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Callan-Symanzik equations for infrared Yang-Mills theory

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Abstract. Dyson-Schwinger equations have been successful in determining the correlation functions in Yang-Mills theory in the Landau gauge, in the infrared regime. We argue that similar results can be obtained, in a technically simpler way, with Callan-Symanzik renormalization group equations. We present generalizations of the infrared safe renormalization scheme proposed by Tissier and Wschebor in 2011, and show how the renormalization scheme dependence can be used to improve the matching to the existing lattice data for the gluon and ghost propagators.

Historically, the first semi-analytical method that has made it possible to access the deep infrared (IR) regime of Landau gauge Yang-Mills theory, have been Dyson-Schwinger equations. The first solutions of these equations that have been found show a scaling behavior of the propagators in the IR [1]. Several years later, another type of solutions of the same equations was discovered [2], with a massive gluon propagator and a finitely enhanced ghost propagator in the IR (decoupling solutions). To date, Dyson-Schwinger equations are still the main tool for the (semi-)analytical exploration of the IR regime of Yang-Mills theory.

In a parallel development that actually initiated much earlier with Gribov's observation of the existence of gauge copies in his famous 1978 paper [3] and was later worked out in great detail by Zwanziger [4], the theoretical foundations of IR Yang-Mills theory in the Landau gauge were laid. While this so-called Gribov-Zwanziger scenario seemed to confirm the existence of the scaling solutions, a more recent refinement [5] favors the decoupling type of solutions. Finally, the results of simulations of Yang-Mills theory in the Landau gauge on huge lattices [6] have been interpreted by most workers in the field as confirming the realization of the decoupling type of solutions in three and four space-time dimensions.

In this contribution, we will present a different technique for the exploration of the IR regime of Yang-Mills theory. We intend to reproduce the results of lattice simulations in the Landau gauge that restrict the gauge field configurations to the (first) Gribov region, but make no attempt to reach the fundamental modular region. The restriction to the Gribov region implies the breaking of BRST invariance in the continuum formulation [4]. The most important consequence of the broken BRST invariance for the IR regime is the appearance of a mass term for the gluon field [5]. Indeed, the addition of a gluonic mass term to the Yang-Mills action has been shown to reproduce all the solutions (two types of scaling solutions and the decoupling solution) found before with the help of Dyson-Schwinger equations when solving the Callan-Symanzik equations for this theory in the IR regime in an epsilon expansion around the upper critical space-time dimension [7]. This analysis also shows that

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the decoupling solution is the one that is physically realized since it is the only one that corresponds to an IR attractive fixed point (for dimensions above two).

For the rest of this contribution, we will focus on the decoupling solution. Tissier and Wschebor [8] have shown that straightforward one-loop perturbation theory applied to Yang-Mills theory with a gluon mass term yields results for the gluon and ghost propagators that reproduce surprisingly well the propagators found on the lattice in three and four space-time dimensions in the IR regime. It is apparent, however, that a renormalization group improvement is necessary for a quantitative description of the ultraviolet (UV) regime in four space-time dimensions, and also for a quantitatively good fit to the propagators in the IR regime in three dimensions. In renormalization group language, what is required is a precise description of the crossover between the UV attractive Gaussian fixed point and the IR attractive (high-temperature) fixed point. It turns out that an action that contains all the terms that are relevant or marginal with respect to one or the other fixed point, is sufficient to describe the complete crossover, see Eq. (1) below.

Two comments are in order before we move on: first, the BRST invariance of Yang-Mills theory is essential for the standard construction of the Hilbert space of the theory, the proof of unitarity and also the Kugo-Ojima confinement criterion [9]. It is not clear at present how these constructions could be adapted to the case of broken BRST symmetry which we have assumed here from the outset. In fact, adding a gluon mass term to the Yang-Mills action gives rise to a Curci-Ferrari model [10] which is perturbatively renormalizable, but is commonly considered not to be unitary [11]. We should like to point out that the statement of non-unitarity refers to the standard construction of the Hilbert space, and it has actually not been shown that no Hilbert space construction exists that would lead to a unitary theory. We refer, in this context, to the recent efforts in Refs. [12, 13] for the Curci-Ferrari model and the (refined) Gribov-Zwanziger action. In this contribution, we will not be concerned with this fundamental and important issue, but rather focus our attention on calculating the n-point functions of the theory in the most efficient and accurate way.

The second comment concerns the gluon propagator in the presence of a mass term: it has been shown in numerous occasions that a gluon propagator of the decoupling type violates spectral positivity (just as in the more obvious case of the scaling type), and it thus does not describe the propagation of a physical particle, contrary to the case of mass generation through the Higgs mechanism. Also, gobal color symmetry remains an exact symmetry of the theory even though BRST invariance is broken.

We shall now set up renormalization schemes for the theory that allow for a consistent formulation of Callan-Symanzik equations, i.e., that avoid the appearance of a Landau pole. Let us remark that we use the term "Callan-Symanzik equations" to refer to the equations that quantify the change of the renormalized correlation functions with respect to variations of the renormalization scale (not with respect to variations of the mass parameter), in accord with the use of the term in modern textbooks on quantum field theory.

We begin with the Euclidean action of Yang-Mills theory in Landau gauge, including, in addition, a mass term for the gluon field (we then arrive at a Curci-Ferrari model). It is convenient to introduce the Nakanishi-Lautrup auxiliary field B^a to implement the gauge condition. Then the action is, in four space-time dimensions,

$$S = \int d^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} A^a_\mu m^2 A^a_\mu + \partial_\mu \bar{c}^a D^{ab}_\mu c^b + i B^a \partial_\mu A^a_\mu \right) . \tag{1}$$

Note that there are (at this point) *two* parameters in the theory, the gluon mass parameter m and the coupling constant g that appears in the covariant derivative D^{ab}_μ and in the field strength tensor $F^a_{\mu\nu}$. The mass parameter m can be thought of as representing the Gribov parameter that is introduced in order to implement the restriction to the Gribov region [3]. The Gribov parameter, and as a consequence the mass parameter, is in principle fixed through the horizon condition [4] in terms of the unique scale of the theory, $\Lambda_{\rm QCD}$ (equivalently, in terms of the value of g at an arbitrary scale). Since the exact relation between m and $\Lambda_{\rm QCD}$ is not known, at least not beyond the lowest perturbative order, we treat m here as an independent parameter that has to be fixed by comparison with the "experiment" (lattice simulations), just like the mass of a physical particle in a theory without confinement is set equal to its physical value.

We emphasize that the breaking of BRST invariance and the appearance of a mass term for the gluon field should not alter the well-known perturbative behavior of Yang-Mills theory in the UV regime. In general, fine-tuning conditions will have to be implemented in the UV limit to make sure that the usual perturbative results and the BRST symmetry are recovered in this limit. These conditions express the fact that we are describing a gauge theory within a wider class of effective models.

The first renormalization scheme we shall consider is defined through the conditions (in momentum space)

$$\Gamma^T_{AA}(p^2 = \mu^2) = \mu^2 + m^2 \,, \tag{2}$$

$$\Gamma^L_{AA}(p^2 = \mu^2) = m^2 \,, \tag{3}$$

$$\Gamma_{c\bar{c}}(p^2 = \mu^2) = \mu^2 \,, \tag{4}$$

imposed on the renormalized proper two-point functions. These conditions determine the field renormalization constants Z_A and Z_c and the mass parameter m at the renormalization scale μ. The symbols Γ^T_{AA} and Γ^L_{AA} refer to the transverse and longitudinal part, respectively, of the proper gluonic two-point function. We have suppressed the delta functions for momentum conservation and the trivial color factor δ^{ab} resulting from global color symmetry in all two-point functions. Note that there is no trace of the gauge fixing function left in the longitudinal part because of our use of the auxiliary field B^a in the action (1). Contrary to the proper two-point function, the gluon propagator is exactly transverse (as it should be in the Landau gauge).

The normalization conditions (2)–(4) were first proposed by Tissier and Wschebor in Ref. [14], where they were written in a different, but equivalent form. For the definition of the renormalized coupling constant, Tissier and Wschebor use the renormalized ghost-gluon vertex in the Taylor limit where the ghost momentum vanishes. In this limit, the quantum corrections to the vertex vanish exactly [15] which makes the calculation of the beta function in this renormalization scheme particularly simple.

For a first assessment of the renormalization scheme dependence we propose to consider the ghost-gluon vertex at the symmetry point for the definition of the renormalized coupling constant, instead of using the Taylor limit. The calculation of the beta function in this scheme involves an explicit evaluation of the one-loop corrections to the ghost-gluon vertex (at the symmetry point). In Fig. 1 we compare the running of the strong fine structure constant $\alpha(\mu) = g^2(\mu)/4\pi$ and the square of the mass parameter $m^2(\mu)$ as functions of the renormalization scale, obtained from the integration of the corresponding beta functions, in the two renormalization schemes. For the integration of the differential equations, we have been careful to use initial conditions (at $\mu = 3$ GeV) in the two schemes that correspond to the same bare theory, i.e., that are physically equivalent. Although the qualitative behavior of the curves is the same in both schemes, a considerable quantitative dependence on the renormalization scheme is seen.

We should like to clarify that at any fixed order of perturbation theory, the results for all correlation functions have to be exactly the same in any renormalization scheme (except for possible

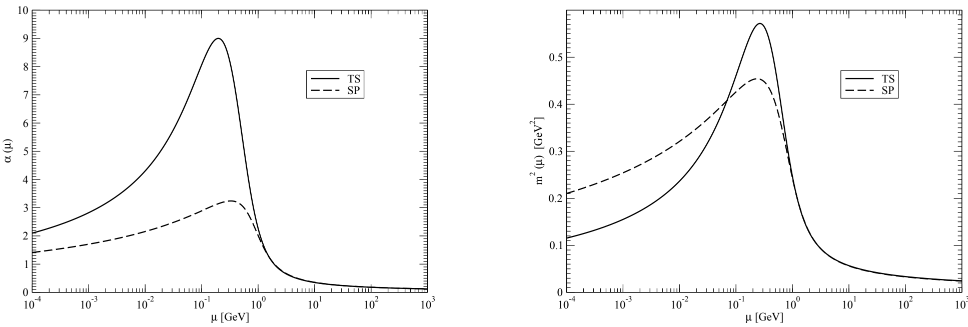

Figure 1. Running fine structure constant (to the left) and running mass parameter (to the right) in the Taylor scheme (TS) vs. the symmetry point scheme (SP).

constant rescalings of the fields, and assuming that physically equivalent values of the parameters g and m are chosen in all schemes). However, the results of applying Callan-Symanzik equations to different renormalization schemes can be different since they sum up parts of higher-loop diagrams by "extrapolating" the information obtained from the correlation functions to a given loop order in different ways.

Notice from Fig. 1 that the running coupling constant (or, rather, the fine structure constant) tends logarithmically to zero in the large-momentum limit as it should (asymptotic freedom). The same is true for the mass parameter. However, in the beta functions it is really the dimensionless combination m^2/μ^2 that matters, so the UV limit of a massless and BRST-invariant theory is actually reached much faster than logarithmically.

Interestingly, both the coupling constant and the mass parameter also tend logarithmically to zero in the IR limit of zero momentum, in four space-time dimensions. In the case of the coupling constant, this means that the IR fixed point is trivial. The triviality of the fixed point explains the success of straightforward perturbation theory in the IR regime (in four dimensions) [8]. On the other hand, the fact that the mass parameter tends to zero does not mean that one recovers a massless theory in the deep IR: the scale dependence of the gluon field renormalization constant leads to a finite limit of the gluon propagator at zero momentum. The trivial IR fixed point is thus essentially different from the trivial UV fixed point.

Let us now compare the results for the gluon and ghost propagators obtained from the integration of the Callan-Symanzik equations in the two different renormalization schemes with the data of lattice simulations. To this end, we have to determine those values of the parameters g and m at some (arbitrary) scale that lead to the best possible fits to the lattice data. The actual values of these parameters in the two renormalization schemes will generally not be physicall equivalent. Our strategy is to fix the two parameters in such a way that we obtain the best possible fit to the ghost propagator and its dressing function, a procedure that has turned out to determine the values of these parameters to a fair precision. Using the same values of the parameters in the calculation of the gluon propagator and comparing the result for the gluon propagator and its dressing function to the corresponding lattice data then allows to assess the degree to which the renormalization scheme is able to reproduce the lattice results (the dressing functions are the ratios of the full propagators to the tree-level propagators, the latter taken to be massless in the case of the gluon propagator).

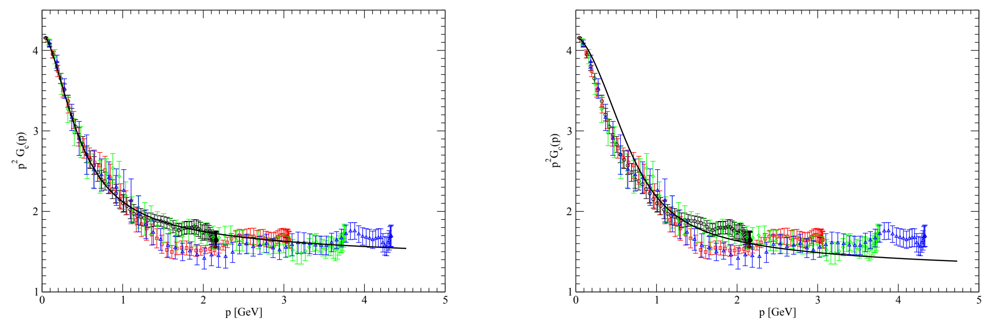

Figure 2. The ghost dressing function $F_c(p^2) = p^2 G_c(p^2)$, where $G_c(p^2) = 1/\Gamma_{c\bar{c}}(p^2)$ is the ghost propagator, with the definition of the renormalized coupling constant in the Taylor scheme (or Tissier-Wschebor's original scheme, to the left) and in the symmetry point scheme (to the right), compared to the lattice results of Ref. [16]. The different colors of the lattice data visible in the online version correspond to different directions on the lattice.

There are two circumstances that complicate the fits, or the determination of the free parameters. One is the fact that the lattice propagators are not normalized in any specific way, hence, for the comparison to the propagators that are obtained from the integration of the Callan-Symanzik equations, they are still to be multiplied with arbitrary overall constants (in this sense, there are two more parameters to be fitted). The other problem is the comparatively lower precision of the lattice data in the UV regime, in particular, the breaking of (continuous) rotational invariance on the lattice which becomes visible in this regime, cf. the plots of the ghost and gluon dressing functions below. In addition, one has to keep in mind that our results are renormalization group improvements of perturbative one-loop expressions, while it is well-known that, for a reliable fit to the UV data, renormalization group improved two-loop results are needed.

In Figs. 2–4 we show the best fits to the lattice data we have obtained in the original Tissier-Wschebor scheme with the definition of the renormalized coupling constant in the Taylor limit of the ghost-gluon vertex, and in the scheme that uses the symmetry point of the vertex for the definition of the coupling constant. The results obtained from the integration of the Callan-Symanzik equations for the ghost dressing function in the Tissier-Wschebor (or Taylor) scheme in Fig. 2 (to the left) are slightly different from the ones obtained by Peláez, Tissier and Wschebor in Ref. [18]. The difference lies in their fitting of the renormalization group improved result for the ghost dressing function to the lattice data for momenta in the directions $(0, 0, 1, 1)$ and $(1, 1, 1, 1)$ (red and blue data points in the online version), in the momentum regime where the breaking of rotational invariance on the lattice is visible, while we have found better overall fits by adjusting our curves to the data for momentum directions $(0, 0, 0, 1)$ and $(0, 1, 1, 1)$. Considering the data in the latter directions, our fit in Fig. 2 for the Tissier-Wschebor scheme is perfect for momenta up to 3.7 GeV, in the sense that the renormalization group improved curve stays within the statistical error bars for all data points.

In contrast, in the renormalization scheme that uses the ghost-gluon vertex at the symmetry point for the definition of the renormalized coupling constant, no such perfect fit to the ghost dressing function (or the ghost propagator, for that matter) can be achieved. Furthermore, the fit to the gluon propagator and the gluon dressing function in Fig. 4 is clearly poorer than the corresponding fit in Tissier-Wschebor's original (or Taylor) scheme. It is, hence, the latter choice that leads to better results for the gluon and ghost propagators. Notice how the plots of the gluon propagator give a

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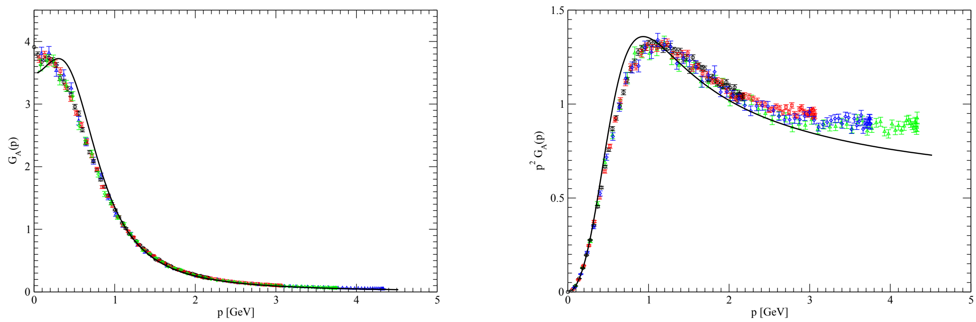

Figure 3. The gluon propagator $G_A(p^2) = 1/\Gamma^T_{AA}(p^2)$ (to the left) and the gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$ (to the right) in the original Tissier-Wschebor or Taylor scheme, compared to the lattice results of Ref. [17].

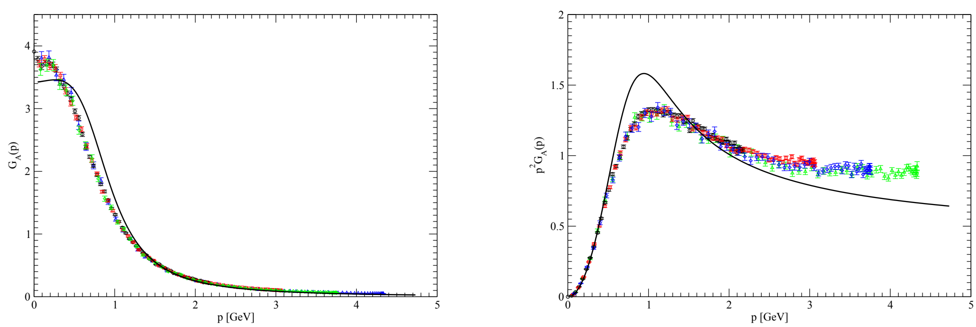

Figure 4. Gluon propagator (to the left) and gluon dressing function (to the right) in the symmetry point scheme, compared to the lattice results.

clearer picture of the quality of the fits in the extreme IR regime, while the plots of the gluon dressing function emphasize the mid- and large-momentum regimes.

We have also considered a different class of renormalization schemes that use the p^2-derivatives of the proper two-point functions in order to fix the field renormalization constants. Note that Z_A is determined in the Tissier-Wschebor scheme from the difference of Eqs. (2) and (3),

$$\left[\Gamma^T_{AA}(p^2) - \Gamma^L_{AA}(p^2)\right]\Big|_{p^2=\mu^2} = \mu^2 \,. \tag{5}$$

In the derivative schemes, we replace the latter condition with

$$\frac{\partial}{\partial p^2}\left[\Gamma^T_{AA}(p^2) - \zeta\,\Gamma^L_{AA}(p^2)\right]\Big|_{p^2=\mu^2} = 1\,, \tag{6}$$

where we have, furthermore, introduced an *a priori* arbitrary parameter ζ, taking advantage of the fact that the "classical" longitudinal part of the proper two-point function, i.e., the tree-level contribution

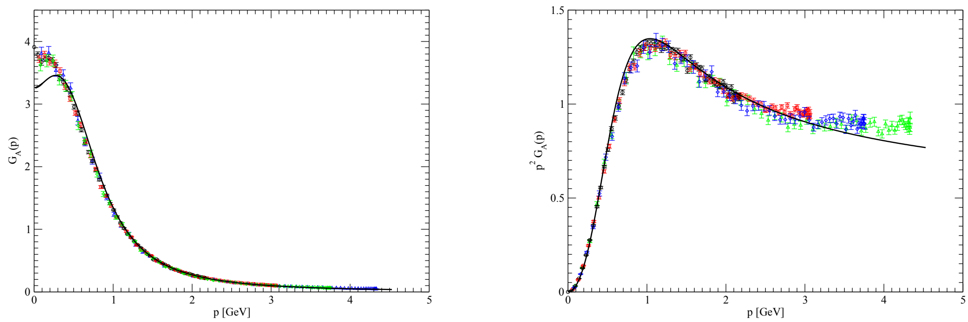

Figure 5. The gluon propagator $G_A(p^2) = 1/\Gamma^T_{AA}(p^2)$ (to the left) and the gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$ (to the right) in the $\zeta = 1$ derivative scheme, compared to the lattice results of Ref. [17].

to the longitudinal part of the proper gluonic two-point function that is derived from Eq. (1), is p^2-independent. We keep the normalization condition (3) for the mass, but replace condition (4) with

$$\frac{\partial}{\partial p^2} \Gamma_{c\bar{c}}(p^2)\Big|_{p^2=\mu^2} = 1\,, \tag{7}$$

in analogy with Eq. (6). Given our previous experience with the two "natural" definitions of the renormalized coupling constant, we will use the Taylor limit of the ghost-gluon vertex for its definition in the derivative schemes [we have actually checked that its definition from the symmetry point of the vertex leads to poorer fits to the gluon and ghost propagators, just as in the case of the normalization conditions (2)–(4)].

The IR limit of the beta function in the derivative schemes is (for N colors)

$$\beta_g = \mu \frac{d}{d\mu}\, g(\mu) = \frac{3\zeta - 1}{12} \frac{N g^2}{(4\pi)^2}\, g\,. \tag{8}$$

In order to avoid a Landau pole in the integration of the renormalization group equations, one needs the beta function to be positive in the IR, hence $\zeta > 1/3$ (in particular, for $\zeta = 0$ a Landau pole is inevitable). The "critical" value $\zeta = 1/3$ still leads to a finite (non-zero) value of $g(\mu)$ [and also of $m^2(\mu)$] in the limit $\mu \to 0$.

We have plotted our best fits to the gluon propagator and dressing function for the derivative schemes with $\zeta = 1$ and $\zeta = 1/3$ in Figs. 5 and 6, employing the same strategy for the fits as before. The corresponding fits to the ghost dressing function are practically identical to the one for the original Tissier-Wschebor scheme in Fig. 2 (to the left), perfect for momenta up to 3.7 GeV in the sense explained above. We conclude that the renormalization group improved results in the derivative schemes, in particular for the critical case $\zeta = 1/3$, fit the lattice data better than in the original Tissier-Wschebor scheme. Our fits should also be compared to more recent results of the solution of the Dyson-Schwinger equations or the in many aspects similar functional renormalization group equations, see Ref. [19].

Note that there is a decrease of the gluon propagator towards the extreme IR in the renormalization group improved curves, for all renormalization schemes considered. This decrease is not seen in the lattice data, however, a slight decrease is not excluded by the data, either. The decrease gets more

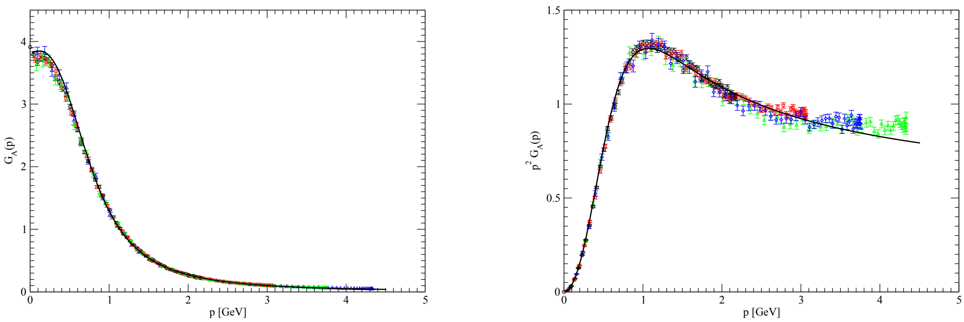

Figure 6. Gluon propagator (to the left) and gluon dressing function (to the right) in the critical $\zeta = 1/3$ derivative scheme, compared to the lattice results.

pronounced in the derivative schemes with increasing value of the parameter ζ, which is why the critical value $\zeta = 1/3$, the smallest possible, gives the best fit to the lattice data.

In conclusion, we have demonstrated that a renormalization group improvement of the perturbative one-loop results is sufficient to successfully reproduce the lattice data for the gluon and ghost propagators in Landau gauge Yang-Mills theory over the whole momentum range from the far UV to the deep IR, even better in a quantitative sense than through the application of Dyson-Schwinger equations (or functional renormalization group equations) [19], if one allows for a non-zero gluon mass parameter in the "classical" action. The usual massless Yang-Mills theory and its well-known behavior including the BRST symmetry is recovered in the UV regime, while the running mass parameter avoids the appearance of a Landau pole. In the Callan-Symanzik approach, only two parameters, the values of the coupling constant and the mass parameter at a given renormalization scale, have to be adjusted to achieve a good fit to the lattice results. The renormalization scheme dependence of the results can be used to improve the matching to the lattice data. Within the class of renormalization schemes we have considered, the best fits are obtained in the derivative scheme with the critical value $1/3$ of the ζ parameter [see Eq. (6)].

One of the important advantages of the Callan-Symanzik approach, as compared to the use of Dyson-Schwinger equations, or functional renormalization group equations for that matter, is the possibility of systematically improving the results by going to higher loop orders, without introducing any additional parameters. The calculation of the flow functions is analytical (the evaluation of some of the appearing Feynman diagrams may require numerical integration, but at least the IR and UV limits can usually be extracted analytically), and the renormalization group equations constitute a set of coupled ordinary first-order differential equations which can be handled numerically without difficulty.

The application of the Callan-Symanzik equations is not limited to the gluon and ghost propagators, but can be extended to the vertex functions. The advantages of the approach are even more evident in this case: there is no need to guess at the most important structures or to construct *ansätze* for the vertex functions, everything is in principle determined (to the order considered) by the corresponding Feynman diagrams once the renormalization scheme and the two free parameters are fixed, compare, e.g., Ref. [18] to Ref. [20]. As an example, the analytical evaluation in the IR limit of the one-loop corrections to the ghost-gluon and the three-gluon vertices at the respective symmetry points (with the tensor structures of the tree-level vertices, which give the dominant contributions in the UV as well as in the IR limit) shows that the ghost-gluon vertex function tends towards a non-zero finite constant, while the three-gluon vertex function becomes proportional to $\ln(p^2/\Lambda_1^2)$ for small momenta (at the symmetry point) $p_1^2 = p_2^2 = p_3^2 = p^2$, and Λ_1 is a new IR scale where the three-gluon vertex function changes sign. This sign change ("zero crossing") has been obtained both in the Callan-Symanzik and the Dyson-Schwinger approaches [18, 20], and it has been observed in lattice simulations in three space-time dimensions, while the data in four dimensions are still inconclusive [21]. The new scale Λ_1 is obtained as a function of the two free parameters when the Callan-Symanzik equations are integrated completely (the value of the coupling constant at a given renormalization scale may be expressed through Λ_{QCD}).

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References

[1] L. von Smekal, A. Hauck and R. Alkofer, Phys. Rev. Lett. **79**, 3591 (1997); C. Fischer and R. Alkofer, Phys. Lett. B **536**, 177 (2002)

[2] A.C. Aguilar and A.A. Natale, JHEP**0408**, 057 (2004); P. Boucaud, T. Brüntjen, J.P. Leroy *et al.*, JHEP**0606**, 001 (2006); M. Frasca, Phys. Lett. B **670**, 73 (2008)

[3] V.N. Gribov, Nucl. Phys. B **139**, 1 (1978)

[4] D. Zwanziger, Nucl. Phys. B **323**, 513 (1989); Nucl. Phys. B **399**, 477 (1993)

[5] D. Dudal, J.A. Gracey, S.P. Sorella *et al.*, Phys. Rev. D **78**, 065047 (2008)

[6] I.L. Bogolubsky, E.M. Ilgenfritz, M. Müller-Preussker and A. Sternbeck, Proc. Sci. LAT2007, 290 (2007); A. Cucchieri and T. Mendes, Proc. Sci. LAT2007, 297 (2007); A. Sternbeck, L. von Smekal, D.B. Leinweber and A.G. Williams, Proc. Sci. LAT2007, 340 (2007)

[7] A. Weber, Phys. Rev. D **85**, 125006 (2012); A. Weber, P. Dall'Olio and F. Astorga, Int. J. Mod. Phys. E **25**, 1642002 (2016)

[8] M. Tissier and N. Wschebor, Phys. Rev. D **82**, 101701 (2010)

[9] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979)

[10] G. Curci and R. Ferrari, Nuovo Cim. A **32**, 151 (1976)

[11] G. Curci and R. Ferrari, Nuovo Cim. A **35**, 1 (1976); I. Ojima, Z. Phys. C **13**, 173 (1982); J. de Boer, K. Skenderis, P. van Nieuwenhuizen and A. Waldron, Phys. Lett. B **367**, 175 (1996)

[12] K.-I. Kondo, K. Suzuki, H. Fukamachi *et al.*, Phys. Rev. D **87**, 025017 (2013); M. Schaden and D. Zwanziger, Phys. Rev. D **92**, 025001 (2015)

[13] D. Dudal and S.P. Sorella, Phys. Rev. D **86**, 045005 (2012); M.A.L. Capri, D. Dudal, D. Fiorentini *et al.*, Phys. Rev. D **94**, 025035 (2016)

[14] M. Tissier and N. Wschebor, Phys. Rev. D **84**, 045018 (2011)

[15] J.C. Taylor, Nucl. Phys. B **33**, 436 (1971)

[16] A. Cucchieri and T. Mendes, Phys. Rev. D **78**, 094503 (2008)

[17] A. Cucchieri and T. Mendes, Phys. Rev. Lett. **100**, 241601 (2008)

[18] M. Peláez, M. Tissier and N. Wschebor, Phys. Rev. D **88**, 125003 (2013)

[19] C.S. Fischer, A. Maas and J.M. Pawlowski, Ann. Phys. **324**, 2408 (2009); M.Q. Huber and L. von Smekal, JHEP**1304**, 149 (2013)
[20] A.L. Blum, M.Q. Huber, M. Mitter and L. von Smekal, Phys. Rev. D **89**, 061703 (2014)
[21] A. Cucchieri, A. Maas and T. Mendes, Phys. Rev. D **77**, 094510 (2008)