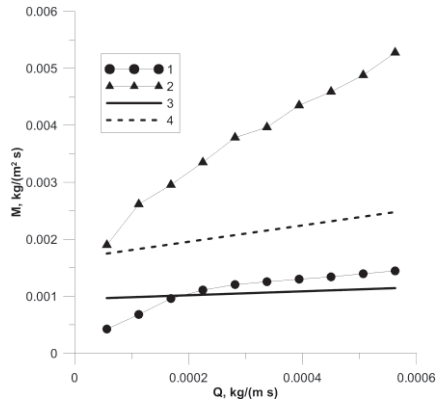
Fig. 2. Dependence of the evaporation intensity on the gas flow rate. A_1 = -100 K/m, h = 0.3·10^{-2} m, l = 0.3·10^{-2} m (1 experimental data at T = 20 ºC; 2 experimental data at T = 30 ºC; 3 calculations at T = 20 ºC; 4 3 calculations at T = 30 ºC).

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