EPJ Web of Conferences **161**, 02023 (2017) DOI: 10.1051/epjconf/201716102023
PECS-2017

Exact dynamics of a two-level atom beyond the rotating wave approximation

Vitalii Semin*

Samara National Research University, Department of nanoengeeniring, 34 Moskovskoe Shosse, Samara, 443086 Russia

Abstract. Interaction Hamiltonians of some models beyond the rotating wave approximation are just a product of two commuting operators. The evolution operator of such models can be transformed into product of two independent chronological exponents with the help of Hubbard-Stratonovich transformation. We use such a representation of the evolution operator to exactly describe a two-level atom in a photonic thermostat.

Description of dynamics of an open quantum system is an extremely difficult task. Today there are only a few models allowing one to find the exact dynamics. Among these are a quantum harmonic oscillator in a thermostat of oscillators [1], Garraway's model [2], pure dephasing of a two-level atom [3], a fermionic oscillator in a bath of fermionic oscillators [4], spin star models [5]. The characteristics of the abovementioned models that allow us to find the exact dynamics are some symmetries of the model Hamiltonian or a possibility to truncate the Hilbert space of the model.

In this paper we consider the model of a free relaxation of a two-level atom beyond the rotating wave approximation. The interaction Hamiltonian in the interaction pictures writes as a product of two independent operators, namely

$$H_I(t) = \left(\sigma_- e^{-i\omega_0 t} + \sigma_+ e^{i\omega_0 t}\right)\left(\sum_k g_k \left(b_k e^{-i\omega_k t} + b_k^+ e^{i\omega_k t}\right)\right) = A(t)B(t), \quad (1)$$

where $\sigma_\mp$ is the Pauli matrix, ω_0 is the atomic transition frequensy, $b_k(b_k^+)$ is annihilation (creation) operator of kth photon in the bath with frequency ω_k and g_k is the interaction constant.

The exact dynamics is described by the reduced density operator

$$\rho_A = Tr_B\big(U(t)\rho(0)U^+(t)\big), \quad (2)$$

where the trace is taken over the bath and the evaolution operator satisfies the Schroedinger equation with the Hamiltonian (1). We also assume that initially the bath and the atoms are uncorrelated and the bath is in the thermal equilibrium state.

Now we can use the fact of commuting operators in (1). For such operators the evolution operators in (2) can be represented as a product of two independent chronological exponents with the help of the Hubbard-Stratonovich transformation. After the transformation the trace in (2) can be calculated explicitly and the result expression has the form

$$\rho_S(t) = \int \mathcal{D}\big(z_1(t), z_2(t)\big)\overleftarrow{T}\exp\left[-i\int_0^t z_1(s)A(s)ds\right]\rho_A(0)\overrightarrow{T}\exp\left[i\int_0^t z_1(s)A(s)ds\right], \quad (3)$$

* Corresponding author: semin@ssau.ru

where the arrows indicate the time-ordering and $\mathcal{D}$ is some functional measure. The expression (3) is exact and its calculation is the main result of this research.

References

[1] B.L. Hu, J. Paz, Y. Zhang, Phys. Rev. D **45**, 1576 (1992)

[2] B.M. Garraway, Phys. Rev. A **55**, 2290 (1997)

[3] W.G. Unruh, Phys. Rev. A **51**, 992 (1995)

[4] A. Ghosh, S.S. Shinha, D.S. Ray, Phys. Rev. E **86**, 011138 (2012)

[5] H.P. Breuer, D. Burgarth, F. Petruccione, Phys. Rev. B **70**, 045323 (2004)