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Ground-state level spin systems within Bernasconi model

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Abstract. The design problem for law autocorrelation binary sequences (LABS) is a notoriously difficult computational problem which is numbered as the problem number 005 in CSPLib. In statistical physics LABS problem can be interpreted as the energy of N interacting Ising spins. This is a Bernasconi model for a one N-dimensional chain of quantum particles. Ground-state level quantum system within the frame of Bernasconi model is binary sequence with lowest level of peak-sidelobes of aperiodic autocorrelation function. The new results of an exhaustive search for optimal binary sequences with minimum peak sidelobe up to length N=86 are demonstrated. The Bernasconi model exhibits features of a glass transition like a jump in the specific heat and slow dynamics and aging. The Bernasconi model the high temperature phase of Ising spin system reproduces exactly approximation.

Let us consider the one-dimensional spin lattice $\mathbf{S}=(s_0, s_1, ..., s_{N-1})$ consisting of N particles with an equal distances between neighbouring particles. Within one dimensional Ising model [1] the spin can assume one of two values $s_n = \pm 1$ in each lattice node with a number n. The energy of such quantum system consists of pairwise of exchanged interaction between neighbouring atom spins and the interaction of the spins and an external magnetic field:

$$E(\mathbf{S}) = -\sum_{n,\tau} J_{n\tau} s_n s_\tau - \sum_n h_n s_\tau \quad (1)$$

where indices n and τ denote the number of lattice nodes, h_n is the strength of the external magnetic field in the n-th node, and $J_{n\tau}$ is the energy of interaction for spins located in nodes n and τ.

In [2], Bernasconi pointed out the connection between the problem of searching for low energy states of a simplified the one dimensional Ising model (2) and that of constructing binary sequences $\mathbf{S}=(s_0, s_1, ..., s_{N-1})$ of N length with low levels of autocorrelation.

There are two criteria for the optimality of binary sequences with low levels of aperiodic autocorrelation. The first is the minimax criterion, in which the maximum level of side lobe PSL

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$$PSL(\mathbf{C}) = \max_{1 \le \tau \le N-1} |C_\tau| \quad (2)$$

must be minimal:

$$MPS = \min_{\mathbf{S}} PSL\,. \quad (3)$$

The second is the MF (merit factor) coefficient characterizing the ratio of the main frame energy to that of the side lobes of the aperiodic autocorrelation function:

$$MF(\mathbf{C}) = \frac{N^2}{2\sum_{\tau=1}^{N-1}[C_\tau]^2}\,. \quad (4)$$

Table 1 shows the constructed binary sequences with the length $N = 86$ and $PSL = 5$ with the maximum values of the coefficient MF.

Table 1. Optimal binary sequences with low level of side lobes

Binary sequence	*MF*
000010011110110101111011010011101100000100100111001100100000011100001 11001010101010000	7,125
000000001000100011011101010011001011001100011110110000101101101010110 00011101011110000	7,017
000000000001011010100101010011011100010101110011001010011011000100111 10010110011110000	6,522
000000001110000010000111101111011010100010101010011101011010011001010 01110010011011000	6,522
000000000001011010100101010011011100010101110011001010011011000100111 10010110011110000	6,522
000000000001110011100001010011100110011001101100100100101101010101010 11110000011110000	6,431
000000000100101011010110001001110101010100011000111100110001001101001 01100100111110000	6,431
000001001011001110011110110000001010001001010010101000001011110100110 00101110011111000	6,431
000000000001110011100001010011100110011001101100100100101101010101010 11110000011110000	6,431

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