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Mass generation in Yang-Mills theories*

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Abstract. In this talk we review recent progress on our understanding of the nonperturbative phenomenon of mass generation in non-Abelian gauge theories, and the way it manifests itself at the level of the gluon propagator, thus establishing a close contact with a variety of results obtained in large-volume lattice simulations. The key observation is that, due to an exact cancellation operating at the level of the Schwinger-Dyson equations, the gluon propagator remains rigorously massless, provided that the fully-dressed vertices of the theory do not contain massless poles. The inclusion of such poles activates the well-known Schwinger mechanism, which permits the evasion of the aforementioned cancellation, and accounts for the observed infrared finiteness of the gluon propagator both in the Landau gauge and away from it.

1 Introduction

As has been well-established during the past decade by means of high-quality lattice simulations [1–4] and a variety of nonperturbative approaches in the continuum [5–7], the gluon propagator, $\Delta(q^2)$, reaches a finite value in the deep infrared, both in the Landau gauge as well as considerably away from it [8–10]. These results have induced a profound change in our understanding of the infrared sector of Yang-Mills theories, affecting our description of fundamental QCD phenomena such as confinement, chiral symmetry breaking, and bound-state formation [11].

A particular set of physical concepts and formal techniques for casting such a picture in a well-defined theoretical structure has been developed in a series of articles [5, 12, 13] based on the Schwinger-Dyson equations (SDEs) [14] derived within the powerful framework obtained from the fusion of the pinch technique (PT) [15–20] and the background field method (BFM) [21, 22].

In this presentation we outline the key elements of a particular mechanism that accounts for the observed finiteness of the gluon propagator, while preserving intact the fundamental symmetries of the theory [23].

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2 The PT-BFM framework

The PT-BFM formalism provides a manifestly BRST preserving truncation scheme for the gluon propagator SDE [12, 13]. Within this framework one writes the gauge field, A^a_μ, as the sum of a background, B^a_μ, and a quantum, Q^a_μ, part. This introduces mixed Green's functions, describing combinations of B and Q fields. For instance, there are three types of gluon propagators: (*i*) the conventional propagator, with two Q-type gluons (Q^2), denoted by $\Delta^{ab}_{\mu\nu}(q)$, (*ii*) the background-quantum propagator, with one Q- and one B-type gluon (QB or BQ), denoted by $\widetilde{\Delta}^{ab}_{\mu\nu}(q)$, and (*iii*) the background propagator, with two B-type gluons, denoted by $\widehat{\Delta}^{ab}_{\mu\nu}(q)$.

The Q^2 gluon propagator, $\Delta^{ab}_{\mu\nu}(q) = \delta^{ab}\Delta_{\mu\nu}(q)$, is given by

$$\Delta_{\mu\nu}(q) = -i\left[\Delta(q^2)P_{\mu\nu}(q) + \xi\frac{q_\mu q_\nu}{q^4}\right], \qquad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\,. \tag{1}$$

where the function $\Delta(q^2)$ is related to the gluon self-energy $\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q^2)$ through

$$\Delta^{-1}(q^2) = q^2 + i\Pi(q^2). \tag{2}$$

Similar expressions hold for the QB and BB propagators. For example, in the former case one has $\widetilde{\Delta}^{ab}_{\mu\nu}(q)$, with $\Delta(q^2) \to \widetilde{\Delta}(q^2)$ and $\Pi(q^2) \to \widetilde{\Pi}(q^2)$. Additionally, $\Delta(q^2)$ relates to $\widetilde{\Delta}(q^2)$ and $\widehat{\Delta}(q^2)$ by the so called "background-quantum identities" [24–26] which establishes that

$$\Delta(q^2) = [1 + G(q^2)]\widetilde{\Delta}(q^2)\,, \qquad \widetilde{\Delta}(q^2) = [1 + G(q^2)]\widehat{\Delta}(q^2), \tag{3}$$

where $G(q^2)$ is an auxiliary function typical of the PT-BFM scheme [13]. In the ensuing analysis we will concentrate on the QB self-energy, $\widetilde{\Pi}(q^2)$, given by the sum of diagrams given in Figs. 1, 2 and 3.

3 Ward Identities

3.1 From Takahashi to Ward identities

The vertices appearing in the definition of the self-energy, $\widetilde{\Pi}(q^2)$, will be denoted by $\widetilde{\Gamma}_{\mu\alpha\beta}$ (BQ^2), $\widetilde{\Gamma}_\alpha$ ($B\bar{c}c$) and $\widetilde{\Gamma}^{abc}_{\mu\alpha\beta\gamma}$ (BQ^3). When contracted with the momentum carried by the B gluon entering the diagrams of the Figs. 1, 2 and 3, such vertices satisfy the following Takahashi identities

$$\begin{aligned}
q^\mu\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) &= i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p),\\
q^\mu\widetilde{\Gamma}_{\mu}(q,r,-p) &= D^{-1}(q+r) - D^{-1}(r),\\
q^\mu\widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma}(q,r,p,t) &= f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma}(r,p,q+t) + f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha}(p,t,q+r)\\
&\quad + f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta}(t,r,q+p)\,,
\end{aligned} \tag{4}$$

where $D(q^2)$ represents the fully dressed ghost propagator, and the vertices $\Gamma_{\alpha\beta\gamma}$ on the r.h.s. of the last equation are the conventional three-gluon vertices (Q^3).

Here we are interested in the behaviour of the gluon propagator at the origin, $\Delta(0)$ [or equivalently $\widetilde{\Delta}(0)$ - see Eq. (3)]. Therefore, the relevant Ward identities (WI) are those obtained by taking the limit of the corresponding identity of Eq. (4) when the background gluon momentum q is taken to vanish.

After carrying out a Taylor expansion around $q = 0$ on both sides of the first two identities of Eq. (4) while assuming the absence of $1/q^2$ poles in the form factors of the vertices, we obtain the following WIs for BQ^2 e $B\bar{c}c$,

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,-p,p) = i\frac{\partial\Delta^{-1}_{\alpha\beta}(p)}{\partial p^\mu}; \qquad \widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu}; \tag{5}$$

$$\widetilde{\Gamma}_{\mu}(0,-p,p) = i\frac{\partial D^{-1}(p^2)}{\partial p^\mu}; \qquad \widetilde{\Gamma}_{\mu}(0,r,-r) = -i\frac{\partial D^{-1}(r^2)}{\partial r^\mu}. \tag{6}$$

The corresponding WI for the BQ^3 vertex is somewhat more complicated and reads

$$\begin{aligned}\widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma}(0,r,p,-r-p) &= \left(f^{mne}f^{esr}\frac{\partial}{\partial r^\mu} + f^{mre}f^{ens}\frac{\partial}{\partial p^\mu}\right)\Gamma_{\alpha\beta\gamma}(r,p,-r-p),\\ \widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma}(0,-r,-p,r+p) &= -\left(f^{mne}f^{esr}\frac{\partial}{\partial r^\mu} + f^{mre}f^{ens}\frac{\partial}{\partial p^\mu}\right)\Gamma_{\alpha\beta\gamma}(-r,-p,r+p).\end{aligned} \tag{7}$$

3.2 Ward identities and vertex form factors

The WIs just derived are bound to affect the form factors appearing in the tensorial decomposition of the corresponding vertices. Consider for example the simplest case of the gluon-ghost vertex, $\widetilde{\Gamma}^\mu$ ($B\bar{c}c$), whose the most general Lorentz decomposition is given by

$$\widetilde{\Gamma}^\mu(q,r,p) = \widetilde{\mathcal{A}}_1(q^2,r^2,p^2)q^\mu + \widetilde{\mathcal{A}}_2(q^2,r^2,p^2)r^\mu, \tag{8}$$

with q denoting the momentum carried by the background gluon. In agreement with the assumptions made when deriving the WIs of Eqs. (5), (6) and (7), we consider that the form factors $\widetilde{\mathcal{A}}_1$ and $\widetilde{\mathcal{A}}_2$ do not contain (kinematic or dynamical) poles in q^2; therefore, when $q = 0$, Eqs. (6) and (8) imply that

$$\widetilde{\mathcal{A}}_2(r^2) = -2i\frac{\partial D^{-1}(r^2)}{\partial r^2}. \tag{9}$$

Next, we consider the three-gluon vertex $\widetilde{\Gamma}^{\mu\alpha\beta}$, whose tensorial decomposition may be written as

$$\widetilde{\Gamma}^{\mu\alpha\beta}(q,r,p) = \sum_{i=1}^{14}\widetilde{A}_i(q^2,r^2,q\cdot r)\,b_i^{\mu\alpha\beta}, \tag{10}$$

where the basis $b^i_{\mu\alpha\beta}$ is chosen to be

$$\begin{aligned}&b_1^{\mu\alpha\beta} = q^\mu g^{\alpha\beta}; && b_2^{\mu\alpha\beta} = q^\mu q^\alpha q^\beta; && b_3^{\mu\alpha\beta} = q^\mu q^\alpha r^\beta; && b_4^{\mu\alpha\beta} = q^\mu r^\alpha q^\beta; && b_5^{\mu\alpha\beta} = q^\mu r^\alpha r^\beta,\\ &b_6^{\mu\alpha\beta} = r^\mu g^{\alpha\beta}; && b_7^{\mu\alpha\beta} = r^\mu q^\alpha q^\beta; && b_8^{\mu\alpha\beta} = r^\mu q^\alpha r^\beta; && b_9^{\mu\alpha\beta} = r^\mu r^\alpha q^\beta; && b_{10}^{\mu\alpha\beta} = r^\mu r^\alpha r^\beta,\\ &b_{11}^{\mu\alpha\beta} = q^\alpha g^{\beta\mu}; && b_{12}^{\mu\alpha\beta} = q^\beta g^{\alpha\mu}; && b_{13}^{\mu\alpha\beta} = r^\alpha g^{\beta\mu}; && b_{14}^{\mu\alpha\beta} = r^\beta g^{\alpha\mu}.\end{aligned} \tag{11}$$

As we assume that the form factors $\widetilde{A}_i$ do not contain poles in q^2, only q-independent tensors, *i.e.*, b_6, b_{10}, b_{13} and b_{14}, will survive in the limit $q \to 0$, therefore

$$\widetilde{\Gamma}^{\mu\alpha\beta}(0,r,-r) = \widetilde{A}_6(r^2)r^\mu g^{\alpha\beta} + \widetilde{A}_{10}(r^2)r^\mu r^\alpha r^\beta + \widetilde{A}_{13}r^\alpha g^{\beta\mu} + \widetilde{A}_{14}r^\beta g^{\alpha\mu}. \tag{12}$$

Then, using the identity (5) together with the inverse of the gluon propagator

$$-i\Delta^{-1}_{\mu\nu}(q) = \Delta^{-1}(q^2)P_{\mu\nu}(q) + \xi^{-1}q_\mu q_\nu, \tag{13}$$

and matching the corresponding tensorial structures, lead us to the relations

$$\widetilde{A}_6(r^2) = 2\frac{\partial\Delta^{-1}(r^2)}{\partial r^2} \qquad \widetilde{A}_{10}(r^2) = -2\frac{\partial}{\partial r^2}\left(\frac{\Delta^{-1}(r^2)}{r^2}\right) \qquad \widetilde{A}_{13}(r^2) = \widetilde{A}_{14}(r^2) = \frac{1}{\xi} - \frac{\Delta^{-1}(r^2)}{r^2}. \tag{14}$$

The analogous procedure involving the four-gluon vertex, $\widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma}(q,r,p,t)$, would be particularly cumbersome in virtue of the large number of tensorial structures involved in its Lorentz decomposition. Luckily however, this decomposition is not needed in the computation of $\Delta(0)$, which is all we are interested in.

4 Seagull Identity

A second crucial ingredient in this analysis is the so-called seagull identity [27]. To derive it, let us consider the class of vector functions

$$\mathcal{F}_\mu(k) = f(k^2)\,k_\mu, \tag{15}$$

with $f(k^2)$ an arbitrary scalar function. Since $\mathcal{F}_\mu(-k) = -\mathcal{F}_\mu(k)$, we have that

$$\int_k \mathcal{F}_\mu(k) = 0; \quad \text{where} \quad \int_k \equiv \frac{\mu^\epsilon}{(2\pi)^d}\int \mathrm{d}^d k, \tag{16}$$

with $d = 4 - \epsilon$ and μ the 't Hooft mass.

If we now assume that the function $f(k^2)$ vanishes rapidly enough when $k^2 \to \infty$, so that the integral (in spherical coordinates, with $y = k^2$)

$$\int_k f(k^2) = \frac{1}{(4\pi)^{\frac{d}{2}}\Gamma(\frac{d}{2})}\int_0^\infty dy y^{\frac{d}{2}-1} f(y), \tag{17}$$

converges for all positive values d below a certain value d^*. Due to the translational invariance of dimensional regularization, we may shift the argument of the function defined in the Eq. (15) by an arbitrary momentum q without affecting the result of Eq. (16). Then, carrying out a Taylor expansion around $q = 0$, we obtain

$$\begin{aligned}\mathcal{F}_\mu(q+k) &= \mathcal{F}_\mu(k) + q^\nu\left\{\frac{\partial}{\partial q^\nu}\mathcal{F}_\mu(q+k)\right\}_{q=0} + O(q^2)\\ &= \mathcal{F}_\mu(k) + q^\nu\frac{\partial\mathcal{F}_\mu(k)}{\partial k^\nu} + O(q^2),\end{aligned} \tag{18}$$

and, after integrating both sides of the expansion and using Eq. (16), we find

$$q^\nu\int_k \frac{\partial\mathcal{F}_\mu(k)}{\partial k^\nu} = 0. \tag{19}$$

Since the integral above has two free Lorentz indices and no momentum scale, it must be proportional to the metric tensor $g_{\mu\nu}$. Moreover, given that the momentum q is arbitrary, we find that Eq. (19) implies the seagull identity [23],

$$\int_k \frac{\partial\mathcal{F}_\mu(k)}{\partial k^\mu} = 0. \tag{20}$$

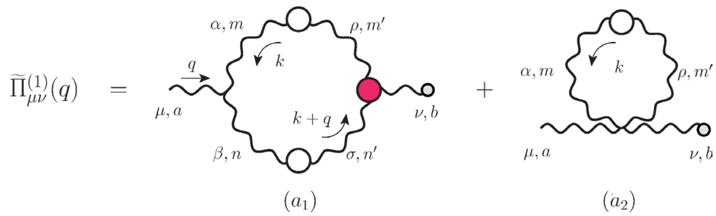

Figure 1. The gluonic one-loop dressed diagrams contributing to the SDE of the QB gluon self-energy, $\widetilde{\Pi}_{\mu\nu}(q^2)$. The white circles indicate fully dressed gluon propagators and the red circle indicates a fully dressed three gluon vertex, $\widetilde{\Gamma}_{\nu\sigma\rho}$ (BQ^2).

5 The zero momentum gluon propagator

The WIs derived in Sect. 3.1 may be used to cast the zero momentum gluon self-energy $\widetilde{\Pi}(0)$ in a form suitable for the application of the seagull identity given by Eq. (20). To do so, consider first the contribution given by the gluonic one-loop dressed diagrams (a_1) and (a_2) of Fig. 1. In particular, at $q = 0$ we have

$$d\,\widetilde{\Pi}^{(1)}(0) = a_1(0) + a_2(0), \tag{21}$$

with

$$a_1(0) = \frac{1}{2}g^2 C_A \int_k \Gamma^{(0)}_{\mu\alpha\beta}(0,k,-k)\Delta^{\alpha\rho}(k)\Delta^{\beta\sigma}(k)\widetilde{\Gamma}^{\mu}_{\sigma\rho}(0,k,-k)\,, \tag{22}$$

$$a_2(0) = -ig^2 C_A(d-1)\int_k \Delta^{\alpha}_{\alpha}(k), \tag{23}$$

where $C_A = N$ represents the Casimir eigenvalue of the adjoint representation for SU(N), while $\Gamma^{(0)}_{\mu\alpha\beta}$ is the (conventional) tree-level three-gluon vertex, which is given by

$$\Gamma^{(0)}_{\mu\alpha\beta}(0,k,-k) = 2k_\mu g_{\alpha\beta} - k_\beta g_{\alpha\mu} - k_\alpha g_{\beta\mu}\,. \tag{24}$$

Then, using Eq. (5), we derive the relation

$$\Delta^{\alpha\rho}(k)\Delta^{\beta\sigma}(k)\widetilde{\Gamma}^{\mu}_{\sigma\rho}(0,k,-k) = -i\frac{\partial}{\partial k^\mu}\Delta^{\alpha\beta}(k), \tag{25}$$

so that we can rewrite Eq. (22) in the form

$$a_1(0) = -\frac{i}{2}g^2 C_A\left\{\int_k \frac{\partial}{\partial k^\mu}[\Gamma^{(0)}_{\mu\alpha\beta}(0,k,-k)\Delta^{\alpha\beta}(k)] - \int_k \Delta^{\alpha\beta}(k)\frac{\partial}{\partial k^\mu}\Gamma^{(0)}_{\mu\alpha\beta}(0,k,-k)\right\}. \tag{26}$$

Finally, observing

$$\frac{\partial}{\partial k^\mu}\Gamma^{(0)}_{\mu\alpha\beta}(0,k,-k) = 2(d-1)g_{\alpha\beta}, \tag{27}$$

we note that the second term of Eq. (26) cancels against the expression for $a_2(0)$, leaving us with the final result

$$d\widetilde{\Pi}^{(1)}(0) = -g^2 C_A(d-1)\int_k \frac{\partial \mathcal{F}^{(1)}_\mu(k)}{\partial k^\mu}; \qquad \mathcal{F}^{(1)}_\mu(k) = \Delta(k^2)k_\mu. \tag{28}$$

Consequently, by virtue of the seagull identity, $\widetilde{\Pi}^{(1)}(0) = 0$.

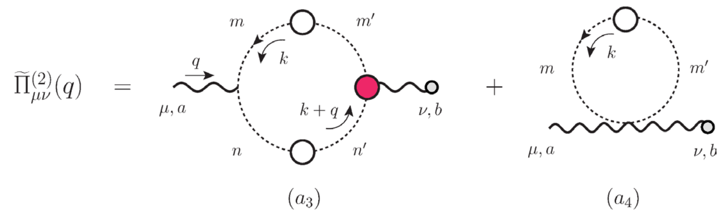

Figure 2. One-loop dressed ghost diagrams contributing to the SDE of the QB gluon self-energy, $\widetilde{\Pi}_{\mu\nu}(q^2)$. White circles indicate fully dressed ghost propagators and the red circle indicates a fully dressed gluon-ghost vertex, $\widetilde{\Gamma}_\mu$ $(B\bar{c}c)$.

Next, consider the ghost loop diagrams of Fig. 2. At $q = 0$, we have

$$a_3(0) = -g^2 C_A \int_k k_\mu D^2(k^2)\widetilde{\Gamma}^\mu(0, k, -k), \tag{29}$$

$$a_4(0) = dg^2 C_A \int_k D(k^2), \tag{30}$$

and from Eq. (6), it is possible to obtain

$$a_3(0) = g^2 C_A \Big\{ \int_k \frac{\partial}{\partial k^\mu}[\Gamma^{(0)}_\mu D(k^2)] - d \int_k D(k^2) \Big\}. \tag{31}$$

Thus, the second term in Eq. (31) cancels against $a_4(0)$ and we are left with

$$d\widetilde{\Pi}^{(2)}(0) = g^2 C_A \int_k \frac{\partial \mathcal{F}^{(2)}_\mu(k)}{\partial k^\mu}; \qquad \mathcal{F}^{(2)}_\mu(k) = D(k^2)k_\mu\,, \tag{32}$$

so that the seagull identity implies that also $\widetilde{\Pi}^{(2)}(0) = 0$

Finally, the gluon two-loop dressed of Fig. 3 at $q = 0$ yield

$$a_5^{ab}(0) = \frac{i}{6} g^4 \Gamma^{(0)amnr}_{\mu\alpha\beta\gamma} \int_k \int_\ell \Delta^{\gamma\tau}(k)\Delta^{\beta\sigma}(\ell)\Delta^{\alpha\rho}(k+\ell)\widetilde{\Gamma}^{brnm}_{\mu\tau\sigma\rho}(0, k, \ell, -k-\ell),$$

$$a_6(0) = \mathcal{N}_{\mu\alpha\beta\gamma} \int_k \Delta^{\gamma\tau}(k)\Delta^{\lambda\delta}(k) Y^{\alpha\beta}_\delta(k)\widetilde{\Gamma}^\mu_{\tau\lambda}(0, k, -k)\,, \tag{33}$$

where we have defined

$$\mathcal{N}_{\mu\alpha\beta\gamma} = \frac{3}{4} i g^4 C_A^2 (g_{\mu\alpha} g_{\gamma\beta} - g_{\mu\beta} g_{\gamma\alpha})\,, \tag{34}$$

and

$$Y^{\alpha\beta}_\delta(k) = \int_\ell \Delta^{\alpha\rho}(\ell)\Delta^{\beta\sigma}(k+\ell)\Gamma_{\sigma\rho\delta}(-k-\ell, \ell, k)\,, \tag{35}$$

which is proportional to the subdiagram nested inside (a_6), and was studied in Ref. [28]. Employing the WI of Eq. (7), one obtains [23],

$$d\widetilde{\Pi}^{(3)}(0) = -i(d-1) g^4 C_A^2 \int_k \frac{\partial \mathcal{F}^{(3)}_\mu(k)}{\partial k^\mu}; \qquad \mathcal{F}^{(3)}_\mu(k) = \Delta(k^2) Y(k^2) k_\mu. \tag{36}$$

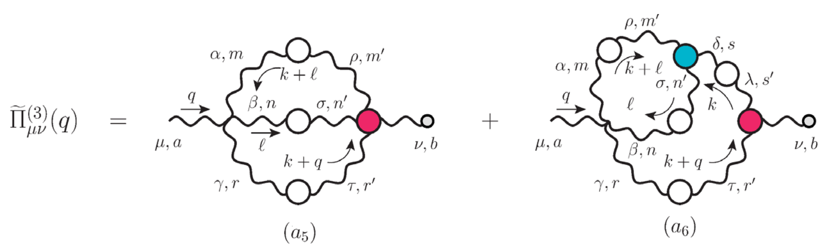

Figure 3. The gluonic two-loop dressed diagrams contributing to the SDE of the QB gluon self-energy,$\widetilde{\Pi}_{\mu\nu}(q^2)$. White circles indicate the fully gluon dressed propagators and the red circle in (a_5) indicates the fully dressed four gluon vertex, $\widetilde{\Gamma}^{abc}_{\mu\alpha\beta\gamma}$ (BQ^3) while in (a_6) is the three gluon vertex, $\widetilde{\Gamma}_{\mu\alpha\beta}$ (BQ^2).

The results of Eqs. (28), (32) and (36) establish that, in the absence of massless poles in the form factors of the fundamental vertices,

$$\widetilde{\Pi}^{(i)}(0) = 0, \quad i = 1, 2, 3; \qquad \Rightarrow \qquad \widetilde{\Pi}(0) = \sum_{i=1}^{3} \widetilde{\Pi}^{(i)}(0) = 0, \tag{37}$$

and since $\widetilde{\Delta}^{-1}(q^2) = q^2 + i\widetilde{\Pi}(q^2)$, we are lead to the conclusion that $\widetilde{\Delta}^{-1}(0) = 0$. The conventional gluon propagator is then obtained by using Eq. (3),

$$\Delta^{-1}(0) = \frac{\widetilde{\Delta}^{-1}(0)}{1 + G(0)}. \tag{38}$$

Now, if we introduce the additional assumption that $1 + G(0)$ is finite for every ξ, we conclude that $\Delta^{-1}(0) = 0$, *i.e.*, *the gluon is massless in the absence of poles*.

We emphasize that the proof elaborated in this section is valid for any value of the gauge-fixing parameter ξ within the class of linear covariant gauges (R_ξ). Additionally, in this class of gauges, the $G(q^2)$ function is related to the ghost dressing function $F(q^2)$ as [26]

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2) + \xi K(q^2), \tag{39}$$

where K represents an auxiliary function related to the antighost coupling with a certain anti-BRST source necessary for formulating the theory in the BFM formalism. In the Landau gauge ($\xi = 0$), we know that $L(0) = 0$, while the ghost dressing function is non-zero at $q^2 = 0$ [29]; thus, we obtain $F^{-1}(0) = 1 + G(0)$ which ensures the finiteness of $G(0)$. However, for $\xi \neq 0$ the situation is more complex. Recent analytical studies [9, 10] indicate that the gluon propagator still saturates in the IR in this case, a result that has been corroborated by lattice simulations [30]. This points towards the finiteness of $1 + G(0)$ even in the case $\xi \neq 0$.

6 Vertices with massless poles

Evading the result established in the previous section requires relaxing the underlying assumption of the absence of massless poles. Thus, in the following we are going to assume that some of the vertices form factors contain such poles. Thus they will be divided in two parts, which will be indicated by a the corresponding superscript: "**p**" (for "pole" parts) or "**np**" (for "no-pole" parts):

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p); \qquad \widetilde{\Gamma}_{\mu}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu}(q,r,p),$$
$$\widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma}(q,r,p,t) = \widetilde{\Gamma}^{\mathbf{np},mnrs}_{\mu\alpha\beta\gamma}(q,r,p,t) + \widetilde{\Gamma}^{\mathbf{p},mnrs}_{\mu\alpha\beta\gamma}(q,r,p,t). \tag{40}$$

Since the pole parts should act like a "dynamical Nambu-Goldstone bosons" and decouple from physical observables, they must be longitudinally-coupled; therefore we can write

$$\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p) = \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p); \qquad \widetilde{\Gamma}^{\mathbf{p}}_{\mu}(q,r,p) = \frac{q_\mu}{q^2}\widetilde{C}(q,r,p),$$
$$\widetilde{\Gamma}^{\mathbf{p},mnrs}_{\mu\alpha\beta\gamma}(q,r,p,t) = \frac{q_\mu}{q^2}\widetilde{C}^{mnrs}_{\alpha\beta\gamma}(q,r,p,t). \tag{41}$$

In order to keep the BRST invariance intact, one requires that the Takahashi identities maintain their exact form in the presence of these poles. Consequently, Eq. (4) is now written as

$$\begin{aligned}
q^\mu\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{C}_{\alpha\beta}(q,r,p) &= i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p),\\
q^\mu\widetilde{\Gamma}^{\mathbf{np}}_{\mu}(q,r,p) + \widetilde{C}(q,r,p) &= iD^{-1}(r^2) - iD^{-1}(p^2),\\
q^\mu\widetilde{\Gamma}^{\mathbf{np},mnrs}_{\mu\alpha\beta\gamma}(q,r,p,t) + \widetilde{C}^{mnrs}_{\alpha\beta\gamma}(q,r,p,t) &= f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma}(r,p,q+t) + f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha}(p,t,q+r)\\
&\quad + f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta}(t,r,q+p).
\end{aligned} \tag{42}$$

Additionally, we derive the equivalent of Eqs. (5), (6) and (7) by carrying out a Taylor expansion of both sides of Eqs. (42) around $q = 0$. Clearly, we see that the zeroth order terms vanish in all cases,

$$\widetilde{C}_{\alpha\beta}(0,r,-r) = 0; \qquad \widetilde{C}(0,-r,r) = 0; \qquad \widetilde{C}^{mnrs}_{\alpha\beta\gamma}(0,r,p,-p-r) = 0, \tag{43}$$

while the terms linear in q^μ yield, for the BQ^2 vertex

$$\begin{aligned}
\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) &= -i\frac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu} - \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,-r-q)\right\}_{q=0};\\
\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,p,-p) &= i\frac{\partial\Delta^{-1}_{\alpha\beta}(p)}{\partial p^\mu} - \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,-p-q,p)\right\}_{q=0}.
\end{aligned} \tag{44}$$

Similarly, for the $B\bar{c}c$ vertex, we have

$$\begin{aligned}
\widetilde{\Gamma}^{\mathbf{np}}_{\mu}(0,r,-r) &= -i\frac{\partial D^{-1}(r^2)}{\partial r^\mu} - \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}(q,r,-r-q)\right\}_{q=0},\\
\widetilde{\Gamma}^{\mathbf{np}}_{\mu}(0,p,-p) &= i\frac{\partial D^{-1}(p^2)}{\partial p^\mu} - \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}(q,-p-q,p)\right\}_{q=0},
\end{aligned} \tag{45}$$

and for the BQ^3 vertex,

$$\begin{aligned}
\widetilde{\Gamma}^{\mathbf{np},mnrs}_{\mu\alpha\beta\gamma}(0,r,p,-r-p) &= \left(f^{mne}f^{esr}\frac{\partial}{\partial r^\mu} + f^{mre}f^{ens}\frac{\partial}{\partial p^\mu}\right)\Gamma_{\alpha\beta\gamma}(r,p,-r-p)\\
&\quad - \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}^{mnrs}_{\alpha\beta\gamma}(q,r,p,-q-r-p)\right\}_{q=0},\\
\widetilde{\Gamma}^{\mathbf{np},mnrs}_{\mu\alpha\beta\gamma}(0,-r,-p,r+p) &= -\left(f^{mne}f^{esr}\frac{\partial}{\partial r^\mu} + f^{mre}f^{ens}\frac{\partial}{\partial p^\mu}\right)\Gamma_{\alpha\beta\gamma}(-r,-p,r+p)\\
&\quad + \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}^{mnrs}_{\alpha\beta\gamma}(-q,-r,-p,q+r+p)\right\}_{q=0}.
\end{aligned} \tag{46}$$

Now, we can repeat the calculations of Sect. 5, using the new WIs of Eqs. (44), (45) and (46), which have additional terms containing the derivatives of the $\widetilde{C}$ functions. Consequently, the terms already present in Eqs. (5), (6) and (7) will trigger again the seagull identity, vanishing as before, whereas the contributions originated from the $\widetilde{C}$ functions will survive. As a result, one obtains

$$d\widetilde{\Pi}^{(1)}(0) = -\frac{1}{2}g^2 C_A \int_k \Gamma^{(0)}_{\mu\alpha\beta}(0,k,-k)\Delta^{\alpha\rho}(k)\Delta^{\beta\sigma}(k)\left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\sigma\rho}(q,-k-q,k)\right\}_{q=0}, \tag{47}$$

$$d\widetilde{\Pi}^{(2)}(0) = -g^2 C_A \int_k \Gamma^{(0)}_{\mu}(0,k,-k)D^2(k^2)\left\{\frac{\partial}{\partial q^\mu}\widetilde{C}(q,-k-q,k)\right\}_{q=0}, \tag{48}$$

and

$$\begin{aligned} d\widetilde{\Pi}^{(3)}(0)\delta^{ab} = {} & \frac{1}{6}g^4\Gamma^{(0)amnr}_{\mu\alpha\beta\gamma}\int_k\int_\ell \Delta^{\alpha\rho}(k+\ell)\Delta^{\beta\sigma}(\ell)\Delta^{\gamma\tau}(k)\left\{\frac{\partial}{\partial q^\mu}\widetilde{C}^{brnm}_{\tau\sigma\rho}(q,-k-q,-\ell,k+\ell)\right\}_{q=0} \\ & + i\mathcal{N}_{\mu\alpha\beta\gamma}\delta^{ab}\int_k Y^{\alpha\beta}_{\delta}(k)\Delta^{\gamma\tau}(k)\Delta^{\lambda\delta}(k)\left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\tau\lambda}(q,-k-q,k)\right\}_{q=0}. \end{aligned} \tag{49}$$

Therefore, the presence of $1/q^2$ poles allows for a non-vanishing value of the gluon self-energy at $q=0$, so that we can have $\widetilde{\Delta}^{-1}(0) \neq 0$.

Let us end this section by observing that the extended WIs of Eqs. (44), (45) and (46) yield interesting results for the vertices form factors. To begin with, notice that the condition of longitudinal coupling implies that only some of the vertex form factors may contain massless poles; specifically one has

$$\begin{aligned} &\widetilde{\mathcal{A}}_1 = \widetilde{\mathcal{A}}^{\mathbf{np}}_1 + \widetilde{\mathcal{A}}^{\mathbf{p}}_1; && \widetilde{\mathcal{A}}_2 = \widetilde{\mathcal{A}}^{\mathbf{np}}_2, \\ &\widetilde{A}_i = \widetilde{A}^{\mathbf{np}}_i + \widetilde{A}^{\mathbf{p}}_i, \quad i = 1,...,5; && \widetilde{A}_i = \widetilde{A}^{\mathbf{np}}_i, \quad i = 6,...,14. \end{aligned} \tag{50}$$

Then, using the relation

$$\left\{\frac{\partial}{\partial q^\mu}\widetilde{C}(q,r,p)\right\}_{q=0} = 2r_\mu \widetilde{C}'(r^2), \tag{51}$$

we see that Eq. (9) gets modified to

$$\widetilde{\mathcal{A}}^{\mathbf{np}}_2(r^2) = -2\left[i\frac{\partial}{\partial r^2}D^{-1}(r^2) + \widetilde{C}'(r^2)\right], \tag{52}$$

Similarly, assuming that out of the five possible tensorial structures of $\widetilde{C}_{\sigma\rho}$ only the one proportional to $g_{\alpha\beta}$ develops a pole in q^2, one has

$$\left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,p)\right\}_{q=0} = 2r_\mu g_{\alpha\beta}\widetilde{C}'_1(r^2). \tag{53}$$

so that the relation (14) for the A_6 form factor gets modified into

$$\widetilde{A}^{\mathbf{np}}_6(r^2) = 2\left[\frac{\partial}{\partial r^2}\Delta^{-1}(r^2) - \widetilde{C}'_1(r^2)\right]. \tag{54}$$

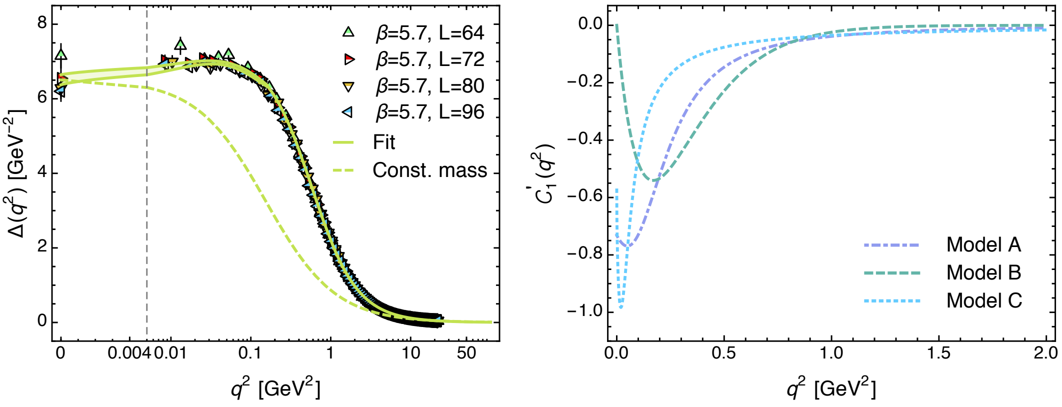

Figure 4. Left panel: The quenched SU(3) lattice data for the gluon propagator, $\Delta(q^2)$, renormalized at $\mu = 4.3$ GeV (symbols) and its corresponding fit (continuous line). Right panel: The functions $\widetilde{C}'_1(y)$ corresponding to: Model A with $a = -9.21\,\mathrm{GeV}^{-4}$, $b = 0.94\,\mathrm{GeV}^{-2}$, $c = -0.47$, Model B with $a = -25.13\,\mathrm{GeV}^{-2}$, $b = 0.17\,\mathrm{GeV}^{2}$, and Model C with $a = -12.79\,\mathrm{GeV}^{-2}$, $b = 3.59\,\mathrm{GeV}^{-1}$, $c = -0.60$.

7 Numerical analysis

In order to obtain some quantitative understanding of the equations derived, we finally carry out a numerical analysis under some simplifying hypotheses. Specifically, we assume that the contributions coming from the poles of $\widetilde{\Gamma}^{\mathbf{p}}_{\mu}(q,r,p)$ and $\widetilde{\Gamma}^{\mathbf{p},mnrs}_{\mu\alpha\beta\gamma}(q,r,p,t)$ are suppressed with respect to that of $\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p)$, so that they can be neglected. Additionally, we consider only the $\widetilde{C}_1$ component of $\widetilde{C}_{\alpha\beta}$, that is only the term proportional to the metric tensor.

In this way, Eq. (47) results in

$$d\widetilde{\Pi}^{(1)}(0) = 2g^2 C_A \int_k (d-1)k^2\Delta^2(k^2)\widetilde{C}'_1(k^2), \tag{55}$$

which yields (in Euclidean space and spherical coordinates with $k^2 = y$)

$$\Delta^{-1}(0) = -\frac{3C_A\alpha_s}{8\pi}F(0)\int_0^\infty \mathrm{d}y\, y^2\Delta^2(y)\widetilde{C}'_1(y). \tag{56}$$

The precise form of the function $\widetilde{C}'_1(y)$ is undetermined at this level (we only know that it must be negative as $\Delta^{-1}(0)$ and $F(0)$ are positive). Thus, in order to proceed with our analysis we will consider three possible models describing its functional form, inspired by the solutions obtained from the Bethe-Salpeter equations governing the formation of massless poles [31, 32]:

$$\widetilde{C}'_1(y) = \begin{cases} 1/(ay^2+by+c), & \text{Model A} \\ ay\exp(-y/b), & \text{Model B} \\ 1/(ay+b\sqrt{y}+c), & \text{Model C} \end{cases} \tag{57}$$

with a, b and c being suitable parameters which will be adjusted in order to satisfy Eq. (56).

Solutions of Eq. (56) are finally obtained when a realistic model for the gluon propagator is used. To this purpose we use the results for the gluon propagator from SU(3) quenched lattice simulations in the Landau gauge [3]. On the left panel of Fig. 4, we present a fit for the propagator (continuous

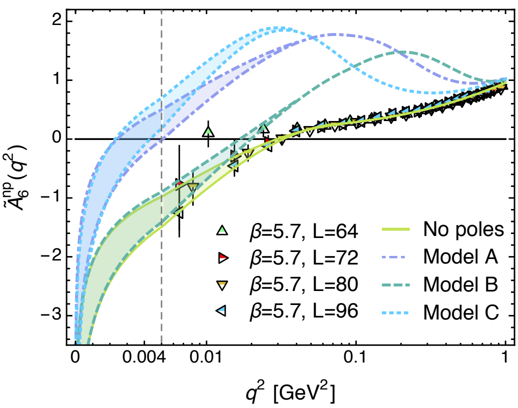

Figure 5. The form factor $\widetilde{A}_6^{\mathbf{np}}$. In the absence of massless poles, this quantity is proportional to the derivative of the inverse propagator (here evaluated directly from the lattice data). When massless poles are generated, the presence of the function $\widetilde{C}_1'$ modifies this dependence according to Eq. (54), and a positive maximum appears in the region $q^2 < 1$ GeV^2, whose height and exact location depend on the details of the model.

line), along with the lattice data renormalized at 4.3 GeV. On the right panel of the same figure we plot instead the typical solutions obtained corresponding to each of the model considered.

We conclude by presenting a short qualitative description of how the study of the form factor $\widetilde{A}_6^{\mathbf{np}}(q^2)$ can confirm or invalidate the need for longitudinally-coupled massless poles. Recall that, unlike the gluon, the ghost remains massless: its propagator behaves as $1/p^2$ in the IR multiplied by a dressing function that saturates at a finite nonvanishing value. This implies that [33]: (*i*) it must display a maximum in the deep IR (see again the left panel of Fig. 4), and (*ii*) the derivative of its inverse will display a logarithmic divergence and a zero crossing:

$$\frac{\partial}{\partial q^2}\Delta^{-1}(q^2) \underset{q\to 0}{\sim} \log q^2. \tag{58}$$

Therefore, if the finiteness of the gluon propagator was not due to the massless poles, Eq. (14) implies that the form factor $\widetilde{A}_6^{\mathbf{np}} \equiv \widetilde{A}_6$ would display the same IR behavior as that of Eq. (58). However, the presence of massless poles will change the shape of $\widetilde{A}_6$ [see Eq. (54)], forcing it to have a positive maximum, as can be appreciated in Fig. 5.

8 Conclusions

A unified framework for the self-consistent treatment of the IR finiteness of the gluon propagator at the level of the SDEs of the theory has been formulated within the PT-BFM framework. Particular attention has been dedicated to the extensive cancellations induced by the WIs of the theory, leading to the necessity of introducing massless poles in order to achieve the desired effect in a self-consistent way. The analysis has been carried out for a general value of the gauge-fixing parameter, reverting to the Landau gauge only in order to simplify the numerical analysis.

Particularly relevant is the observation that the presence of these poles is bound to affect not only the two-point but also the three-point sector of the theory. Therefore, determining special form factors of, *e.g.*, the (background) three-gluon vertex by means of lattice techniques, has the potential to confirm or discard massless poles as the IR mechanism underlying the dynamical generation of a gluon mass.

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