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Quantum spin correlations in relativistic Møller scattering

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Abstract. We present the relativistic spin correlation function (and the corresponding probabilities) for a pair of polarized electrons originating from the Moller scattering. This particular state is easy to prepare experimentally; therefore, the results are discussed in view of a possible measurement. We also discuss the state after the Moller scattering in terms of entanglement and polarization transfer.

1 Introduction

1.1 The EPR paradox

In 1935 Albert Einstein, Boris Podolsky and Nathan Rosen (EPR) formulated their famous paradox [1] which, according to their reasoning, indicated that quantum mechanical description of physical reality was incomplete. It considered two entangled particles in such a state, that the distance between them was precisely known and equal to L, and their total momentum was equal to 0. Now if the pair was investigated by two arbitrairly distant ovservers, Alice and Bob, each one having access to only one particle, we could imagine that Alice performed the position measurement on her particle. But knowing the distance between the particles, in that very moment she would have gained information about the position of the Bobs particle, too. On the other hand she could have peformed the momentum measurement, simultaneously determining the momentum of Bob's particle. EPR claimed that nothing that happens to Alices particle can have impact on the state of the Bobs one (which can be arbitrarily distant) because no information or interaction can propagate with infinite speed. It would however mean that the state (in this case – both position and momentum) of the Bobs particle has been determined all the time, which is not reflected in the quantum mechanical description of physical reality, so (i) either qunatum mechanical description of physical reality is incomplete and the complete state of the system (results of all possible measurements) is precisely determined at all times or (ii) EPR were wrong and measurements performed on one particle of the entangled pair can alter the state of the other particle instantaneously. Einstein strongly disagreed with the second possibility, which he referred to as "spooky action at the distance".

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1.2 Bell type inequalities and the correlation function

For nearly thirty years the discussion whether spooky action at a distance exists was an academic debate, until in 1964 John Stuart Bell formulated his famous theorem [2] stating that every theory including the principle of local realism (i.e. not containing spooky action at the distance) must fulfill a certain class of inequalities, which are now called the Bell-type inequalities. In other words, outcomes of measurements performed on two space-like separated systems cannot be correlated arbitrarily strong. The strength of this correlation can be "measured" by means of so called *correlation function*. In a special case of a dychotomic observable, whose eigenvalues can take values ± 1, the correlation function can be defined as the difference between the probability that Alice and Bob get the same outcome and the probability that their outcomes are opposite

$$C(\vec{a}, \vec{b}) = P_{++} + P_{--} - P_{+-} - P_{-+}, \tag{1}$$

where $P_{\pm\pm}$ is the joint probability that both Alice and Bob measure ± 1.

Bell noticed that if theory includes three assumptions of (i) locality (nothing that happens with one particle can influence the other one instantanously), (ii) realism (outcomes of unperformed measurements do exist despite we don't know them), (iii) free will (Alice and Bob can freely choose what kind of experiment they want to perform), the following inequality must be fullfilled[1]

$$|C(\vec{a}, \vec{b}) + C(\vec{c}, \vec{b} + C(\vec{c}, \vec{d}) - C(\vec{a}, \vec{d}))| \leq 2. \tag{2}$$

It turns out that quantum mechanics can violate the Bell-type inequalities, which means that it is a non–local theory so it is either incorrect (and not only incomplete, as stated by EPR) or the principle of local realism is wrong. In particular the CHSH inequality is violated for a singlet state of two fermions, for which the correlation function has a simple form $C(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b}$, and which has bin described by David Bohm in 1951 in his spin version of the EPR paradox [4]. It can be seen in Fig. 1

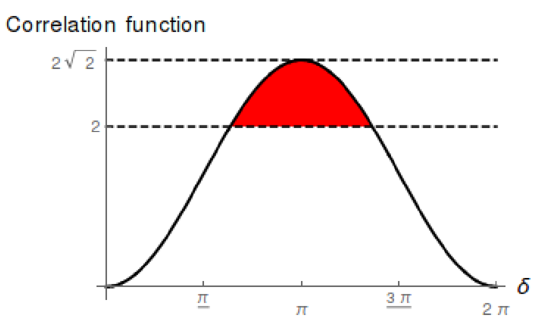


Figure 1: The left-hand-side of the inequality (2) as a function of the angle δ between the X axis and $\vec{d}$, where $\alpha = \pi/4$, $\beta = \pi/2$, $\gamma = 3\pi/4$, for a singlet state of two fermions. It reaches its maximum equal to $2\sqrt{2}$ for $\delta = \pi$.

that when the directions $\vec{a}$, $\vec{b}$. $\vec{c}$ and $\vec{d}$ are separated with $\pi/4$ angle, respectively, the value of the left-hand-side of the inequality (2) reaches value $2\sqrt{2} > 2$.

[1] It is not the original Bell inequality, which is valid only for maximally entangled state, but more general CHSH inequality [3].

This outcome raised a question, whether real quantum systems violate the Bell-type inequalities or not. The first experiment to verify this was performed with use of fully entangled photons by Alain Aspect [5, 6] in 1982 and its outcome was that the Bell-type inequalities are violated by 5 standard deviations. It was a beautiful confirmation of correctness and completeness of quantum mechanics – its non–locality is not just a feature of the formalism, but a fundamental property of Nature.

2 Relativistic correlation functions

If one wanted to perform a Bell-type experiment using massive fermions (as in Bohm version of the paradox) instead of photons, they would have to take into account that they should move apart with great velocities to assure their space–like separation during the measurement and calculate the theoretical correlation function in relativistic regime.

It has been shown that the relativistic correlation function exhibits a significantly different behavior than in the non—relativistic case. It depends on particles momenta [7–9], moreover in some energy ranges this dependence may be non—monotonic (in systems containing at least one massive particle) [8–11]. It can be seen in Fig 2.

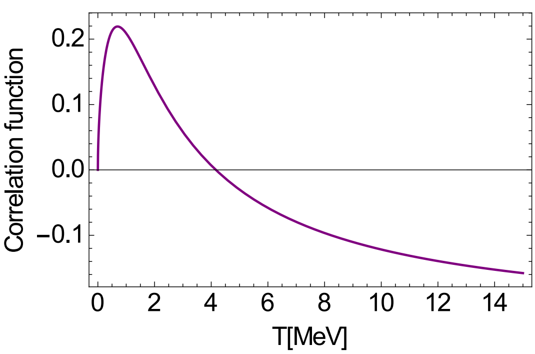


Figure 2: The relativistic correlation function for a pure singlet state as a function of kinetic energy of one particle.The kinetic energy of the second particle equals 5MeV, $\vec{a} = (0, 0, 1)$, $\vec{b} = (1, 0, 0)$, $\frac{\vec{p}}{|\vec{p}|} = \frac{1}{\sqrt{2}}(0, 1, 1)$ and $\frac{\vec{k}}{|\vec{k}|} = \left(-\frac{1}{4}, \frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)$. The non-monotonic behaviour of the function can be clearly seen.

None of the experiments performed so far could observe relativistic effects in correlation function. Up to our best knowledge, only three correlation experiments using massive fermions have been performed so far [12–14]. They were all focused on measuring the violation of the Bell-type inequalities for maximally entangled protons in a singlet state. In this case the correlation function reads:

$$C^{k,p}(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} + \frac{(\vec{k} \times \vec{p})}{m^2 + kp} \cdot \left((\vec{a} \times \vec{b}) + \frac{(\vec{a} \cdot \vec{k})(\vec{b} \times \vec{p}) - (\vec{b} \cdot \vec{p})(\vec{a} \times \vec{k})}{(k^0 + m)(p^0 + m)} \right), \quad (3)$$

where k and p are the four-momenta of Alices and Bobs particle, respectively. To measure the relativistic correction to the non-relativistic correlation function $-\vec{a} \cdot \vec{b}$, the kinetic energies of the particles used should be at least of order of their rest mass. In all the aforementioned experiments the energies were to low to distinguish between the relativistic and the non-relativistic case.

3 2POL experiment

In this situation the groups from University of Lodz, University of Warsaw and Technische Universität Darmstadt decided to perform a pioneering experiment that could allow for distinguishing between the relativistic and non-relativistic correlation function for the first time.

3.1 Initial state preparation

The first problem was the choice of the initial state in which the correlation function is to be measured. It would be optimal to use a pure singlet state, as in the aforementioned experiments with protons. It can be achieved (for protons) by using nuclear reactions, but in such a case the energy of the pair cannot be high enough. This is why we decided to use a state originating from Møller scattering of a polarized electron beam on a target. It is neither maximally entangled nor pure, but it is well defined and it has been shown [15] that the difference between relativistic and non-relativistic correlation function is measurable in this particular case.

Although the state before the scattering is fully separable, after the scattering the particles become entangled. If we take negativity, N, as an entanglement measure

$$N(\hat{\rho}^{in}) = \Sigma_i \frac{|\lambda_i| - \lambda_i}{2}, \tag{4}$$

where λ_i are the eigenvalues of the density matrix $\hat{\rho}^{in}$, and analyze its dependence on the scattering angle and the beam kinetic energy (in the simplest case of an unpolarized target), we can see that for a given beam energy negativity reaches its maximum for the symmetric scattering angle. On the other hand if we focus on the symmetric scattering only, the degree of the entanglement increases with the decrease of the beam kinetic energy (see Fig. 3). Although due to their entanglement, Møller electrons do not have a well determined polarization, it is possible to calculate their mean polarization vectors using the spin density matrix of the final state. It can be shown that although before the scattering one of the electrons was unpolarized, after the Møller scattering both electrons have nonzero mean polarization vectors (see Fig 4). In case of the symmetric scattering (due to their indistinguishability) they are equally polarized. Moreover, in this specific case the length of the mutual polarization vector takes its smallest value (see Fig. 5), which corresponds to maximum of negativity. In section 4 it will be shown that this configuration maximizes also the correlation function.

3.2 Spin-projection measurement

To be able to measure and compare the relativistic correlation function to the theoretical predictions, one must have (i) a proper relativistic spin-projection observable, $\hat{\vec{S}}$, and (ii) a corresponding method of measuring spin projection on a given direction. Problem of defining a proper relativistic spin operator no satisfactionary solution so far and has been widely discussed elsewhere [16, 17]. It can however been shown that only one of the operators proposed in the literature (i) reduces to non-relativistic form in rest frame: $\hat{\vec{S}} = \frac{\hat{\vec{W}}}{m}$, where $\hat{W}$ is the Pauli-Lubanski four-vector, (ii) is a three-vector: $\left[\hat{J}_i, \hat{S}_j\right] = i\epsilon_{ijk}\hat{S}_k$, (iii) fulfills standard commutation relations: $\left[\hat{S}_i, \hat{S}_j\right] = i\epsilon_{ijk}\hat{S}_k$. and (iv) is a linear combination of Pauli-Lubanski four-vector components. It is the Newton-Wigner spin operator of form:

$$\hat{\vec{S}} = \frac{1}{m}\left(\hat{\vec{W}} - \hat{W}^0 \frac{\hat{\vec{P}}}{\hat{P}^0 + m}\right), \tag{5}$$

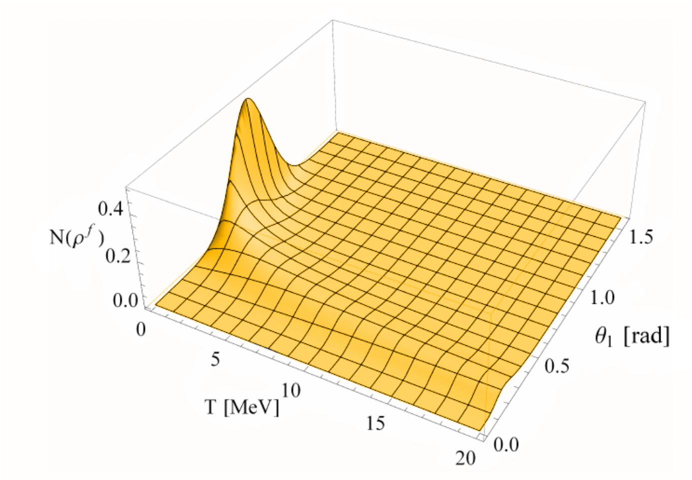
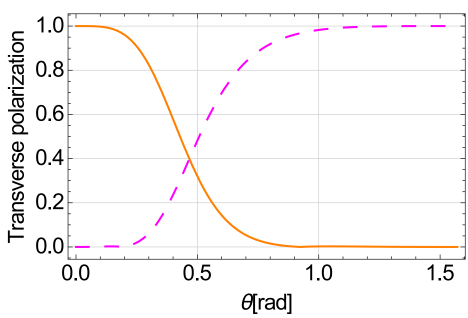
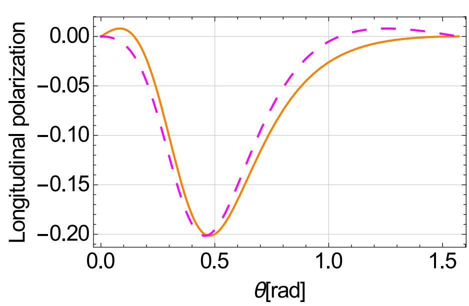


Figure 3: Negativity of a state of two electrons originating from Møller scattering of a polarized beam on an unpolarized target as the function of the scattering angle and the beam kinetic energy. For a given energy, the maximum of entanglement corresponds to the symmetric scattering and for symmetric scattering the less energetic beam, the more entangled the state after the scattering is. Note that before the scattering the beam and the target electrons were in a fully separable state.



(a) transverse polarization in Møller scattering



(b) longitudinal polarization in Møller scattering

Figure 4: The dependence of transverse and longitudinal polarization on the scattering angle in the case of Møller scattering of a 100% transversly polarized beam scattering off an unpolarized target. The scattering angle θ is the angle between the initial beam direction and the direction of the electron with subscript A.

where $\hat{P}$ is the four-momentum operator.

On the other hand a spin-projection measurement procedure had to be chosen. Basically there are two main methods allowing for determining polarization of an electron beam – Møller and Mott [18] polarimetry. For the case of electrons at energies up to about 10 MeV, Mott polarimetry has been established as a standard tool.

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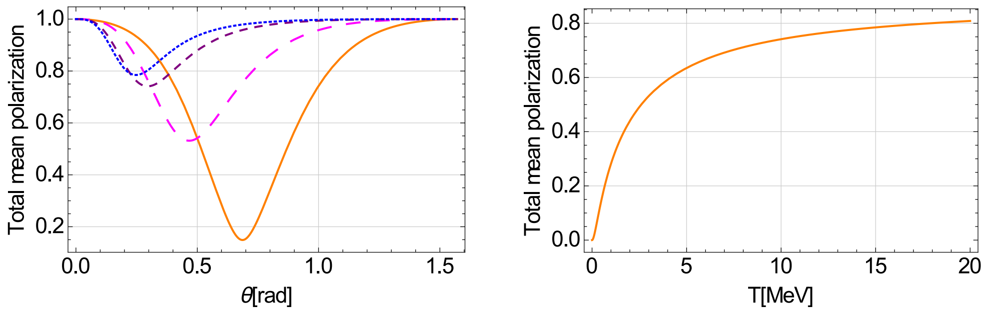

(a) Mutual polarization vector length as a function of scattering angle for beam kinetic energies equal to: 0.5MeV (solid line), 3MeV (large dashing), 10MeV (medium dashing), 15MeV (tiny dashing)

(b) mutual polarization vector lenght as a function of the beam kinetic energy, symmetric scattering

Figure 5: Mutual polarization vector length.

Mott scattering is an electron - nucleus scattering and Mott polarimetry is realized by scattering an electron beam off targets made of heavy elements. For a given scattering angle, Mott cross section depends on the beam polarization due to the spin-orbit coupling. It results in azimuthal asymmetry of scattered electrons. If one places two detectors (conventionally called left and right) symmetrically about the beam axis, the polarization of the beam can be determined. In the correlation experiment a double Mott polarimeter shall be used. The four probabilities P_{++}, P_{+-}, P_{-+} and P_{--}, constituing the correlation function, can be determined by measuring four other probabilities: P_{LL}, P_{LR}, P_{RL} and P_{RR}, where P_{IJ} are the probabilities that the first electron will be scattered towords I-th and the second one towards J-th detector. The latter probabilities can be found by counting corresponding coincidences (LL, LR etc.).

There are two main problems that we need to face in the context of Mott polarimetry. The first one is that the method is most sensitive in case of backscattering, which makes coincidences at a few MeV energies extremely rare. The other one is that it is sensitive only to spin projection on a direction perpendicular to the Mott scattering plane, and in that case the relativistic correction to the correlation function is rather small, as will be shown in the next section.

4 Correlation function in the case of scattering of a polarized beam on a target

In order to design an optimal experiment, the relativistic correlation function for a pair of electrons originatin from scattering of a polarized electron beam off both unpolarized [15] and polarized target has been calculated. In the case of an unpolarized target, the absolute value of the correlation function is small, yet measurable, and reaches its maximum in the case of the symmetric scattering (compare section 3.1). It does not depend on the beam polarization, on the contrary to the joint probabilities P_{++}, P_{+-}, P_{-+} and P_{--} that constitute it. The dependence of the correlation function and the probabilities on the beam kinetic energy for symmetric Møller scattering and vectors $\vec{a}$ and $\vec{b}$ in the Møller scattering plane can be seen in Figs 6 and 7. This configuration has been chosen for experimental reasons for the planned measurement and all the plots in the following sections will be presented in this configuration, unless stated otherwise.

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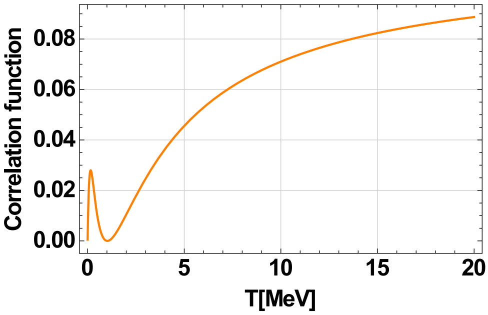


Figure 6: The correlation function vs kinetic energy of the beam, polarized electron beam, unpolarized target. Symmetric Møller scattering, $\vec{a}$ and $\vec{b}$ in the Møller scattering plane. The correlation function does not depend on the polarization.

The dependence of the probabilities on the beam polarization is the reason why we chose to use polarized electron beam in the experiment, despite the correlation function is not sensitive to it.

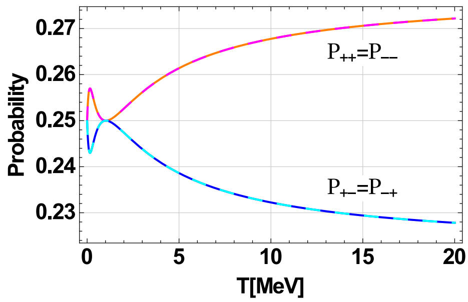
4.1 Polarized target

Although it is possible to observe the relativistic effects by measuring joint probabilities for a few MeV polarized electron beam, one could ask whether polarizing the target would not cause increase of the absolute value of the correlation function, letting perhaps for measuring also the violation of Bell-type inequalities. We analysed the dependence of the correlation function on the beam kinetic energy in the case of targets polarized in (i) 8% and (ii) 85%. The first degree of polarization could be arranged experimentally, the other one is rather unrealistic and we investigated it only for scientific reasons. The results are the following (i) the polarization of the target electron influences the behaviour of the correlation function only if spins of both scattering electrons are paralell to each other, (ii) in case of realisticly polarized target, the absolute value of the correlation function does not change much, so violating theBell-type inequalities is also impossible in this case. It can be seen in Fig 8. The same conclusions are valid for the probabilities, see Fig 9 If the target could be polarized to the same amount as the beam itself, the increase of the absolute value of the correlation function would be much more profaund.

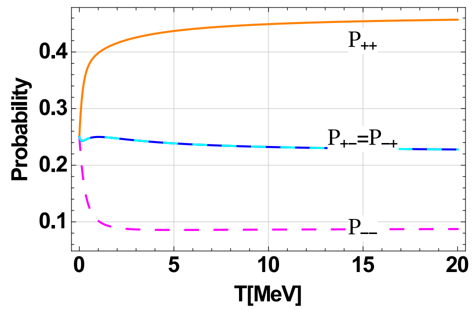
5 Conclusions

Our aim is to design and perform a first experiment allowing to distinguish between relativistic and non-relativistic quantum correlations. It is not going to be a Bell-test experiment, rather a verification of the existing theory. We decided that the correlation function will be measured for a pair of electrons originating from Møller scattering of a polarized electron beam off a stationary target, which is in a well-determined and partilally entangled state. For the spin-projection measurement Mott polarimetry will be used.

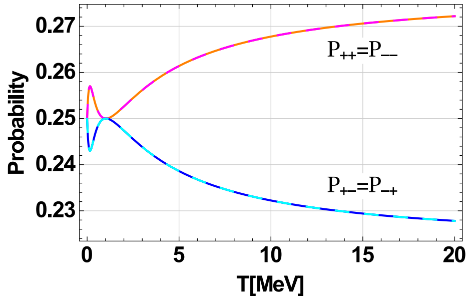
We have shown that in the simple case of an unpolarized target the correlation function is small, yet measurable and although it does not depend on the beam polarization, the probabilities that constitute it, do. For that reason the experiment will be performed at Technische Universität Darmstadt, where up to 85% polarized beam can be provided. We have also shown that polarizing the target would not



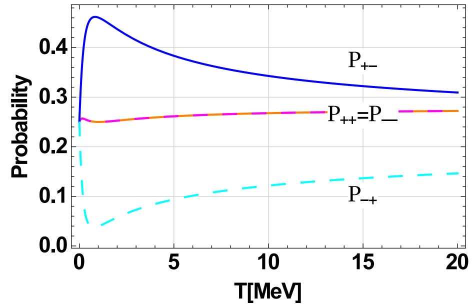
(a) Unpolarized beam



(b) Beam transversly polarized in 85%, polarization vector lying in the Møller scattering plane



(c) Beam transversly polarized in 85%, polarization vector perpendicular to the Møller scattering plane



(d) Beam longitudinally polarized in 85%

Figure 7: Probabilities as functions of kinetic energy for different beam polarizations. Note that the case of beam polarized in direction perpendicular to the Møller scattering plane is equivalent to the case of an unpolarized beam.

bring much improvement to the measurement, this is why the beryllium target we shall use as the Møller target, will be unpolarized.

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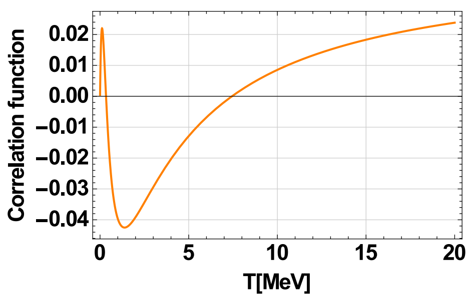
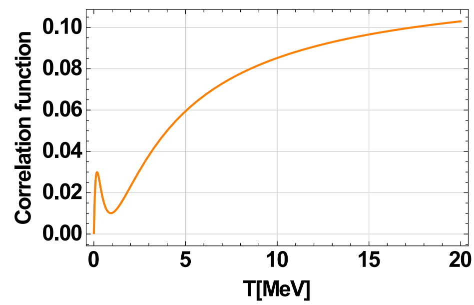
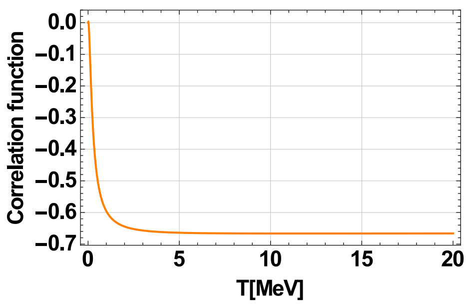


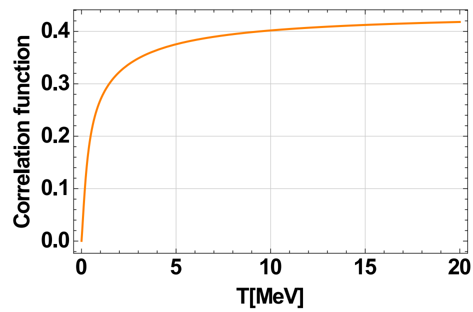
(a) Beam and target electrons transversly polarized in 85% and 8%, respectively, polarization vector lying in the Møller scattering plane.



(b) Beam and target electrons longitudinally polarized in 85% and 8%, respectively.



(c) Beam and target electrons transversly polarized in 85%, polarization vector lying in the Møller scattering plane.



(d) Beam and target electrons longitudinally polarized in 85%

Figure 8: Correlation function in terms of kinetic energy for different beam and target polarizations.

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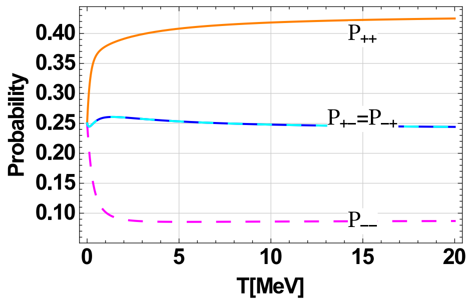
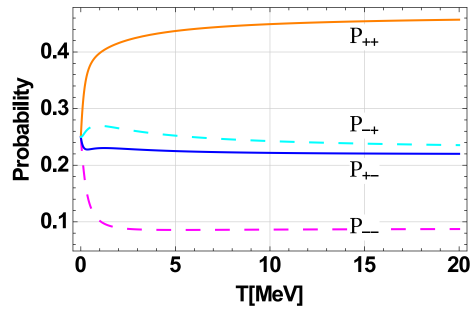
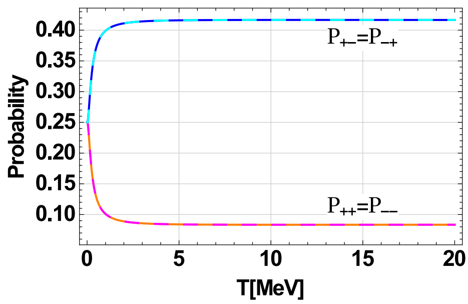


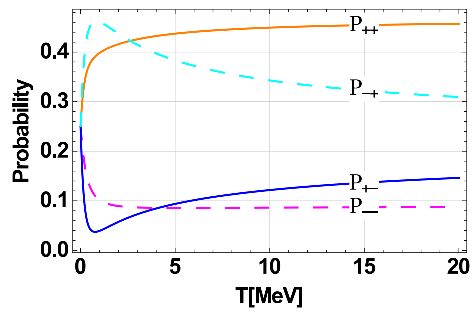
(a) Target electrons transversly polarized in 8%, polarization vector lying in the Møller scattering plane



(b) Target electrons longitudinally polarized in 8%



(c) Target electrons transversly polarized in 85%, polarization vector lying in the Møller scattering plane



(d) Target electrons longitudinally polarized in 85%

Figure 9: Probabilities as functions of the beam kinetic energy for beam transversly polarized in 85%, polarization vector lying in the Møller scattering plane and for different target polarizations

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