

A fitting LEGACY – modelling *Kepler*'s best stars

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Abstract. The LEGACY sample represents the best solar-like stars observed in the *Kepler* mission[5, 8]. The 66 stars in the sample are all on the main sequence or only slightly more evolved. They each have more than one year's observation data in short cadence, allowing for precise extraction of individual frequencies. Here we present model fits using a modified ASTFIT procedure employing two different near-surface-effect corrections, one by Christensen-Dalsgaard[4] and a newer correction proposed by Ball & Gizon[1]. We then compare the results obtained using the different corrections. We find that using the latter correction yields lower masses and significantly lower χ^2 values for a large part of the sample.

1 Introduction

The use of helioseismology has been tremendously successful in mapping the solar interior and the general discipline of asteroseismology has revolutionized our understanding of the physics of stellar interiors, as well as enabled precise determination of crucial global stellar parameters such as the mean density directly inferable from the large frequency separation. The space-based *Kepler* mission has, through long, continuous time-series photometry, provided invaluable data for asteroseismic analysis. *Kepler* has been a huge success in general expanding our knowledge of many types of stars as well as the discovery and characterization of exoplanets. The LEGACY sample focuses on the very best observations of main-sequence (and a few subgiant) solar-like oscillators observed by *Kepler*[5, 8]. It features 66 stars all observed for at least 12 months in short cadence making them some of the highest signal-to-noise solar-like targets in the *Kepler* mission. None of the stars exhibit properties of mixed modes. The details of their oscillation properties are presented in [5], while radii, masses and ages are the topic of [8]. Here we consider one aspect of fitting stellar models to these phenomenal observations, namely the choice of surface-effect corrections.

One of the major challenges in stellar astrophysics has always been the determination of stellar ages. They are extremely relevant in many areas, for example in trying to understand the formation of stellar systems or the evolution of our galaxy. But the age of a star cannot be measured or inferred from observations. Rather it is derived by fitting models of stellar evolution to match asteroseismic

and spectroscopic data. However, stellar evolution models are rather crude, neglecting or oversimplifying key physical processes, which makes the resulting ages somewhat unreliable. Silva Aguirre et al.[7] determine stellar ages of *Kepler* exoplanet host stars to within a median statistical error of 14 per cent - an order of magnitude larger than the corresponding errors on radius, density, mass and distance. Here we apply essentially the same model fitting procedure using both the same surface-effect correction as well as a more recent correction and compare the outcome of the two.

2 Model Fitting

We use a model-fitting procedure, which combines the Aarhus stellar evolution code[2] with the Aarhus adiabatic oscillation package[3] to fit individual model frequencies within a grid of stellar models. The grid was computed as described for ASTFIT in[7] with the following exceptions: The heavy-element mixture was based on Grevesse & Noels (1993), taking the solar surface ratio between the abundances of heavy elements and hydrogen as $Z_s/X_s = 0.0245$. Furthermore the grid extends in mass from 0.7 to 1.7 M_\odot , in steps of 0.01 M_\odot . The heavy-element abundance ranged from $Z = 0.0032$ to 0.059, relating Y and Z by $\Delta Y/\Delta Z$ values varying between 1 and 2, in steps of 0.2. Here ΔY and ΔZ are the differences in helium and heavy element abundance between today and the time of the big bang. Often this is assumed to be around 1.4. However, since the exact value is unknown, and subject to debate, it is allowed to vary here. Diffusion and settling were not taken into account. Two different surface-effect corrections were used to account for the fact that model frequencies are systematically overestimated:

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a: A correction based on a scaled solar fit (see [4]). This works very well for other stars with masses close to $1M_{\odot}$, but falls increasingly short for heavier, and consequently hotter, stars as shown in Section 3.

b: A recent correction presented by Ball & Gizon (2014);

$$\delta\nu = (c_{-1}(\nu/\nu_{\text{ac}})^{-1} + c_3(\nu/\nu_{\text{ac}})^3)/\mathcal{I}, \quad (1)$$

where $\delta\nu$ is the frequency difference $\nu_{\text{obs}} - \nu_{\text{model}}$ for a given mode, ν_{ac} is the acoustic cut-off frequency, \mathcal{I} is the mode inertia, and c_{-1} and c_3 are the fitting coefficients. The formula is based on calculations originally carried out to explain solar cycle frequency variations, and includes two terms: one proportional to ν^{-1} , which represents a frequency shift due to the increase in pressure scale height that would arise from better modelling of convection, and another term proportional to ν^3 correcting for a frequency shift caused by local changes to the sound speed without changing the density stratification [10]. The correction is weighted by individual mode inertia, for which we use the standard ASTEC calculation, which is normalized by the displacement at the photosphere. This correction formula seems to produce very reasonable fits for all the solar-like stars in the mass and temperature range considered here.

When combined with the aforementioned correction we produce frequency fits of unprecedented quality – even for stars where the surface term differs significantly from that of the Sun. The best fitting model is found by minimizing

$$\chi^2 = \chi_{\text{spec}}^2 + \chi_{\nu}^2, \quad (2)$$

where χ_{spec}^2 indicates the contribution from fitting spectroscopic data (T_{eff} and $[\text{Fe}/\text{H}]$) and χ_{ν}^2 is the contribution from fitting the frequencies as defined in [7]. Our ability to model observations in a precise manner helps constrain the physical parameters of the observed stars as well as pinpoint the aspects of stellar models that still need improvement.

3 Results

Figures 1 and 2 show frequency fits for the best fitting models of two LEGACY stars: KIC 8006161 (top) and KIC 7940546 (bottom). Both **a** and **b** work well for the $0.97M_{\odot}$ star; in fact the scaled solar fit is slightly better here. For the second star the two approaches yield slightly different masses close to $\sim 1.5M_{\odot}$ and effective temperatures of 6275.8K (**a**) and 6313.7K (**b**). The surface effect of this star is very different from the Sun and thus modelled poorly by the scaled solar fit **a**, whereas the inertia-scaled fit **b** captures the overall trends in the curve, which is evident from the much lower χ_{ν}^2 value (see figure captions).

Figure 3 (top) shows the effect on obtained masses of using either approach for the whole LEGACY sample. This is shown as δM , the mean mass from approach **b** minus the mean mass from **a** divided by the combined standard deviation σ , i.e.

$$\delta M = \langle M \rangle_{\text{b}} - \langle M \rangle_{\text{a}}, \quad (3)$$

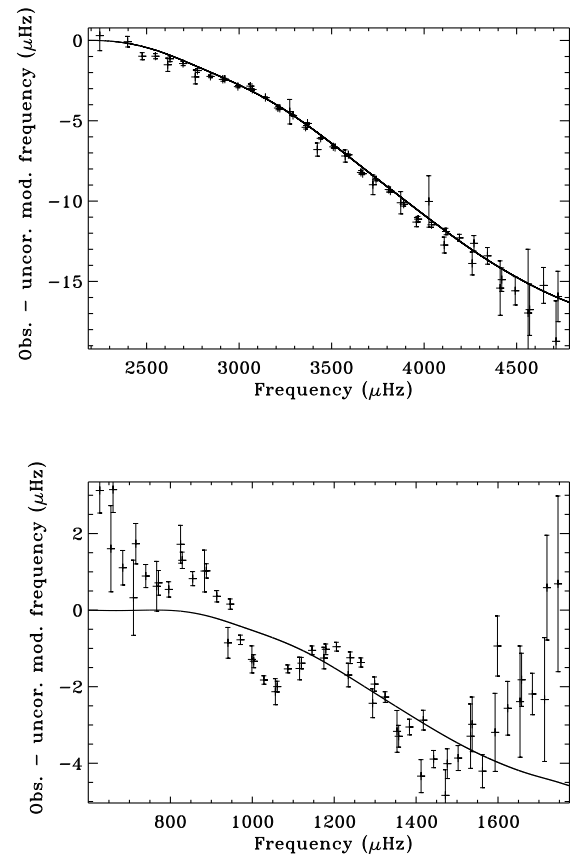


Figure 1: Scaled ‘solar fit’ (**a**) applied to models of KIC 8006161 and KIC 7940546. The fits pertain to the models with the lowest combined χ^2 . **Top panel:** Surface effect correction for a $0.97M_{\odot}$ model with $T_{\text{eff}} = 5465.9\text{K}$. The contribution from the frequency fit to χ^2 is $\chi_{\nu}^2 = 2.64$. **Bottom panel:** Surface effect correction for a $1.47M_{\odot}$ model with $T_{\text{eff}} = 6275.8\text{K}$ and $\chi_{\nu}^2 = 12.60$.

and

$$\sigma(\delta M) = \sqrt{\sigma(M_{\text{b}})^2 + \sigma(M_{\text{a}})^2}. \quad (4)$$

The mean mass is obtained by averaging over many models within a suitable mass range in the grid, where each model is weighted by $\exp(-\chi^2/2)$. For $M \gtrsim 1.3M_{\odot}$ there is a significant difference between the two, where **b** consistently yields lower-mass stellar models, and as a consequence they are older¹.

Figure 3 (bottom) shows the χ^2 ratio between the two approaches. Here χ_{min}^2 (a) is the best fitting model using the scaled solar fit, and (b) is using the Ball & Gizon (2014) surface effect correction. Except for some cases around $\sim 1M_{\odot}$, model fits using the Ball & Gizon (2014) correction have lower χ^2 values. This is thus preferable to any previous correction formulae – even for stars where the surface term differ significantly from that of the Sun. It is worth noting that the lower masses obtained using surface fit **b** are accompanied by lower χ^2 values; not only are

¹This has also been found explicitly in our analysis but is not shown here

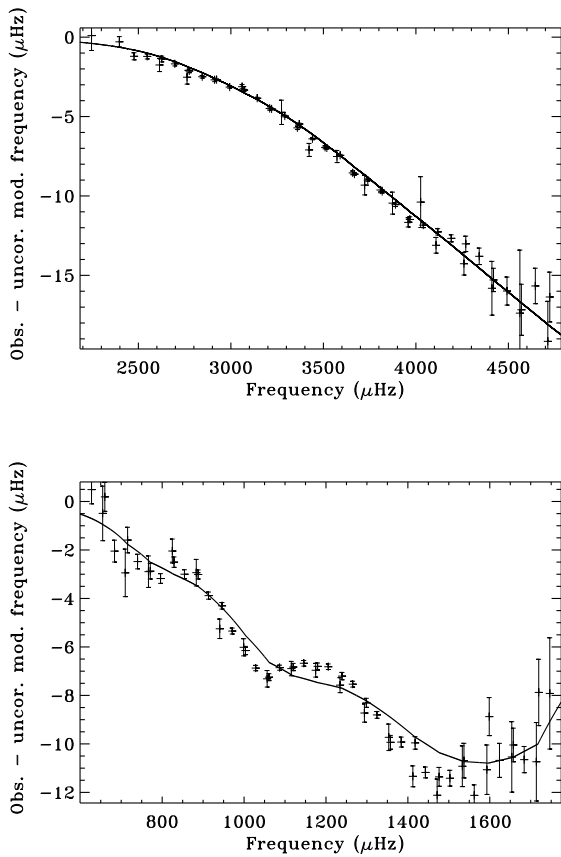


Figure 2: Inertia-scaled two-term fit **(b)** applied to models of the same stars as in Figure 1. Again, the fits shown are from the models with the lowest combined χ^2 . **Top panel:** Surface effect correction for a $0.97M_{\odot}$ model with $T_{\text{eff}} = 5465.8\text{K}$ and $\chi^2_{\nu} = 2.90$. **Bottom panel:** Surface effect correction for a $1.46M_{\odot}$ model with $T_{\text{eff}} = 6313.7\text{K}$ and $\chi^2_{\nu} = 6.73$.

the masses and ages different compared to the previous approach **a**, they are also determined with better precision.

3.1 Inertia scaling

We have established that the inertia-scaled two-term fit is superior for the more massive, and consequently hotter, stars in the sample. We can gain some insight into why this is by looking at the inverse mode inertia used for scaling the fitting terms shown in Figure 4. As has also been pointed out by Trampedach et al.[9], the peculiar features in the frequency differences between $\sim 1000\mu\text{Hz}$ and $\sim 1500\mu\text{Hz}$ coincide with an oscillatory contribution to the inverse mode inertia. We have identified the issue as arising from effects of the second helium ionization zone on the eigenfunctions. There is a good reason for this feature to only show up in the surface layers of hotter stars: There is a significantly larger drop in Γ_1 in the second helium ionization zone, possibly due to the gas density being lower here than in a corresponding zone in a cooler star. This will be elaborated on in an upcoming paper.

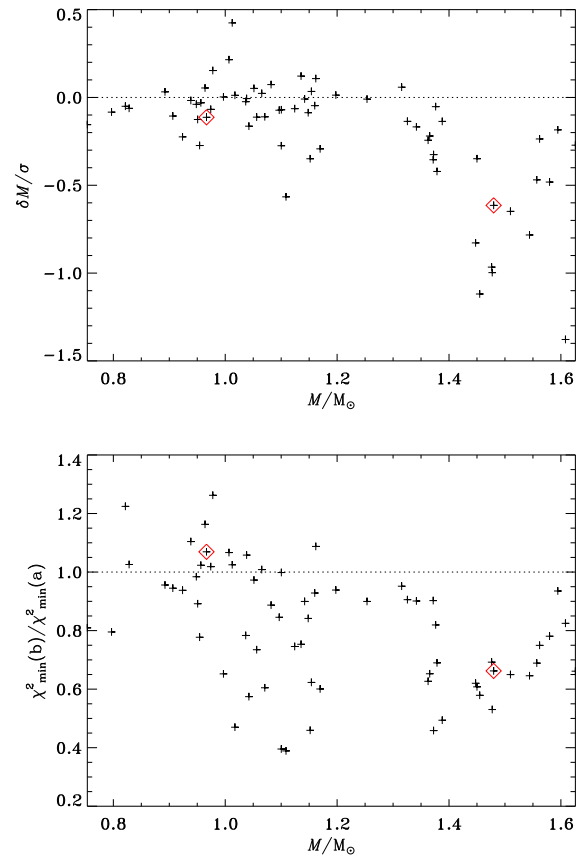


Figure 3: Top: Difference in mean mass obtained with either surface effect correction for all stars in the LEGACY sample. **bottom:** Ratio between χ^2 of the best fitting models. Diamonds highlight the stars in Figures 1 and 2.

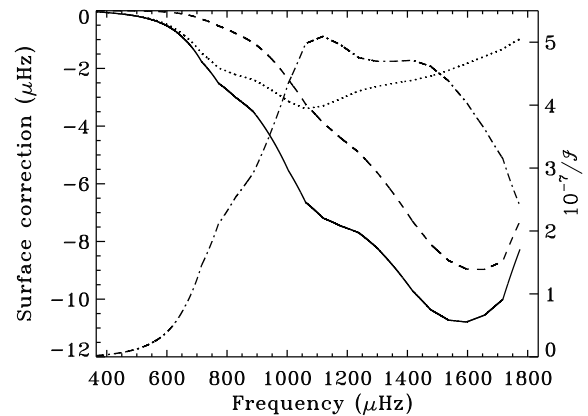


Figure 4: Surface-effect correction and inverse inertia of KIC 7940546. For the left y -axis we present the full line showing the full two-term fit **(b)**, while the dashed and dotted lines show the contributions to the fit from the cubic and inverse frequency terms respectively. On the right y -axis we present the inverse mode inertia (dot-dashed line) used to scale the fitting terms

Table 1: Table of global parameters for the two stars presented in Figures 1 and 2 as well as the fit quality described by χ^2 (see eq. (2))

Correction	KIC ID	$M [M_{\odot}]$	$R [R_{\odot}]$	$T_{\text{eff}} [\text{K}]$	Age [Gyr]	χ^2
a	8006161	0.967 ± 0.018	0.923 ± 0.006	5465.9	5.008 ± 0.463	3.640
b	8006161	0.964 ± 0.018	0.922 ± 0.006	5465.8	5.093 ± 0.460	3.892
a	7940546	1.480 ± 0.026	1.990 ± 0.014	6275.8	2.325 ± 0.104	24.670
b	7940546	1.458 ± 0.025	1.971 ± 0.015	6313.7	2.356 ± 0.106	16.340

4 Conclusions

We have presented a comparison between two surface-effect corrections employed in a grid model fitting scheme. We find that there are significant differences in the obtained global stellar parameters for stellar masses $\geq 1.1M_{\odot}$, the largest differences occurring at $\geq 1.4M_{\odot}$. Here we see that not only are the masses we find using correction **b** systematically lower, they are also determined more accurately as reflected by their lower χ^2 values.

Trampedach et al[9] investigate the appropriateness of eq. (1) across the HR diagram using 1D envelope models patched with averaged 3D atmospheres from hydrodynamical simulations. Their findings support our notion that the fit is very versatile and can be used across the HR diagram for solar-type stars also including the pre-main sequence and red giant phase. The versatility is reflected by the fact that the ratio c_{-1}/c_3 changes, c_{-1} increasing relative to c_3 , when going from cooler to hotter stars. Another study modelling the surface term across the HR diagram was performed by Schmitt & Basu[6]; they also found that, while various correction formulas might suit different types of stars, the two-term fit by Ball & Gizon[1] performs well across the whole range in question (from the main sequence to red giants of models ranging in mass from 0.8 to $1.5M_{\odot}$). The results presented in this paper agree with other recent studies, which suggests that overall the physically motivated correction **b** is the best available surface term to be used for general-purpose model fitting. Of course it requires the structure of the model to be close to that of the observed star in order to get the appropriate mode inertia necessary for eq. (1). However, we do also show that for more specific cases, perhaps other surface terms might also be reasonable and in some cases

even slightly better. As an example the surface term of KIC 8006161 is so close to that of the Sun that it is better described using the scaled solar fit yielding both lower χ^2 and χ^2 .

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