

Accurate $\bar{\rho}$ and $\log g$ of δ Sct stars using Asteroseismology

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Abstract. In this work, we present a new method to determine the surface gravity of δ Sct stars. We used a refined $\Delta\nu$ - $\bar{\rho}$ relation and the stellar parallaxes or luminosities to determine their masses and radii. A comparison with the data obtained from the binary analysis, has shown that the values found by both methods are equivalent, within the uncertainties. Moreover, thanks to the refined relation, the uncertainties in $\log g$ are of the order of those usually estimated with high-resolution spectroscopy. Because of that, this new method to determine the surface gravity is an important step forward to break the degeneracy problem in the spectroscopic analysis.

1 Introduction

The pulsating spectrum of δ Sct stars is one of the most problematic to interpret, despite the large number of observed frequencies [7, 26]. This is because it is complicated to carry out a mode identification or find obvious patterns in their spectra, and because of the difficulties of the theoretical analysis, due to the rapid rotation of most of them.

Nevertheless, recent studies have found patterns in the periodograms of these pulsators [e.g., 2, 8, 11, 23, 33], pointing that they could be related to a large separation in the low radial order regime. In fact, a recent work has found a relation between this periodicity, pattern or large separation and the mean density of the star [9, from now GH15]. All these results agree with theoretical predictions, both based on non-rotating models [29] and on 2D models using a non-perturbative approach [22, 27].

In the work we present here, we describe in detail the methodology to find such separation in the periodogram and we refine the relation found in GH15. The final aim is to use this relation to determine another stellar property, namely the surface gravity. Making use of the parallax or the luminosity, we estimated the stellar masses using the mass-luminosity relation. Based on the $\Delta\nu$ - $\bar{\rho}$ relation, we found the radii and, thus, the surface gravity.

This article is divided into the following sections. In Sec. 2, we present the sample and its more relevant char-

acteristics. Section 3 describes the properties of the large separation-mean density relation, the methodology carried out to determine $\Delta\nu$ and to fit the data. In Sec. 4, we described the procedure we followed to determine $\log g$. And Sec. 5 details the conclusions of this work.

2 The sample

We used a sample of eclipsing binary systems with a δ Sct component. The study of these systems allows us to accurately determine the mass and radius, based only on the Kepler's laws and the transit analysis. In addition, the presence of pulsations in one of the components of the system allows us to check the pulsation theory.

We search in the bibliography for binary systems with such characteristics. We also imposed that they have precise space photometry. In this way, we found 7 systems fulfilling all the constraints. All these systems have precise fundamental parameters (masses and radii) and have been observed by MOST [31], CoRoT [1] or *Kepler* [15]. The sample is composed of 6 eclipsing binary systems and 1 binary system observed with optical interferometry, HD 159561. The δ Sct component of this system is a very fast rotator with $\Omega/\Omega_K = 0.88$.

With all this information, we computed the mean density of the stars. We have also used the frequencies derived from the light curve analysis to obtain the large separation. Table 1 shows the most relevant values used in the present

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work. Additionally, it shows the bibliographic references from where we extracted those data.

3 The $\Delta\nu$ - $\bar{\rho}$ relation

One of the most relevant results of the study in GH15 was the establishment of a relation between the pattern found in the frequency spectrum of a sample of δ Sct stars and their mean density. The method to obtain such a pattern is described in Sec. 3.1. The mean density was obtained from the values provided by the binary analysis. They even took into account the possible deviations from sphericity due to the rotation of the stars to compute their volumes. To that end, they used Roche’s model [e.g., 19]. Following the same procedure and using the same sample of stars, we refined the relation between the mean stellar density and large frequency separation by implementing a Hierarchical Bayesian linear regression (see details in Sec. 3.2). The updated relation is the following:

$$\bar{\rho}/\bar{\rho}_{\odot} = 1.62(\Delta\nu/\Delta\nu_{\odot})^{2.085}. \quad (1)$$

A plot of the data, our fit and a comparison with the solar-like relation and the theoretical relation found by [29] can be seen in the Fig. 1. The simple dependency of the pattern with the mean density and the detection of a trend in the Échelle diagrams is the sign of a large separation (understood as a frequency spacing). However, this large separation is found in the non-asymptotic regime of low-radial order p-modes.

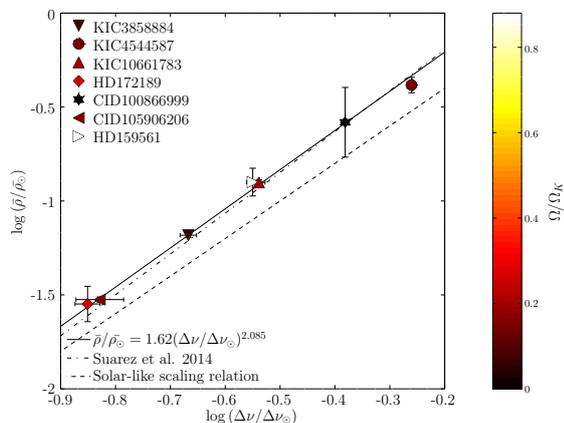


Figure 1: Large separation-mean density relation obtained for the seven binary systems of our sample. A linear fit to the points is also depicted, as well as the solar-like scaling relation [30] and the theoretical scaling relation for non-rotating models of δ Sct stars [29]. Symbols are plotted with a gradient colour scale to account for the different rotation rates.

Equation 1 fits those relations found in previous theoretical works, both using non-rotating models [29] and 2D models using a non-perturbative approach to compute the theoretical frequencies [27]. This relation is essentially the same obtained as that obtained in GH15, although the coefficients are slightly different.

The relation is parallel to that found for solar-type stars, once it is extrapolated to sub-solar densities. The observed shift between both relations is due to the different regimes where they are found. For the solar-like pulsators, this is the asymptotic regime, whereas for $\Delta\nu$ is around the fundamental radial mode.

But the most remarkable consequence, already suggested by previous non-perturbative studies [27], is the invariance of the relation with the rotation of the star. In our sample, we included a very fast rotating star, HD 159561. But, as can be seen in Fig. 1, this star also closely fits the relation. This invariance allows us to derive the mean density of a star independently of its rotation rate, making the large separation a powerful observable for δ Sct stars.

3.1 Obtaining $\Delta\nu$

Although the methodology is already described in GH15, it is convenient to provide some details to guide the reader. In order to obtain the pattern (large separation) from the frequency spectra of the stars in the sample, we followed the prescriptions given by [7, 8] and computed Fourier transforms. Additionally, we made a combined use of histograms of frequencies differences and Échelle diagrams, with the aim of searching for the most appropriated periodicity. An example of these three plots, can be seen in the Fig. 2, 3 and 4. The selected value of the large separation is marked with a dashed-line in the Fourier transform and in the histogram of frequencies differences. This value has then been used to build the Échelle diagram.

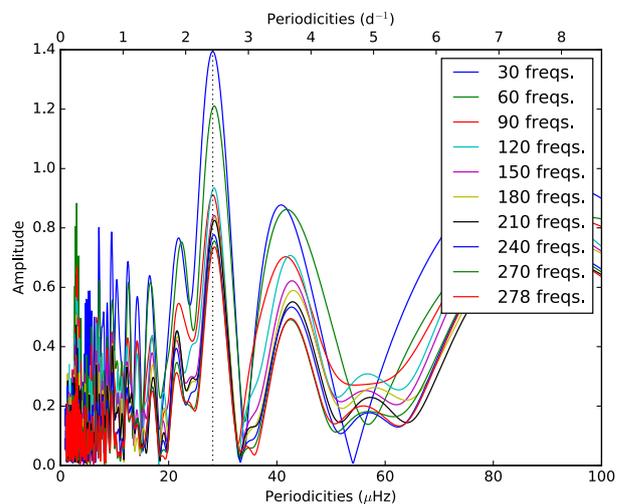


Figure 2: Fourier transform of different subsets of the highest-amplitude frequencies of KIC 3858884. The selected periodicity is marked with a dashed line (see text for details).

Essentially, the method lies in computing the Fourier transform of the frequencies, taking all the amplitudes equal to one (in whatever units). First, it is important to avoid harmonics or combinations between frequencies, frequencies driven by tidal effects, harmonics of the orbital period, and all frequencies above $\sim 50 \mu\text{Hz}$, to avoid possible g modes.

Table 1: Characteristics of the systems taken from the literature (see the reference citation in the first column). The information corresponds to the pulsating component (which is not necessarily the primary).

System	$\Delta\nu$ (μHz)	P_{orb} (d) / ν_{orb} (μHz)	$\bar{\rho}$ (ρ_{\odot})	$v \cdot \sin i$ (km s^{-1})	i ($^{\circ}$)	ν_{rot} (μHz)	Ω/Ω_{K}
KIC 3858884 [20]	29 ± 1	25.953 / 0.44598	0.0657 ± 0.0021	25.7 ± 1.5	88.176 ± 0.002	1.93	0.0754
KIC 4544587 [10]	74 ± 1	2.1890951 / 5.289352	0.414 ± 0.039	75.8 ± 15	87.9 ± 3	10.99	0.172
KIC 10661783 [17]	39 ± 1	1.2313622 / 9.3981481	0.1255 ± 0.0039	78 ± 3	82.39 ± 0.23	6.99	0.200
HD 172189 [5]	19 ± 1	5.70198 / 2.03009	0.0283 ± 0.0061	78 ± 3	73.2 ± 0.6	4.63	0.281
CID 100866999 [4]	56 ± 1	2.80889 / 4.12037	0.262 ± 0.112	–	80 ± 2	–	–
CID 105906206 [28]	20 ± 2	3.6945708 / 3.1365741	0.02986 ± 0.00095	47.8 ± 0.5	81.42 ± 0.13	2.61	0.152
HD 159561 [13, 21]	38 ± 1	3148.4 / 0.0036762	0.123 ± 0.021	239 ± 12	87.5 ± 0.6	19.16	0.88

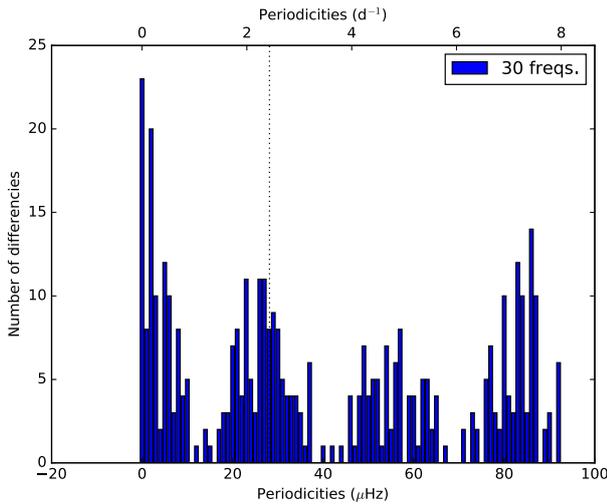


Figure 3: Histogram of frequencies differences for the 30 highest-amplitude frequencies of KIC 3858884. The selected periodicity is marked with a dashed line (see text for details).

Second, the Fourier transform is computed with a certain number of the highest-amplitude frequencies. More frequencies are then added to the set and the Fourier transform is computed again for each subset of frequencies. As a first guess, we took the most prominent peak as the value of the periodicity. The sub-multiples of this peaks might be also visible.

The next step is to determine if the periodicity and its multiples appear in the histogram of frequencies differences. The final verdict is given by the Échelle diagram. The peak in the Fourier transform does not always coincide with the one from the histogram, but a pattern must be detected in the Échelle diagram.

Notice that the periodicities determined in this way are far from the orbital period of the system and from the rotational frequency of the star (see Table 1). The rotational velocity of the star has been calculated using the projected rotational velocity and the inclination angle of the orbital plane, assuming that equatorial and orbital planes are the same. This is a crude estimate of the rotational velocities and are just indicative, so we did not calculate their uncertainties.

One must also have in mind that sometimes the periodicity does not correspond to the value of the large separation. In some cases, the periodicity is a sub-multiple of the

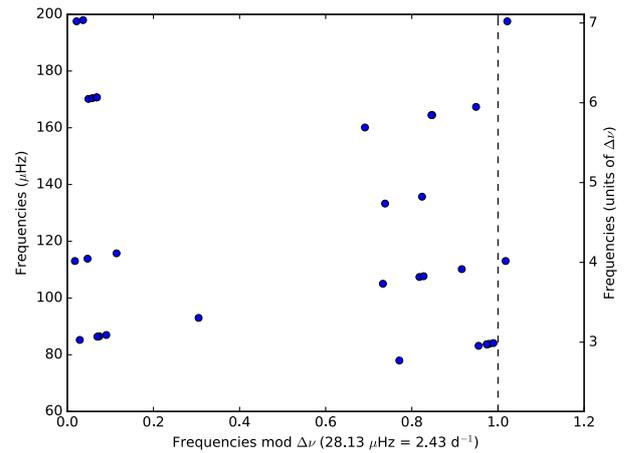


Figure 4: Échelle diagram for the 30 highest-amplitude frequencies of KIC 3858884. The large frequency separation used is that derived from the Fourier transform and the histogram of frequencies differences. Two clear ridges are visible around 0.8.

large separation, such a $\Delta\nu/2$ (as in the case of HD 172189 and HD 159561) or $\Delta\nu/4$ (as in the case of KIC 4544587). The $\Delta\nu/2$ possibility is expected from the theory [18, 27] and correspond to the cases in which $\ell = 1$ modes place between $\ell = 0$ modes at one half of the $\Delta\nu$ distance. The other possibility appears in the Fourier transform when a sub-multiple of $\Delta\nu$ is approximately equal to the rotational splitting or a multiple. To illustrate this, we can look at the case of KIC 4544587, for which $\Delta\nu/4 \sim 2\nu_{\text{rot}}$.

Recently, [24] determined the periodicity of a sample of 90 δ Sct stars observed by CoRoT. They estimated the masses, the radius and the rotational velocities of the stars and tried to place the sample over the $\Delta\nu$ - $\bar{\rho}$ relation given in [29]. The conclusion was that it was necessary to assume that the periodicity was a combination of the large separation and the rotational splitting in most cases in order to put the data over the relation. One should point out that the method they used to determine the periodicity did not use the Fourier transform nor the histograms of frequencies differences.

3.2 Uncertainties of the coefficients

Despite the clear linear relation between the stellar density and the large separation observed in GH15, the uncertainties of coefficients in the relation are likely over-estimated

in their weighted-least-square fitting procedure. We used a hierarchical Bayesian model to fit the relation, which fully takes account of the uncertainties in $\bar{\rho}$ and $\Delta\nu$. The calculation is implemented in the JAGS package [Just Another Gibbs Sampler, 25], and a similar application to fit the mass-radius relation of exoplanets can be found in [32]. We got the following result (C being the coefficient and λ being the exponent):

$$C = 1.62_{-0.14}^{+0.13}$$

$$\lambda = 2.085_{-0.051}^{+0.052}$$

The uncertainties obtained in this fit are one order of magnitude lower for the factor and half of the value for the exponent compared to GH15. The value of the coefficients have varied but they still are within the error-bars of both analysis.

4 Precise measurement of $\log g$ using Asteroseismology

To demonstrate the potential of the large separation as an observable, we tried to obtain the surface gravity of the star. Since most of the δ Sct stars are rapid rotators, we cannot determine their masses by just looking at the modelling. In addition, we wanted our calculations to be independent of any modelling. Since the Gaia mission [3] has just released its first series of data, we decided to use the stellar parallaxes or the luminosities (in case the first quantity is not available), to determine the masses. We thus used the mass-luminosity relation. With this value and the mean density provided by Eq. 1, we can derive the stellar radius, assuming spherical symmetry. We calculated the surface gravity of the stars in the sample and compared this value with the result of the binary analysis. A detailed discussion of the methodology and the results obtained are provided in the following sections.

4.1 Estimating masses

We have revised the bibliography again searching for independent measurements of the parallax, luminosity or absolute magnitude of each star. We just found information about the parallax of KIC 3858884, $p = 2.49 \pm 1.06$ mas, which has been measured by the HIPPARCOS satellite [16]. We used this value, the apparent magnitude, $m_v = 9.4297$ mag, and the bolometric correction from [12], $BC = 0.025$. For the other cases, we used the true luminosity for four stars and the absolute magnitudes for other two, as provided by the main references. Table 2 shows a summary of the used parameters, as well as the bibliographic references where we extracted the values.

Once we have the stellar luminosities, we used a mass-luminosity relation to determine the masses. We thus selected the relation found by [14]. In that work, they used binary systems to derive the relation, expressed as: $L \propto M^{3.20 \pm 0.26}$ (in solar units).

The strongest assumption we made in this case is that this relation can be applied to stars off the main sequence

(although not far) and that it is unaffected by the metallicity. Nevertheless, the mass estimates are not so critical for the calculation of $\log g$, as can be seen in Table 2.

4.2 $\log g$ from binarity vs $\log g$ from $\Delta\nu$ and L

Once we obtained the masses and densities from Eq. 1, it is straightforward to compute the radii. We assumed spherical symmetry, as usually assumed in binary analysis. Thus, we calculated the surface gravity, as shown in Table 2.

A few conclusions can be derived immediately. It is not possible to reach the precision level of the binary analysis, although those values are within the error-bars of the values derived in our work. Moreover, the uncertainties in the $\log g$ determined from $\Delta\nu$ and the stellar luminosities are of the order of those obtained from high-resolution spectroscopy.

At this point, one may see that both quantities correlate. We plotted the values to determine if there a equivalence between both. Fig. 5 shows the result, as well as a linear fit to the data and the relation "y = x". As expected, the equivalence is not perfect but pretty close. Indeed, as in Fig. 1, the dispersion of the points is significantly lower than their uncertainties for the case of the values computed with $\Delta\nu$ (x-axis). The key for obtaining such error-bars is the low uncertainty of the coefficients of the $\Delta\nu$ - $\bar{\rho}$ relation, and, above all, of the exponent.

The only value of $\log g$ not obtained from the binary analysis is that of HD 159561. For this fast rotator, we decided to use the spectroscopic determination, since the most interesting question to solve is how spectroscopy deals with non-spherical stars. It is remarkable that the star still follows closely the identity relation.

Therefore, this methodology can help to obtain a precise surface gravity of δ Sct stars, breaking the degeneracy of this parameters in many spectroscopic analysis. It is remarkable the accuracy in the determination of $\log g$, even with the crude estimate of stellar masses and regardless of the rotation rate of the star.

5 Conclusions

In this work, we presented a methodology to derive the surface gravity of δ Sct stars using just the pulsation frequencies and an estimate of the luminosity. We demonstrated that it is possible to get $\log g$ with the same precision as in high-resolution spectroscopic analysis. The key point is the uncertainties of the $\Delta\nu$ - $\bar{\rho}$ relation.

To achieve these results, we used the eclipsing binary systems from GH15, all of them with a δ Sct component. Thus, we determined the corresponding periodicity from the pulsation modes, using a Fourier transform of the frequencies, a histogram of frequencies differences and Échelle diagrams. We were able to correlate the periodicity and the mean densities of the stars in the sample, minimizing the uncertainties in the coefficients of the relation. This correlation proved that this periodicity is, indeed, a large separation in the low radial-order regime of

Table 2: Characteristics of the systems taken from the literature (named as *bin*) compared to those computed in this work (named as Δv). All the references are the same as in Table 1, except for HD 159561.

System	M_{bin} (M_{\odot})	R_{bin} (R_{\odot})	$\log g_{bin}$ (cgs)	For M-L relation	$M_{\Delta v}$ (M_{\odot})	$\log g_{\Delta v}$ (cgs)
KIC 3858884	1.86 ± 0.04	0.0657 ± 0.0021	3.74 ± 0.01	$\rho = 2.49 \pm 1.06$ mas	1.82 ± 0.04	3.74 ± 0.18
KIC 4544587	1.61 ± 0.06	0.414 ± 0.039	4.33 ± 0.01	$M_b = 1.72 \pm 0.04$ mag	2.03 ± 0.03	4.32 ± 0.21
KIC 10661783	2.100 ± 0.028	0.1255 ± 0.0039	3.938 ± 0.004	$L = 21.62 \pm 1.03 L_{\odot}$	2.19 ± 0.03	3.94 ± 0.19
HD 172189	1.78 ± 0.24	0.0283 ± 0.0061	3.48 ± 0.08	$L = 52.42 \pm 2.85 L_{\odot}$	2.75 ± 0.04	3.54 ± 0.17
CID 100866999	1.8 ± 0.2	0.262 ± 0.112	4.1 ± 0.1	$M_b = 2.3 \pm 0.2$ mag	1.77 ± 0.05	4.13 ± 0.20
CID 105906206	2.25 ± 0.04	0.02986 ± 0.00095	3.53 ± 0.01	$L = 1.53 \pm 1.00 L_{\odot}$	2.46 ± 0.02	3.56 ± 0.20
HD 159561 [6]	$2.40^{+0.23}_{-0.37}$	$R_{eq} = 2.858 \pm 0.015$ $R_{pol} = 2.388 \pm 0.013$	$3.89 \pm 0.20^*$	$L = 31.30 \pm 0.96 L_{\odot}$	2.407 ± 0.019	3.94 ± 0.19

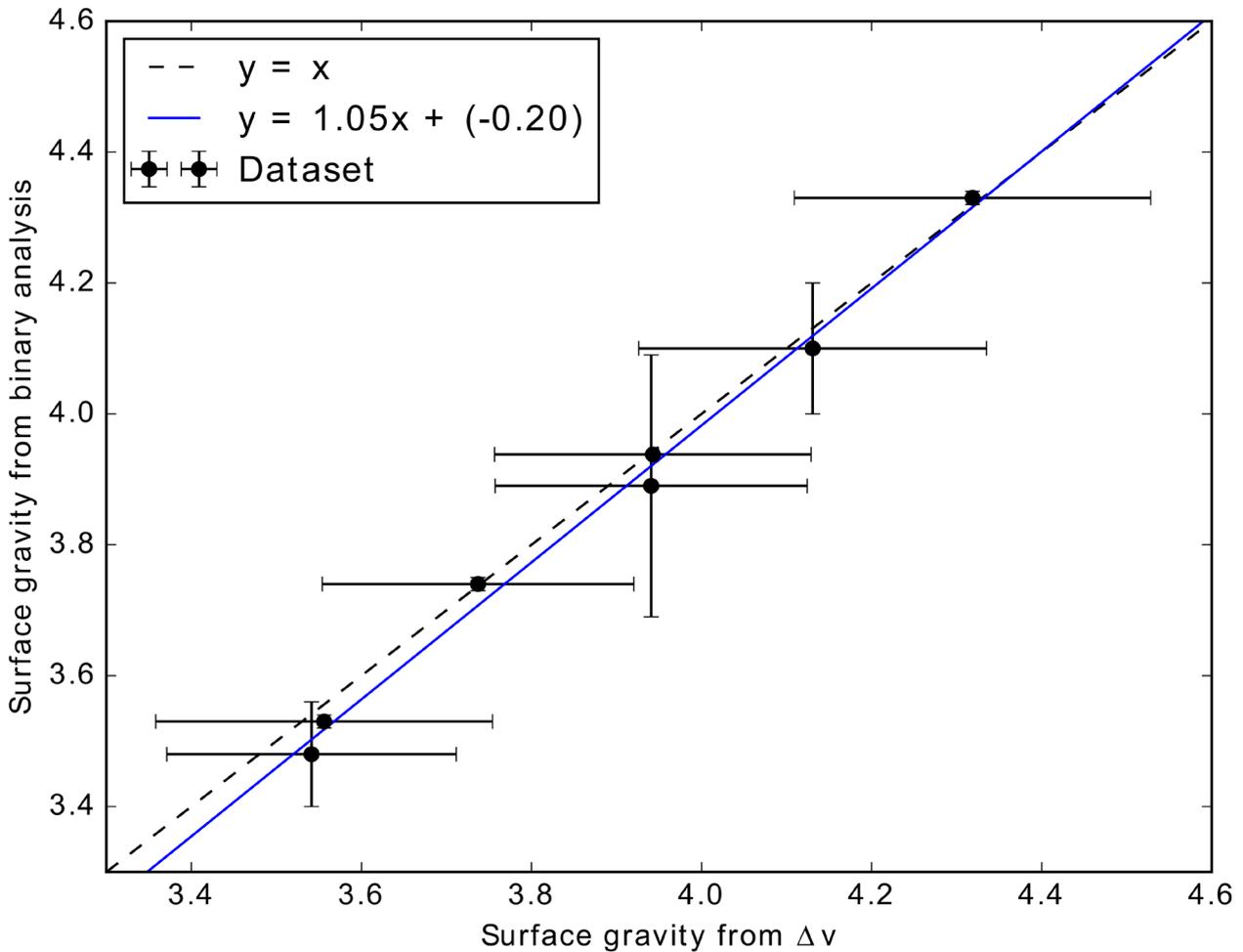


Figure 5: Surface gravity obtained from the analysis carried out in this work versus $\log g$ from the binary analysis (see text for details). A linear fit is also plotted as is the $y = x$ line for reference.

the δ Sct pulsators. The most remarkable characteristic of this relation is its independence with respect to the stellar rotation rate. That makes this large separation a promising observable for the study of δ Sct stars.

Pushing the analysis a step further, we used this relation to determine another physical quantity of the star, namely the surface gravity. We estimated stellar masses through their luminosities. Comparing our result with that obtained from the binary analysis, we found an almost identity relation between both quantities. Moreover, it

is possible to determine $\log g$ with the same precision as in high-resolution spectroscopic analysis, with an uncertainty of around 0.2 dex.

The large separation, Δv , in δ Sct stars has the potential to break the degeneracy in $\log g$ determination from spectroscopic analysis. This will be possible thanks to missions such as Gaia, which will provide accurate measurements of the stellar parallaxes of the whole sky. This will become an important breakthrough in the study of δ Sct pulsations.

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